Modelling MAC-Layer Communications in Wireless Systems (Extended Abstract)

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COORDINATION 2013 - June 4th, Florence

Overview

- Calculus of Collision-prone Communicating Processes (CCCP)
- Contextual Equivalence
- Coinductive Characterisation

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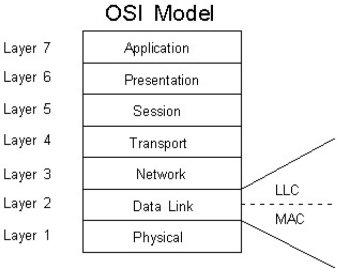
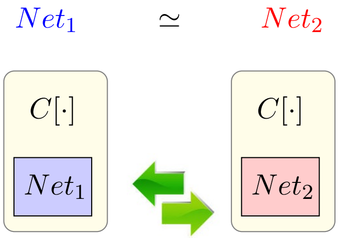
- Calculus of Collision-prone Communicating Processes (CCCP)
  Modelling Networks at the MAC-sublayer of the ISO/OSI reference model
- Contextual Equivalence
- Coinductive Characterisation



Issues: Collisions, Failures of Synchronisations

Overview

- Calculus of Collision-prone Communicating Processes (CCCP)
- Contextual Equivalence
  Interchange two equivalent networks in a larger one without affecting the overall behaviour
- Coinductive Characterisation



Overview

- Calculus of Collision-prone Communicating Processes (CCCP)
- Contextual Equivalence
- Coinductive Characterisation
  Determining if two networks are equivalent without having to quantify over contexts

Bisimulation ($\approx$) based proof principle

- Soundness:
  $Net_1 \approx Net_2$ implies $Net_1 \simeq Net_2$
- Completeness *:
  $Net_1 \simeq Net_2$ implies $Net_1 \approx Net_2$

* Holds for all networks satisfying a natural requirement

Features of CCCP

- *Broadcast Communication*
  Transmitted messages can be
  received by multiple stations
- *Discrete Time*
  Activities happen within time slots
  Transmission of messages can
  require several time slots
- *Collision-prone Communication*
  Error in Reception caused by
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  or by missed synchronisations

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P_1	$c!v$			
P_2		$c!w$		
P_3	$c?\cdot$			

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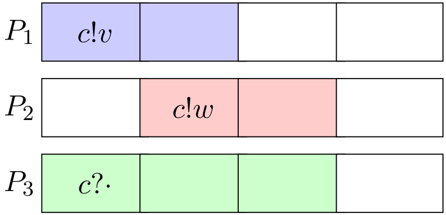

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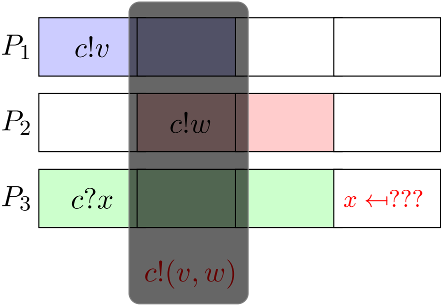

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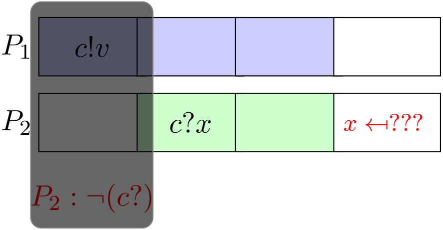

CCCP Formally

- Channel environment: keeps track of transmission related information
  $\Gamma \vdash c : (time, val)$
- System term: contains the code executed by stations
  $W = P_1 \mid \cdots \mid P_n$
- Evolution of a network:
  $\Gamma_1 \triangleright W_1 \rightharpoonup \Gamma_2 \triangleright W_2$

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Constructs:

- Output $c!\langle v \rangle.P$
- Time-out Input $\lfloor c?(x).P \rfloor Q$
- Active Receiver $[c \leftarrow x]P$
- Exposure Check $[\mathsf{exp}(c)]P, Q$
- Standard Constructs

CCCP Formally

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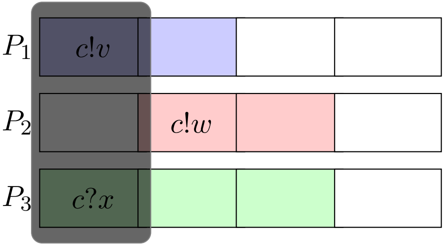
- Evolution of a network:
  $\Gamma_1 \triangleright W_1 \rightarrowtriangle \Gamma_2 \triangleright W_2$

- $\rightarrowtriangle_i$: Instantaneous reduction (within a time slot)
- $\rightarrowtriangle_\sigma$: Passage of time

Example: Collisions



$$
\begin{array}{lll}
\Gamma_0 \triangleright c!\langle v\rangle.\mathsf{nil} \mid \sigma.c!\langle w\rangle.\mathsf{nil} \mid \lfloor c?(x).P \rfloor \mathsf{nil} & \rightarrow_i & \Gamma_0 \vdash c : (0, -) \\
\Gamma_1 \triangleright \sigma^2.\mathsf{nil} \mid \sigma.c!\langle w\rangle.\mathsf{nil} \mid [c \leftarrow x].P & \rightarrow_\sigma & \Gamma_1 \vdash c : (2, v) \\
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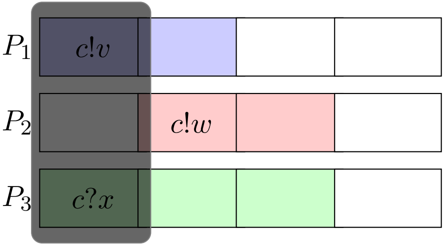

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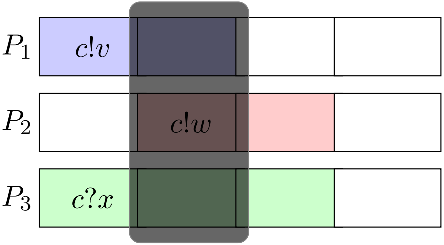

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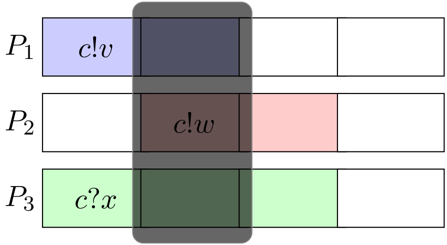

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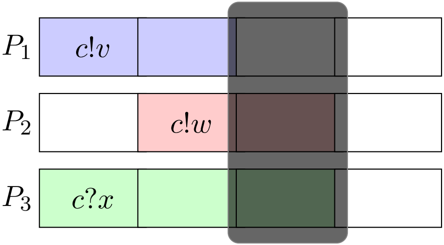

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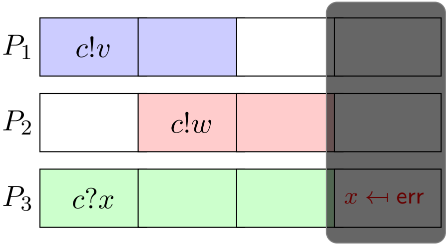

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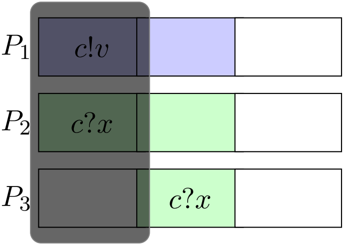
Example: Failure of Synchronisation



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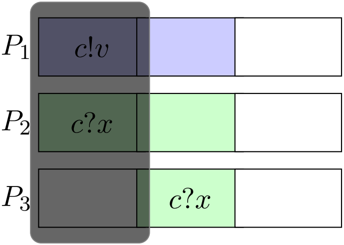

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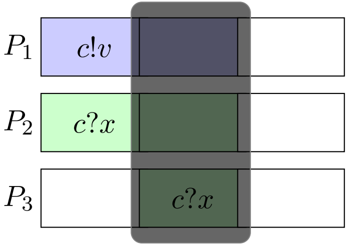

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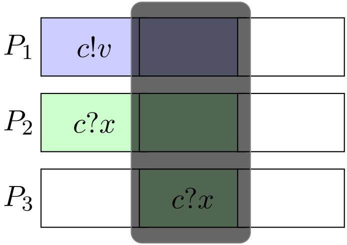

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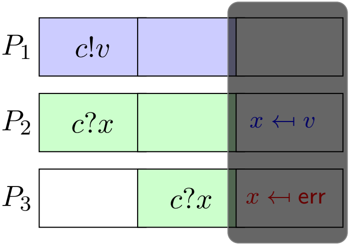

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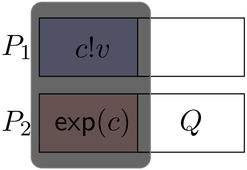
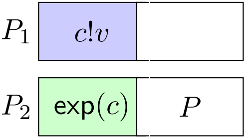
Exposure Check

$$\Gamma_0 \vdash c : (0, -) \quad \Gamma_1 \vdash c : (1, v)$$





- $\Gamma_0 \triangleright c!\langle v\rangle.\mathsf{nil} \mid [\mathsf{exp}(c)]P, Q \twoheadrightarrow_i$
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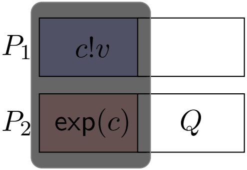

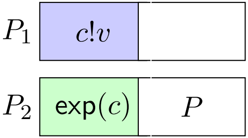

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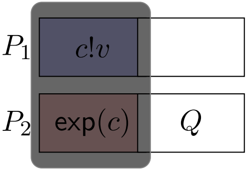

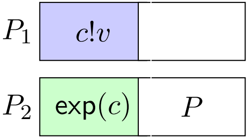

$$\begin{array}{ll}
\Gamma_0 \triangleright c!\langle v\rangle.\text{nil} \mid [\text{exp}(c)]P, Q & \xrightarrow{}_i \\
\Gamma_0 \triangleright c!\langle v\rangle.\text{nil} \mid \sigma.Q & \xrightarrow{}_i \\
\Gamma_1 \triangleright \sigma.\text{nil} \mid \sigma.Q & \xrightarrow{}_\sigma \\
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\end{array}$$

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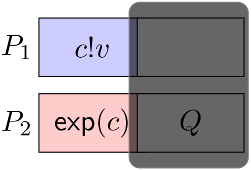

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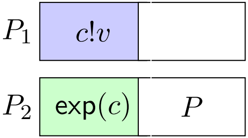

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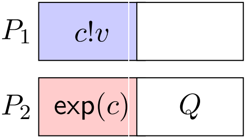

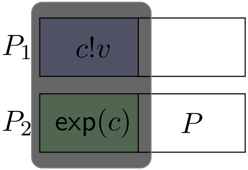

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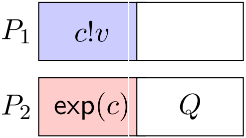

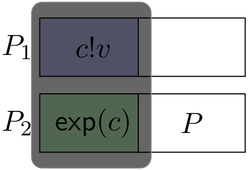

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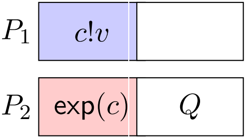

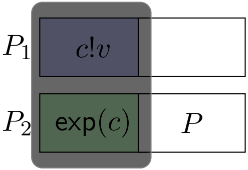

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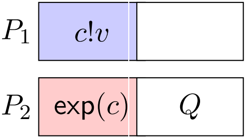

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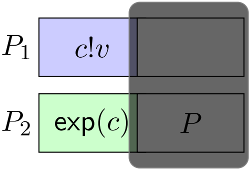

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Basic Observables

Reduction Barbed Congruence

$\Gamma_1 \triangleright W_1 \simeq \Gamma_2 \triangleright W_2$ if they have the same observables under all possible evolutions in all possible contexts

- Standard notion of observable: the ability to output on a channel c
- Observables in CCCP: a channel c being exposed to a transmission

Observables

$\Gamma \triangleright W \downarrow_c$ if $\Gamma \vdash c : (n, v)$, $n > 0$

$\Gamma \triangleright W \Downarrow_c$ if $\Gamma \triangleright W \rightharpoonup^* \Gamma' \triangleright W' \downarrow_c$

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$$\Gamma_1 \triangleright W_1 \simeq \Gamma_2 \triangleright W_2$$
$$\Gamma_1 \triangleright W_1 \rightharpoonup^* \Gamma_1' \triangleright W_1' \text{ implies}$$
$$\Gamma_2 \triangleright W_2 \rightharpoonup^* \Gamma_2' \triangleright W_2' \text{ such that}$$
$$\Gamma_1' \triangleright W_1' \simeq \Gamma_2' \triangleright W_2'$$

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$$\Gamma_1 \triangleright W_1 \simeq \Gamma_2 \triangleright W_2$$
$$\Gamma_1 \triangleright W_1 \mid W \simeq \Gamma_2 \triangleright W_2 \mid W$$

Extensional Semantics

Proving $\Gamma_1 \triangleright W_1 \simeq \Gamma_2 \triangleright W_2$ directly is difficult (contextuality).
Strategy: Give a coinductive characterisation based on the activities of systems observable by some context
Extensional Semantics:

- internal activities: $\overset{\tau}{\mapsto} = \rightharpoonup_i$
- Time: $\overset{\sigma}{\mapsto} = \rightharpoonup_\sigma$
- Input: $\overset{c?v}{\mapsto}$
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Detecting channel c to be idle

$$\frac{\Gamma \vdash c : (0, v)}{\Gamma \triangleright W \overset{\iota(c)}{\mapsto} \Gamma \triangleright W}$$

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- Delivery: $\xmapsto{\gamma(c,v)}$

Detecting the delivery of value v along channel c

$$\frac{\Gamma \triangleright W \rightharpoonup_\sigma \Gamma' \triangleright W' \quad \Gamma \vdash c : (1, v)}{\Gamma \triangleright W \xmapsto{\gamma(c,v)} \Gamma' \triangleright W'}$$

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Weak variant $\stackrel{\alpha}{\Mapsto}$: Abstracting from internal activities

Bisimulations

Bisimulation: largest symmetric relation $\approx$ such that
$\Gamma_1 \triangleright W_1 \approx \Gamma_2 \triangleright W_2$ and $\Gamma_1 \triangleright W_1 \overset{\alpha}{\Mapsto} \Gamma_1' \triangleright W_1'$ implies $\Gamma_2 \triangleright W_2 \overset{\alpha}{\Mapsto} \Gamma_2' \triangleright W_2'$
with $\Gamma_1' \triangleright W_1' \approx \Gamma_2' \triangleright W_2'$

Theorem (Soundness)

$$\Gamma_1 \triangleright W_1 \approx \Gamma_2 \triangleright W_2 \quad \text{implies} \quad \Gamma_1 \triangleright W_1 \simeq \Gamma_2 \triangleright W_2$$

Remark: $\overset{\iota(c)}{\Mapsto}$ and $\overset{\gamma(c,v)}{\Mapsto}$ actions necessary for achieving soundness

Well-Timedness

Is the opposite implication to soundness true?

Idea: provide a distinguishing test W_α for each extensional action $\overset{\alpha}{\Mapsto}$

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Several time slots are required to detect whether the action $\overset{\alpha}{\Mapsto}$ has been performed
But some systems do not allow time to pass!

$$\Gamma \triangleright [c \leftarrow x].P \not\rightarrow_\sigma \text{ if } \Gamma \vdash c : (0, -)$$

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Well-Timedness

$$\Gamma \triangleright W \to^* \Gamma_1 \triangleright W_1 \quad \text{implies} \quad \Gamma_1 \triangleright W_1 \to^* \Gamma_2 \triangleright W_2 \to_\sigma$$

- Natural Requirement
- Can be checked syntactically

Completeness

Is the opposite implication to soundness true?

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For Well-Timed systems:

Theorem (Completeness)

$$\Gamma_1 \triangleright W_1 \simeq \Gamma_2 \triangleright W_2$$

implies

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Conclusions

1. Collision-prone Timed Process Calculus
2. Barbed Congruence
3. Identification of the activities that can be observed
4. Full abstraction (for systems satisfying a natural requirement)
   result achieved for the first time

Thank you


obrigado
Dank U
Merci
mahalo
Köszi
спасибо
Grazie
Thank you
mauruuru
Takk
Gracias
Dziękuję
Děkuju
danke
Kiitos