Parameterised Linearisability

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Joint work with Alexey Gotsman and Hongseok Yang

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A Simple Example

- Converting a sequential data structure into a concurrent one

Trivial Solution:

```
LOCK lock;
do_push(z):
    lock.acquire();
    int retval = push(z);
    lock.release();
    return retval;
```

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LOCK lock;
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- Works for any implementation of push
- but it's inefficient (we can do much better…)

Flat Combiners (Hendler et al, 2010)

Idea: let a single thread handle all requests

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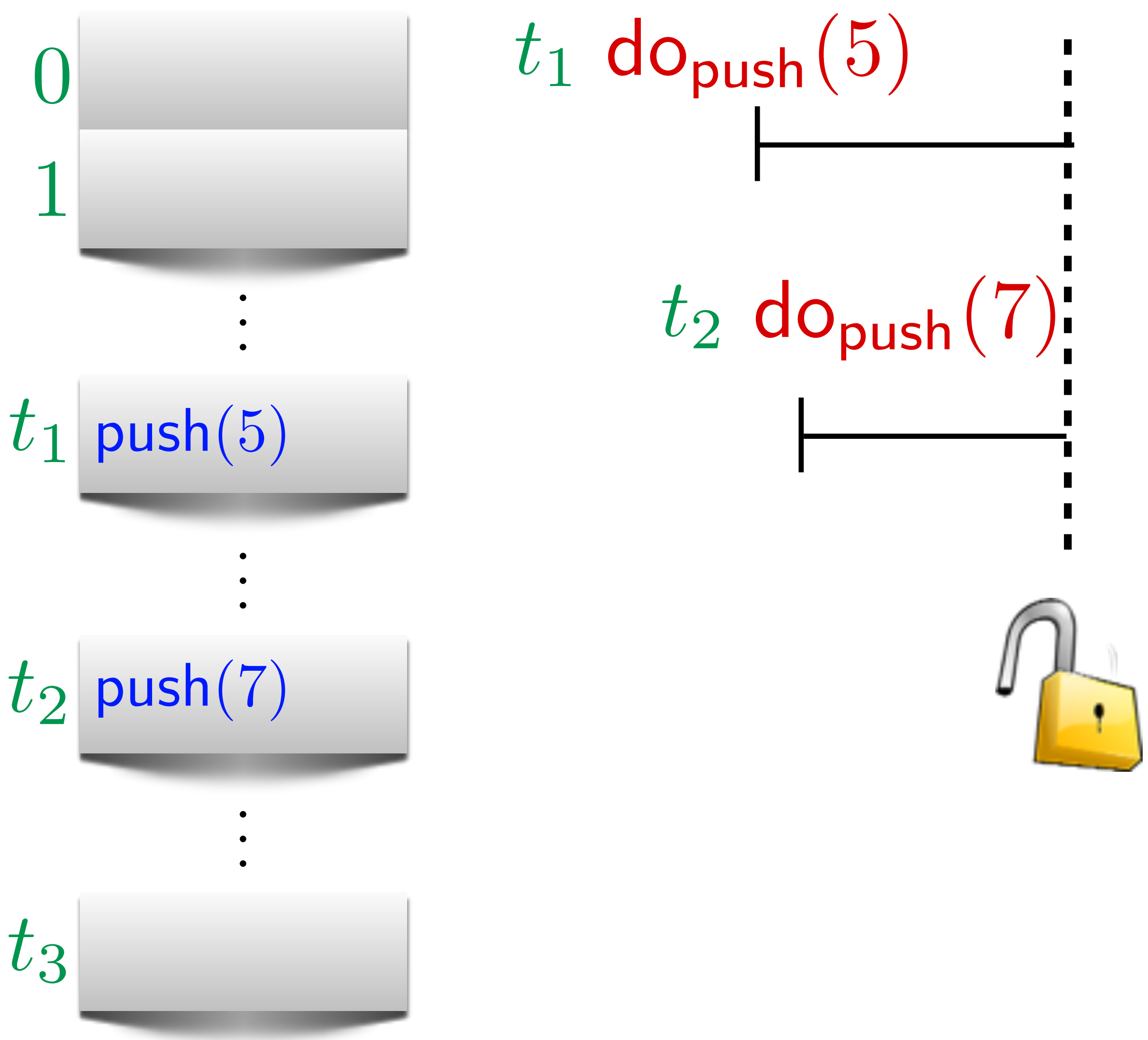
Stack provides methods push, pop to clients

FC_{stack} : access to methods in Stack regulated by do_{push}, do_{pop}



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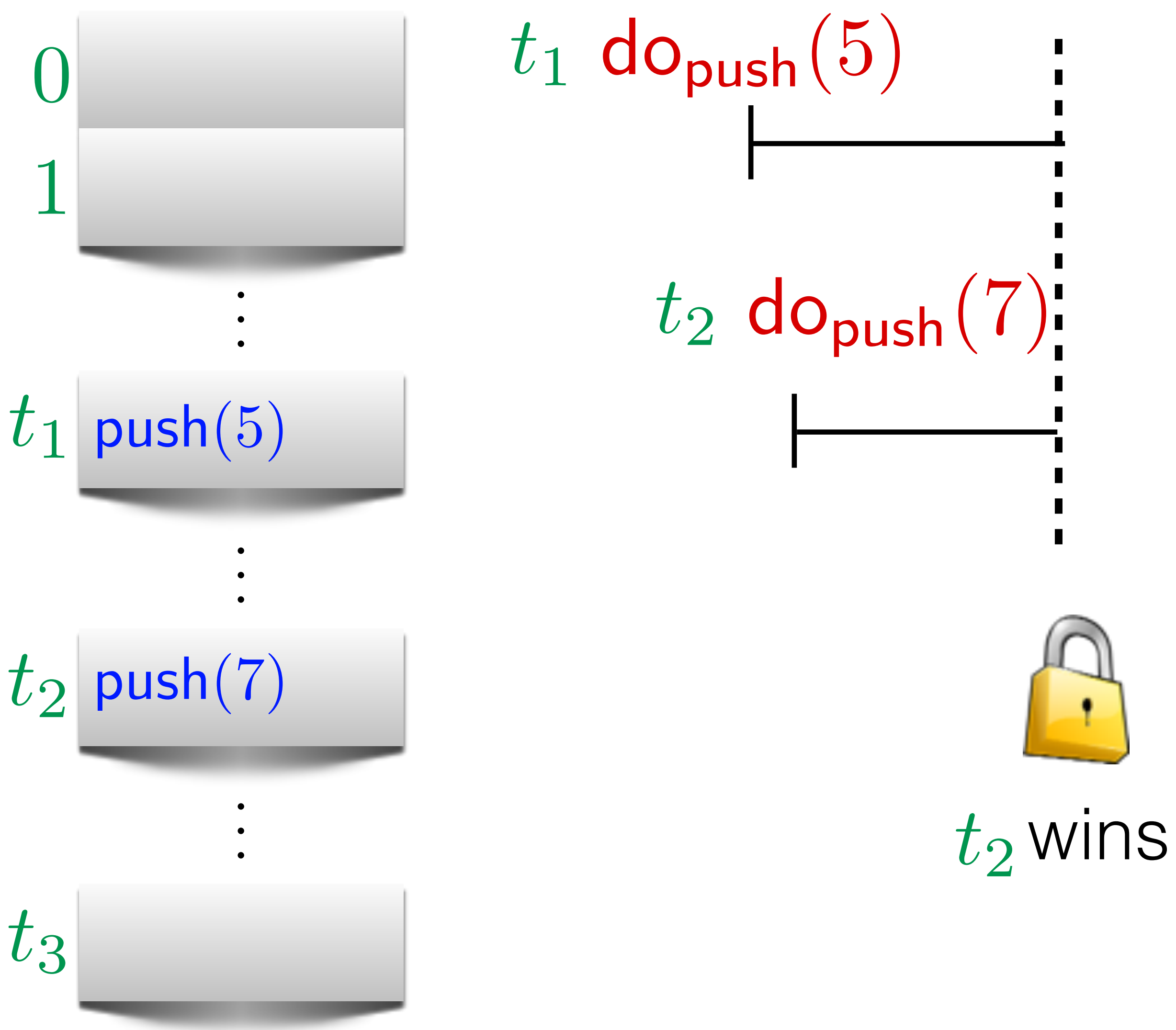

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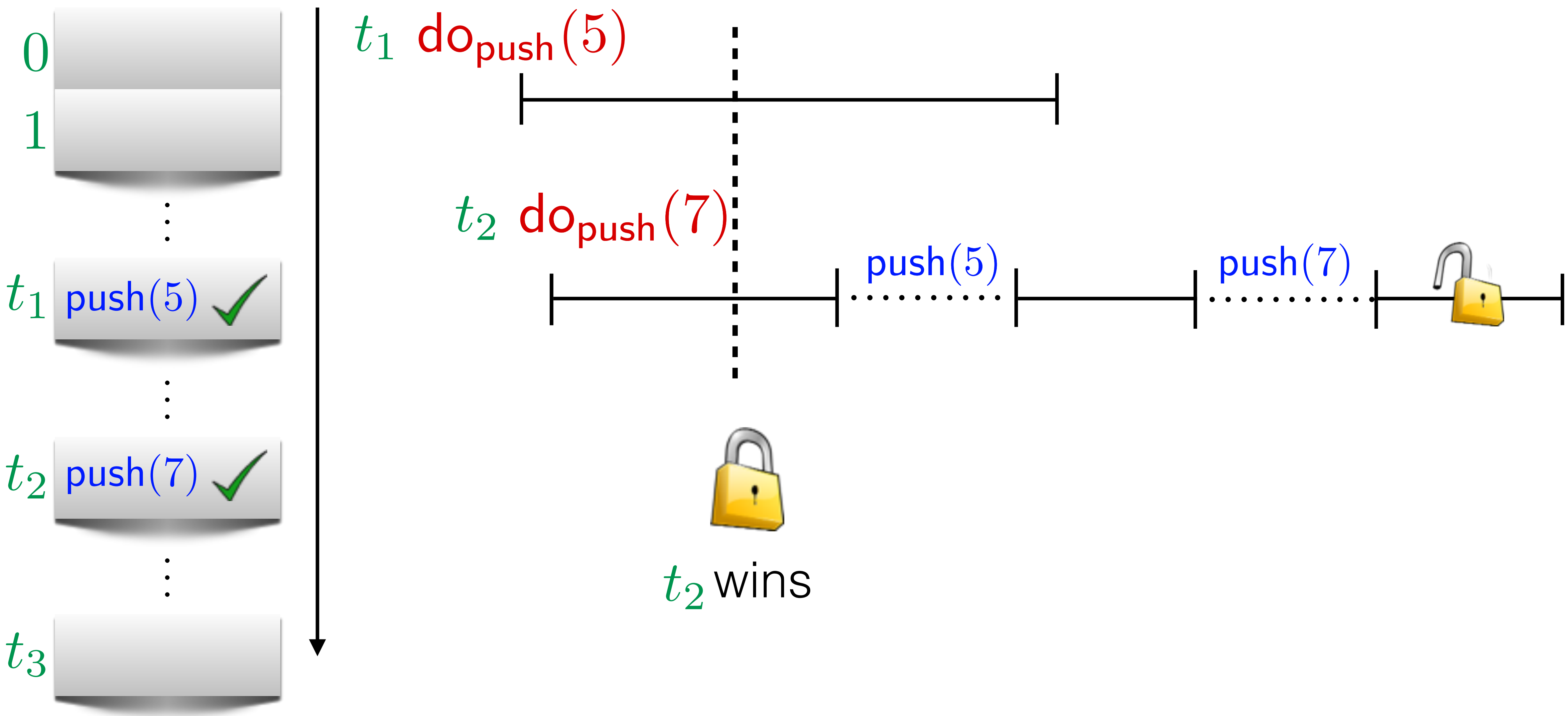

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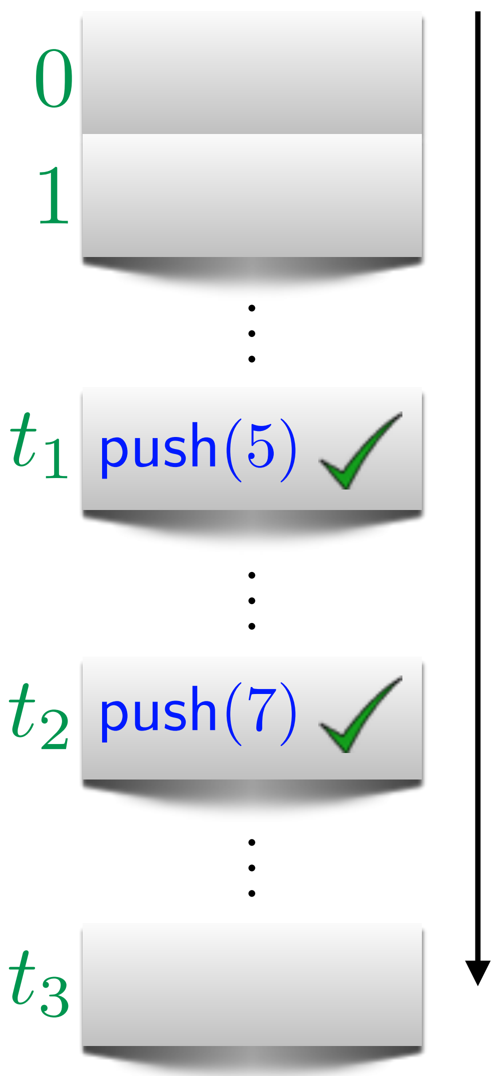

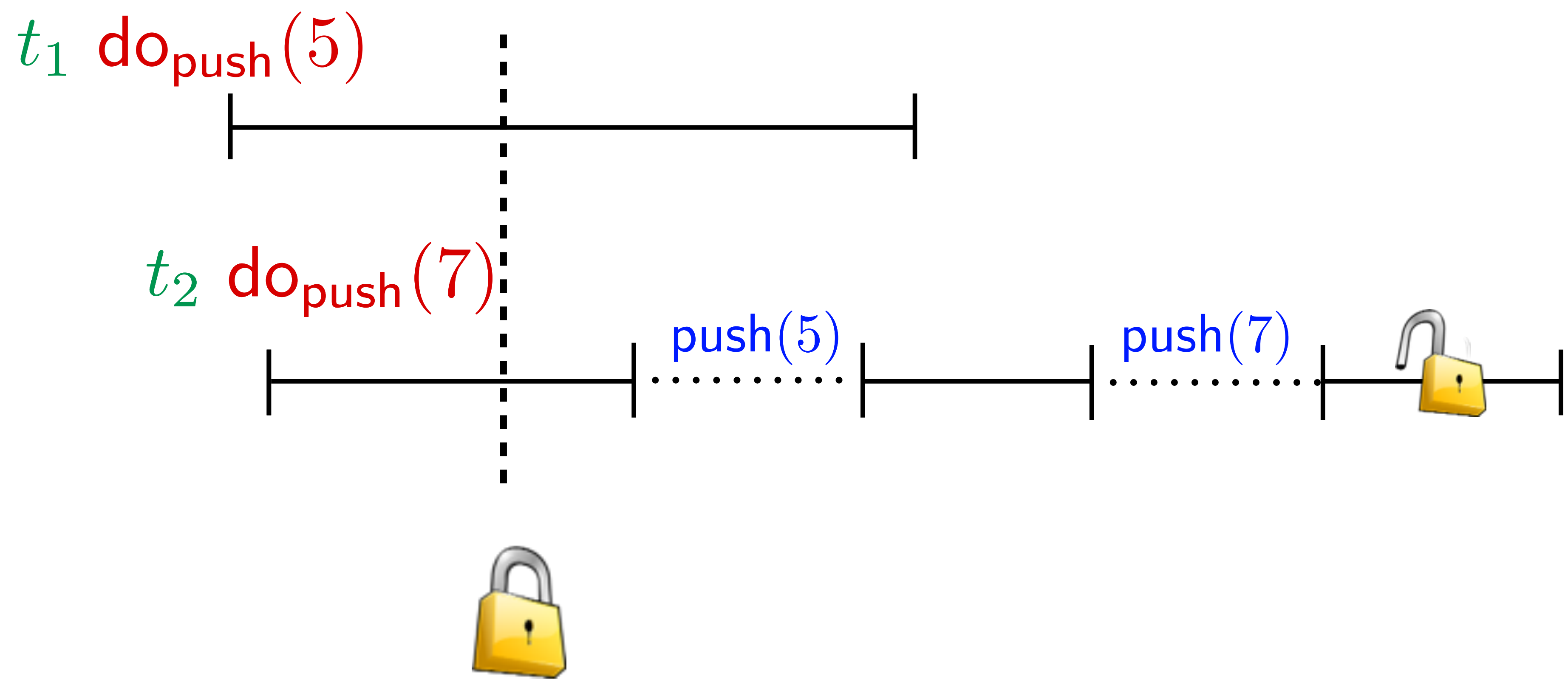

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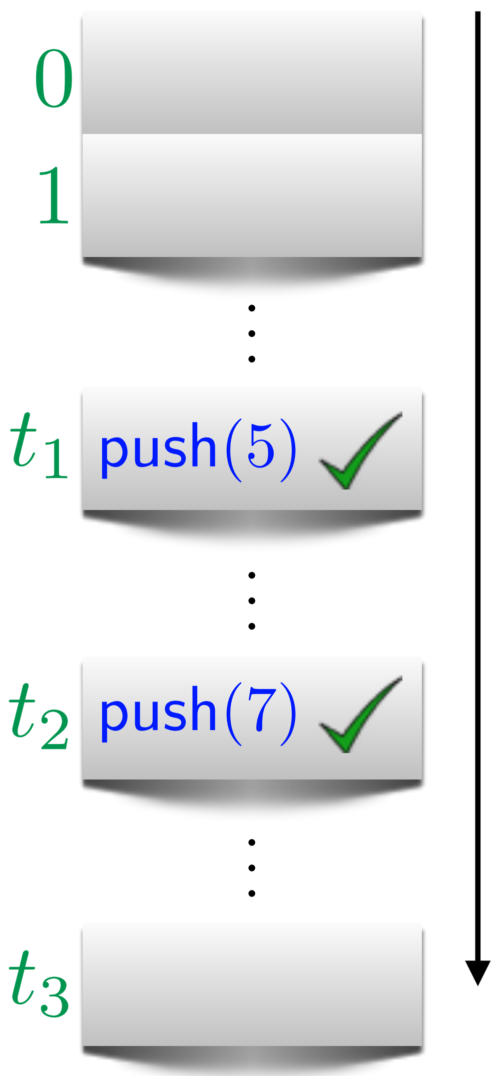

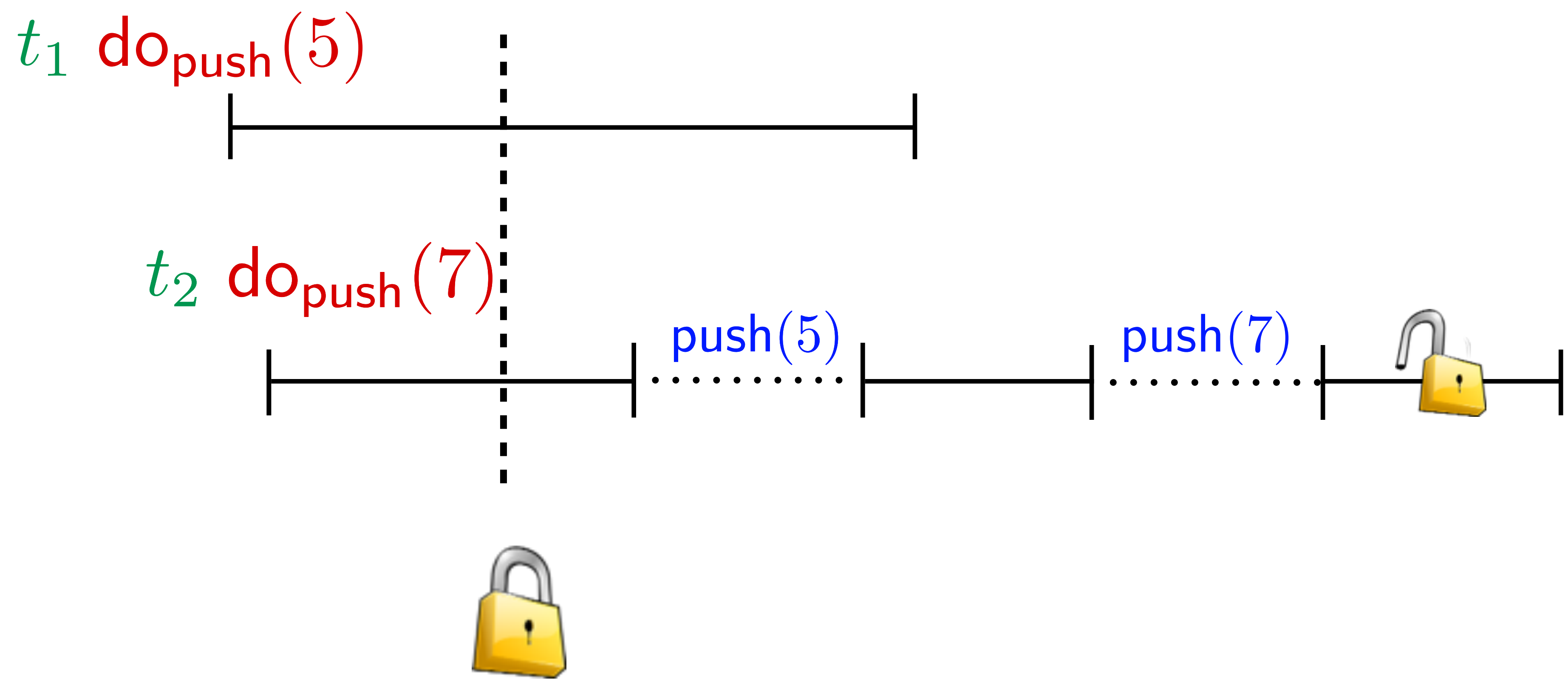

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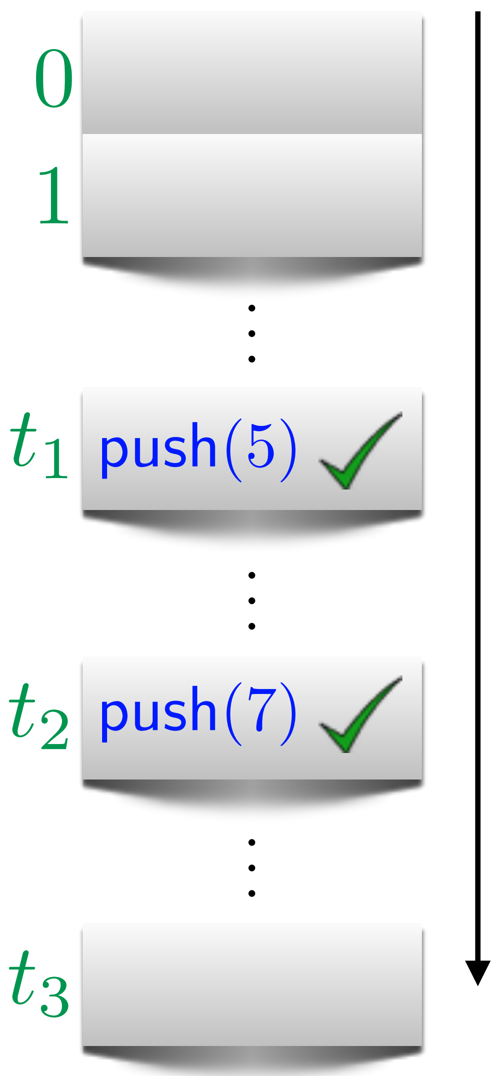
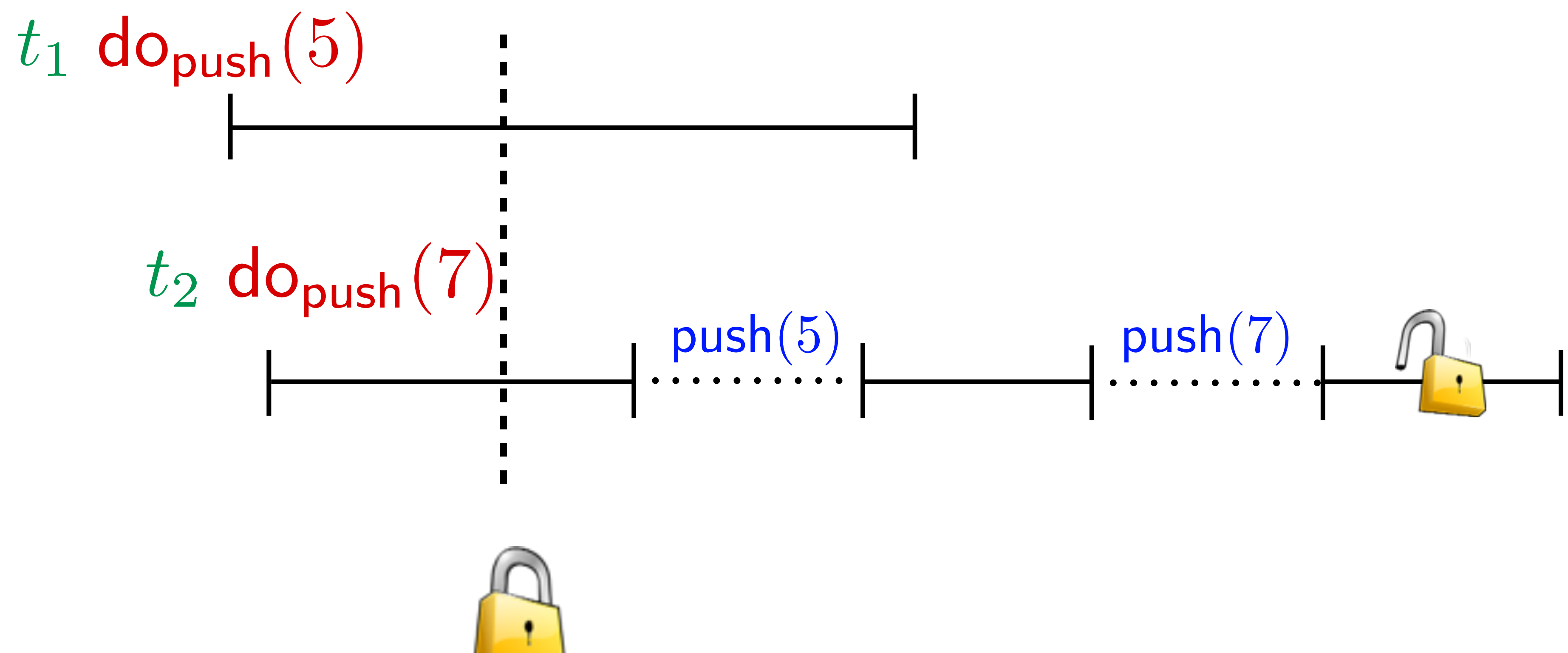




It appears to behave as the naive solution

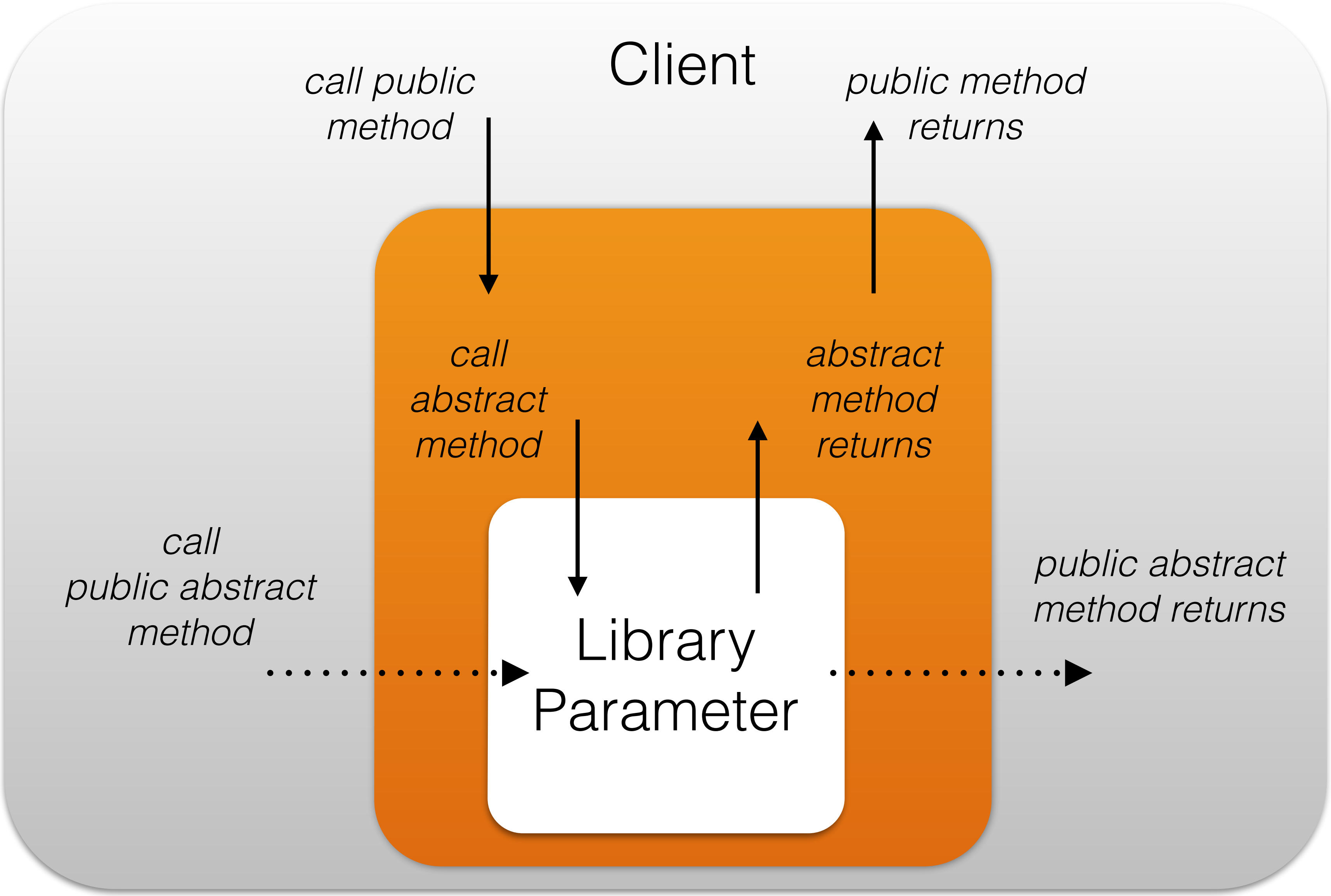
- push, pop nc … ct methods)
- Guaranteed to have correct behaviour for (**almost**) any implementation of abstract methods

Second Order Libraries

Public methods define the operations made available to clients

Abstract methods (without implementation) can be declared

$$L : M' \rightarrow M$$



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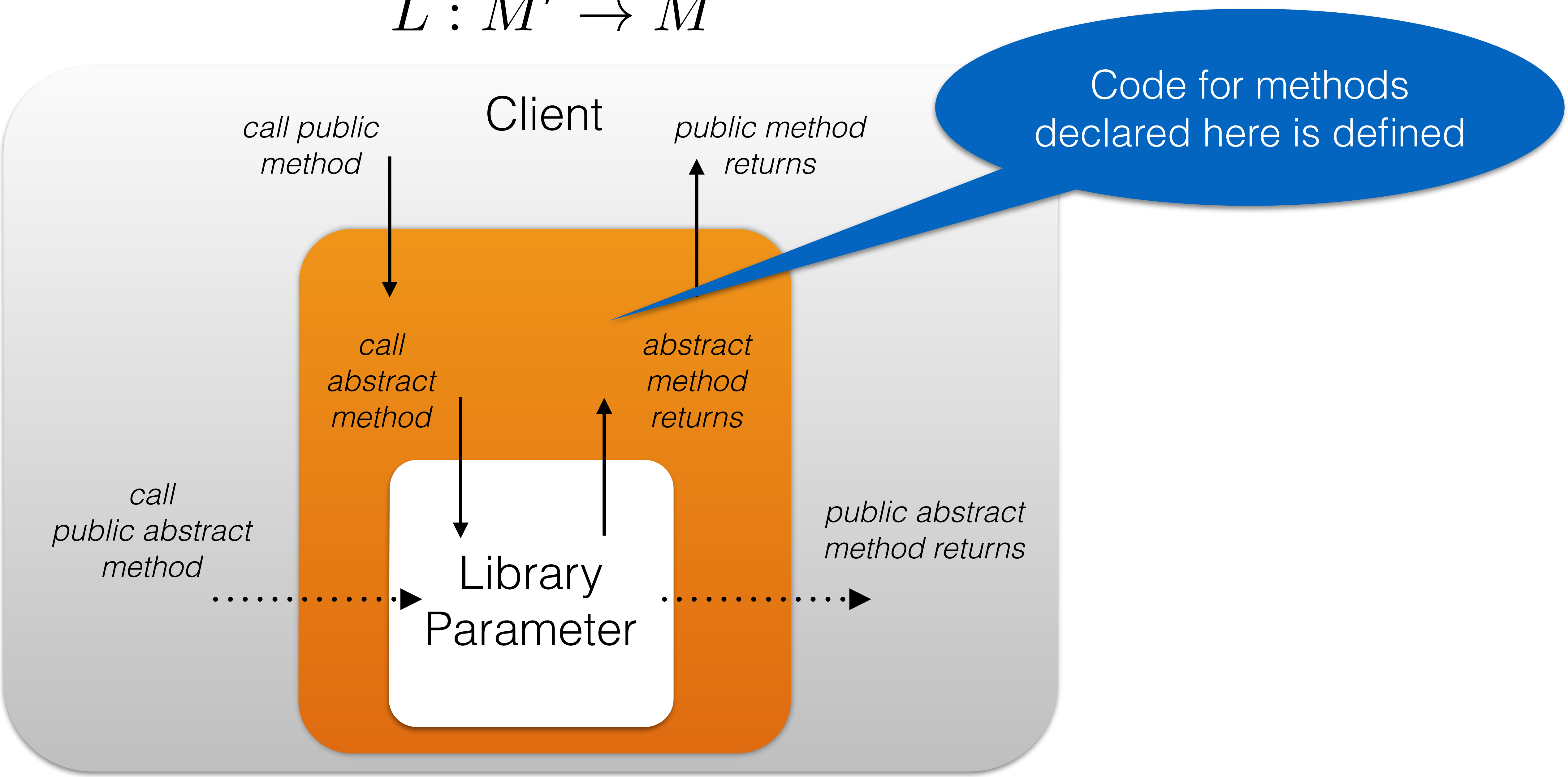

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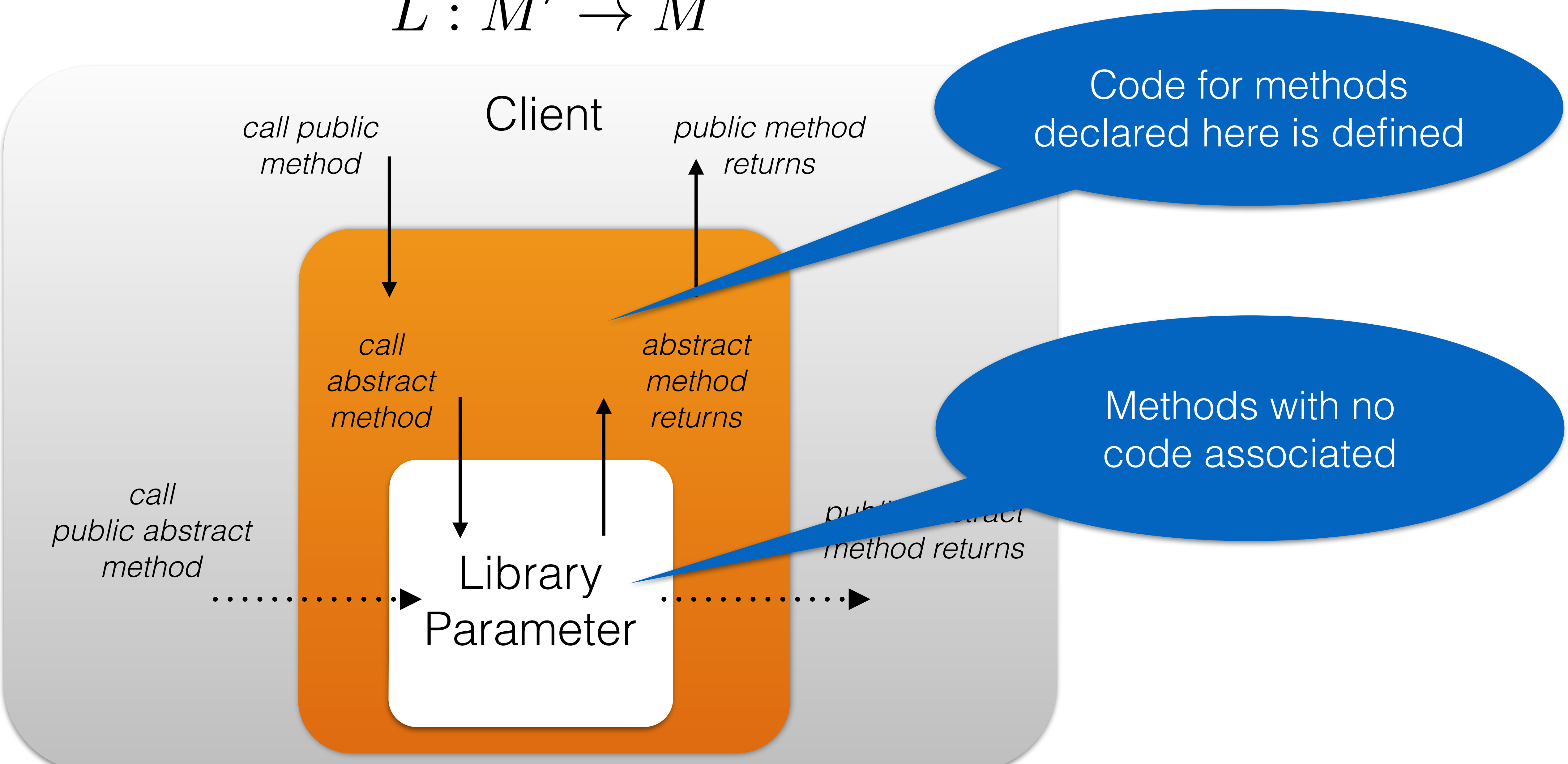

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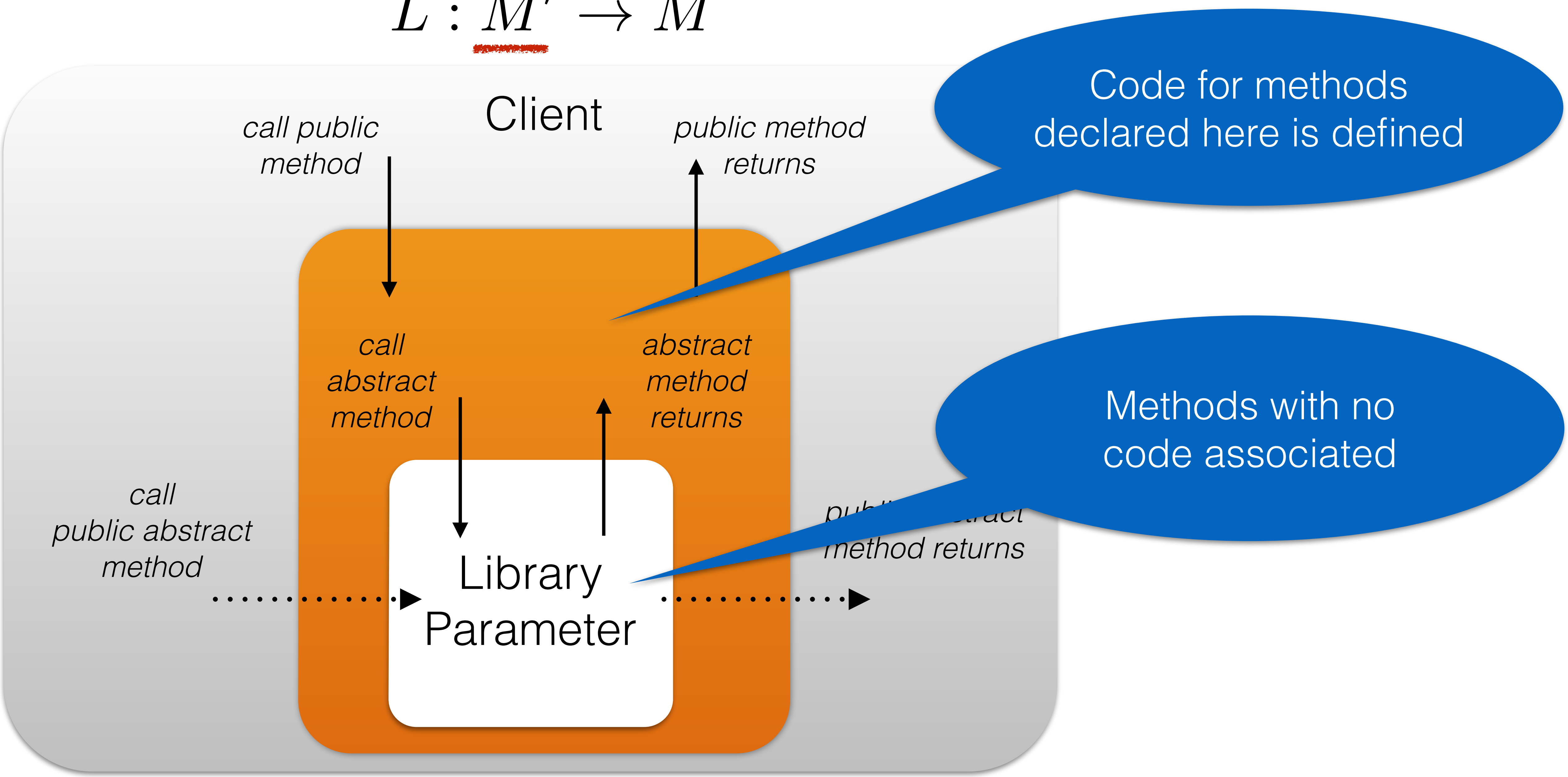

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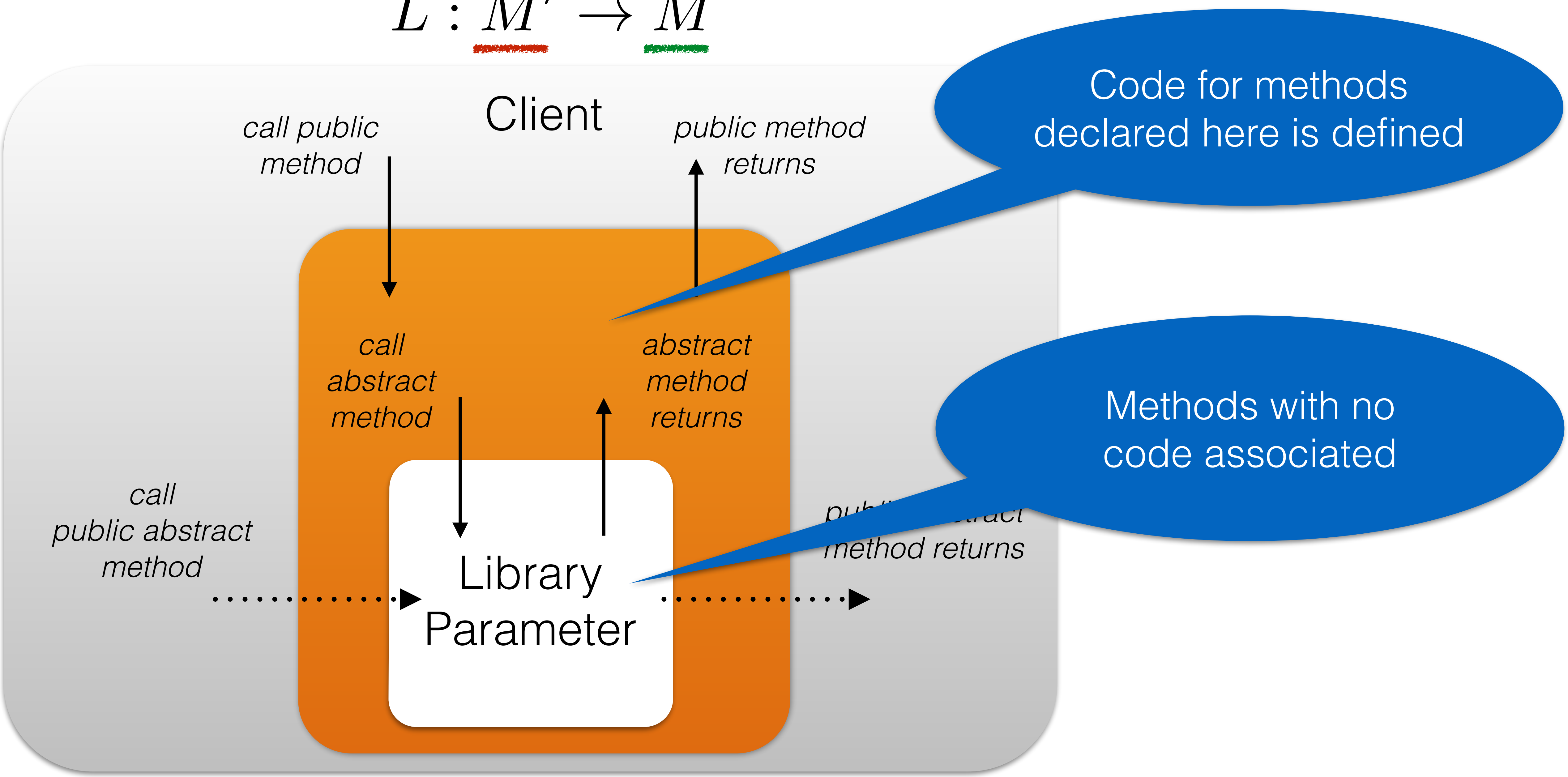
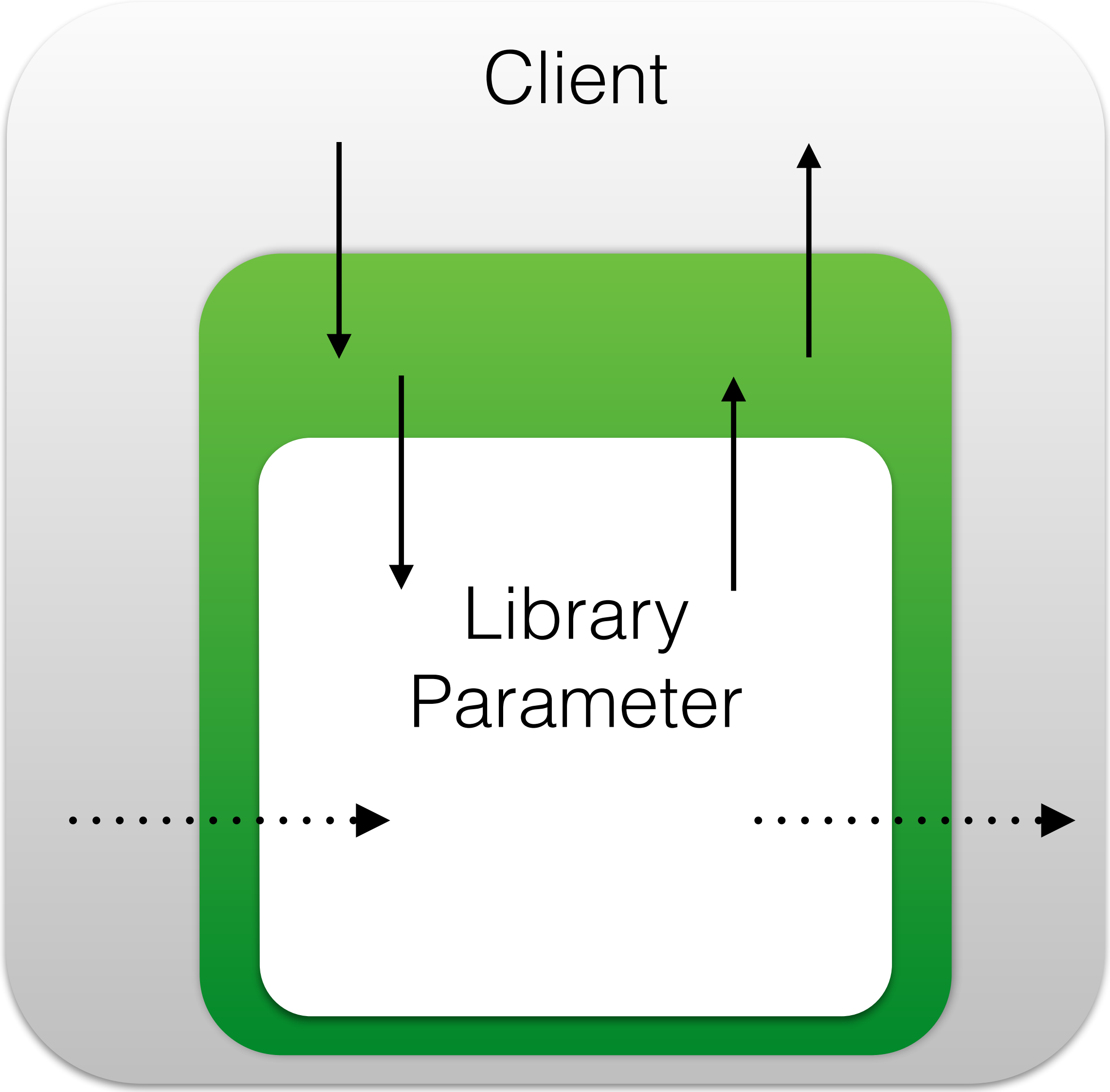


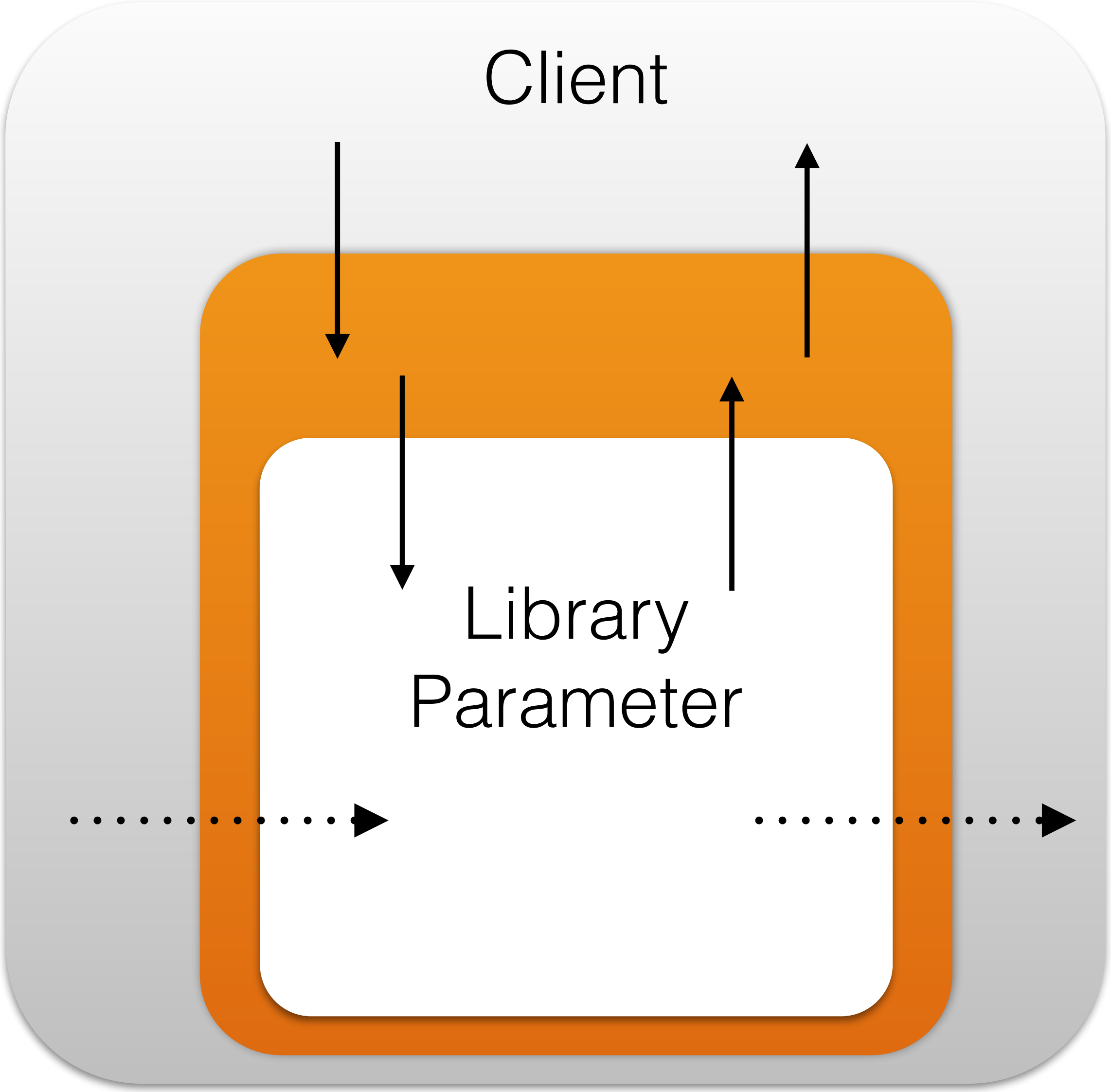
Library Parameter Instantiation

The implementation of abstract methods can be specified by another library

$L_2 : M \rightarrow \underline{M'}$



$L_1 : \underline{M'} \rightarrow M''$



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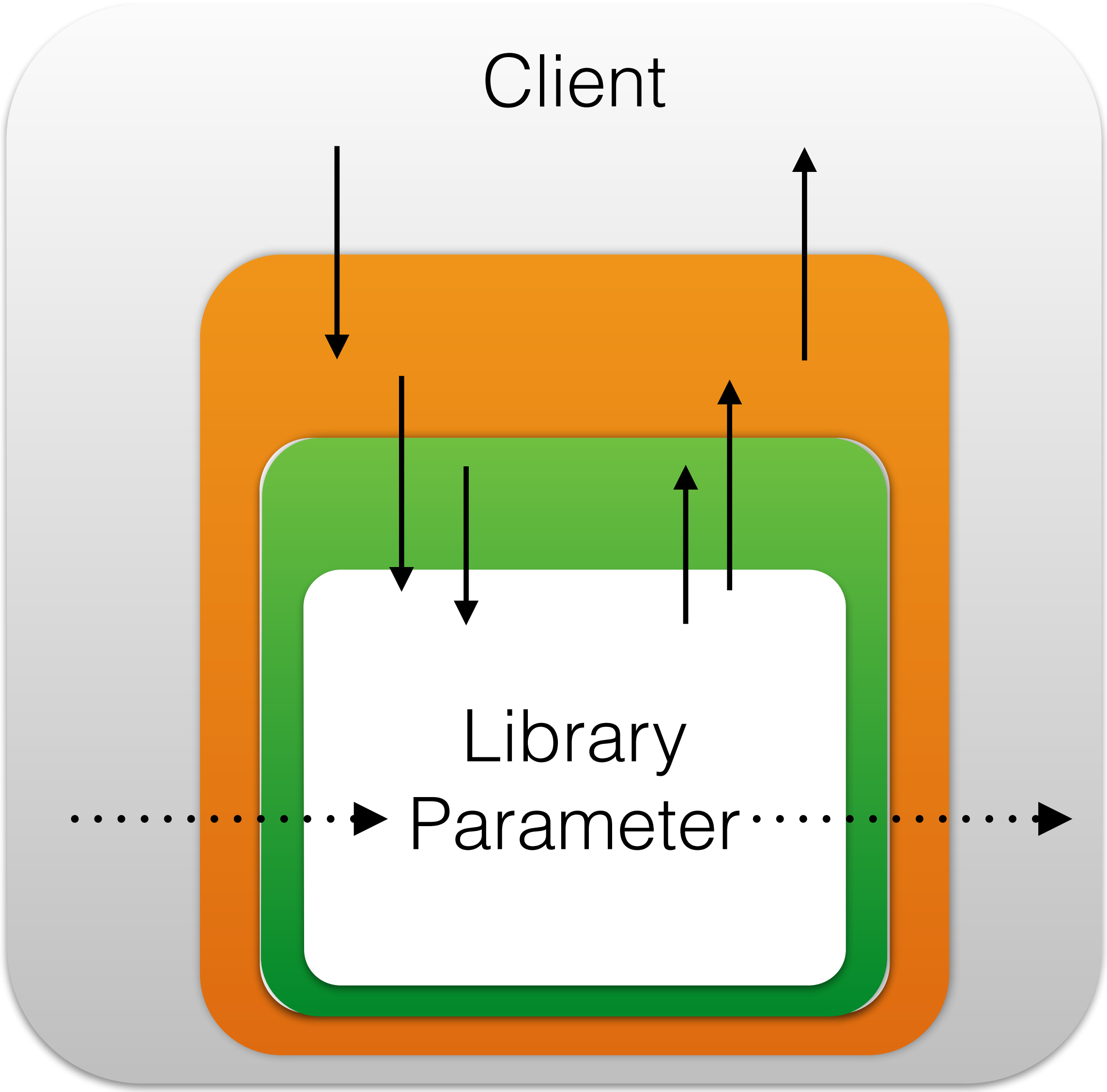

Observational Refinement

Program: let L in $C_1 \| \cdots \| C_n$

only calls public methods implemented in L

L has no abstract methods

Observational Refinement:

inclusion of client traces, for every possible client
and every possible library parameter

$$L_1 \sqsubseteq_{\mathsf{obs}} L_2 : \forall \mathsf{Cl}. \forall L : \emptyset \to M.\mathsf{Obs}(\mathsf{let}\ (L_1 \circ L)\ \mathsf{in}\ \mathsf{Cl}) \subseteq \mathsf{Obs}(\mathsf{let}\ (L_2 \circ L)\ \mathsf{in}\ \mathsf{Cl})$$

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Quantification over clients and library parameters

A proof technique is needed

Specifying the behaviour of libraries

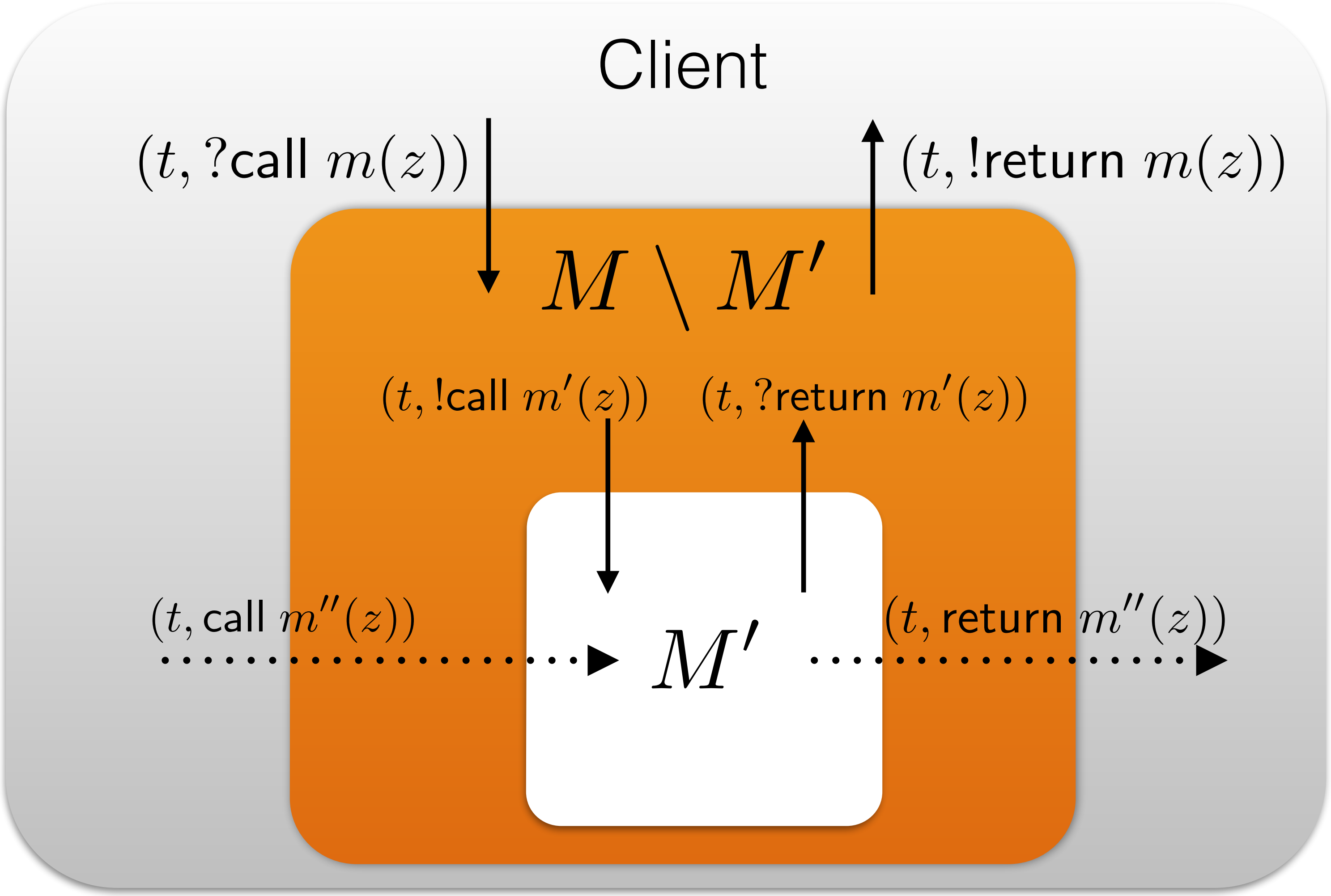
Types for Libraries

$$L : M' \to M$$

Actions:

Observable behaviour of Libraries

(Different entitities execute in different memory spaces)



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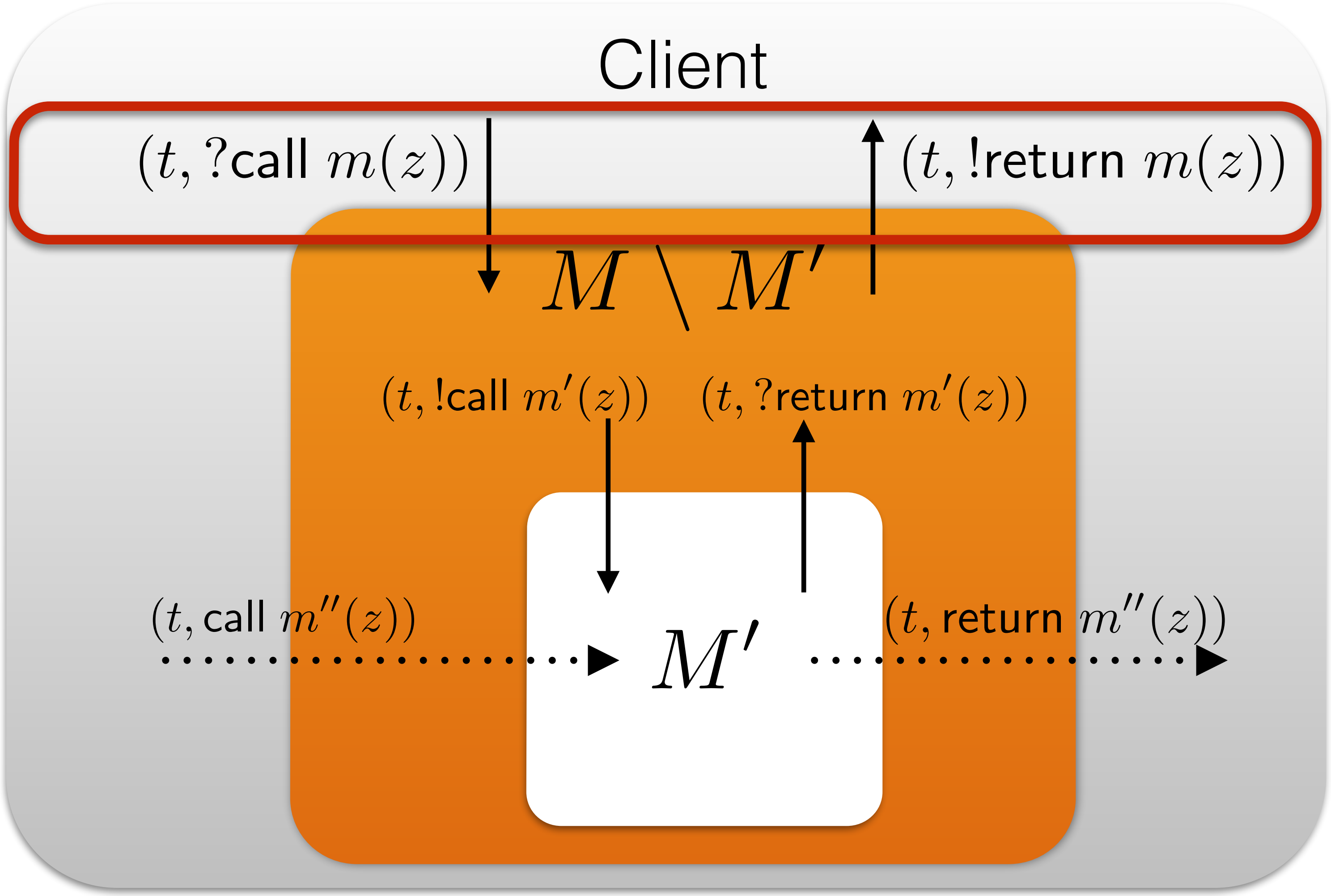

Interactions with the client $(t, ?\text{call } m(z))$ $(t, !\text{return } m(z))$ $m \in M \setminus M'$

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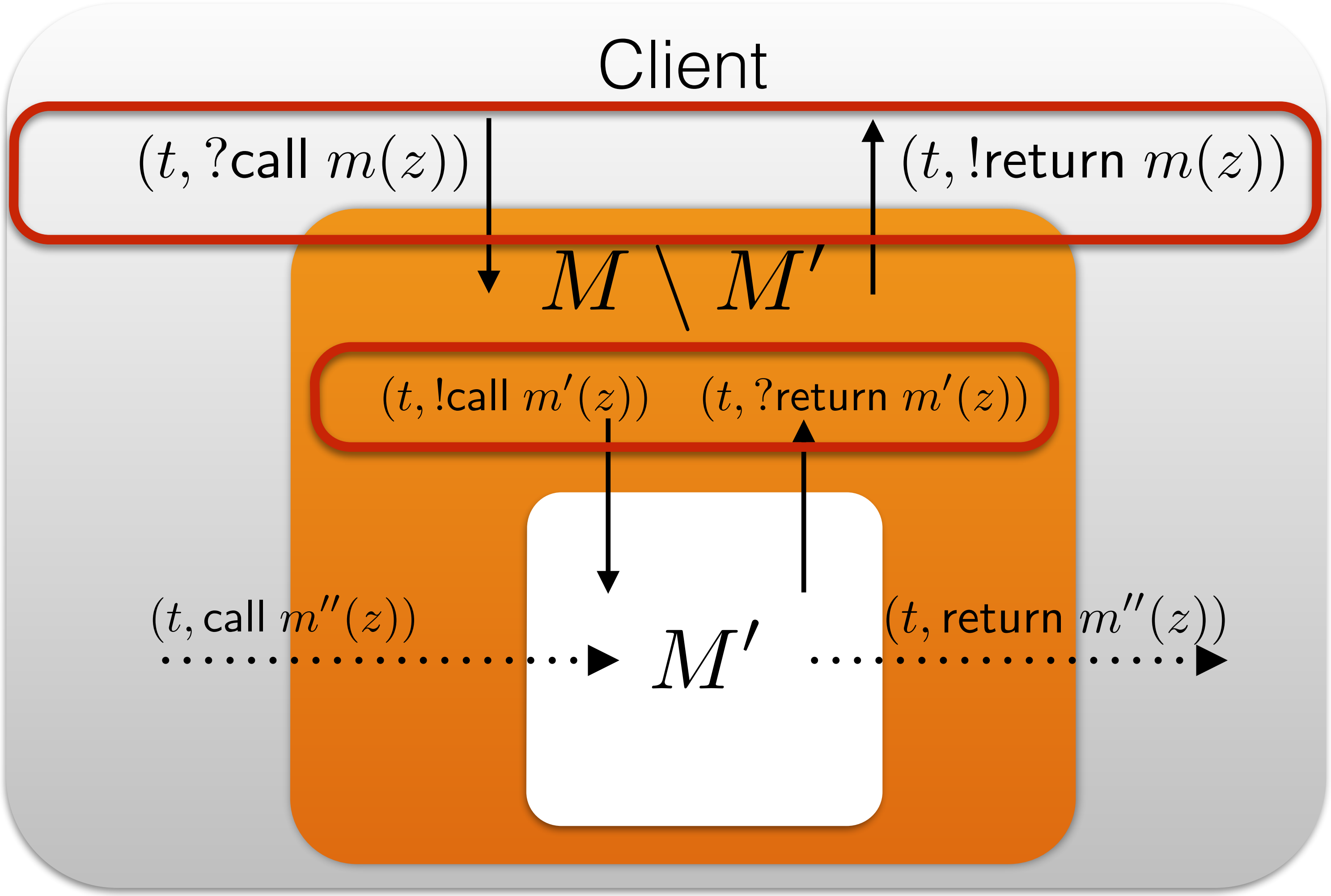

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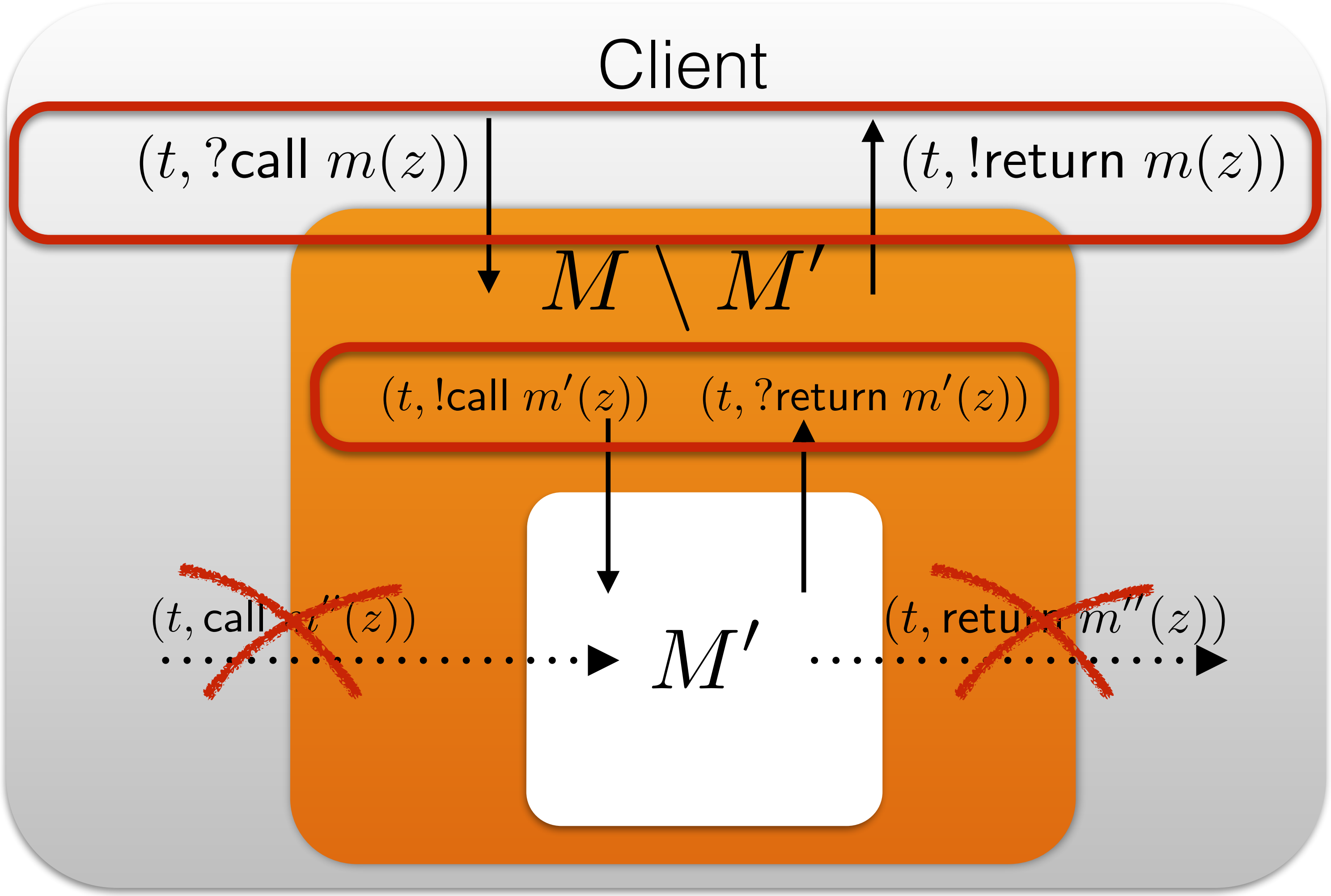

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Abstract methods invoked by the client (not observable) $m \in M \cap M'$

Specifying the behaviour of libraries

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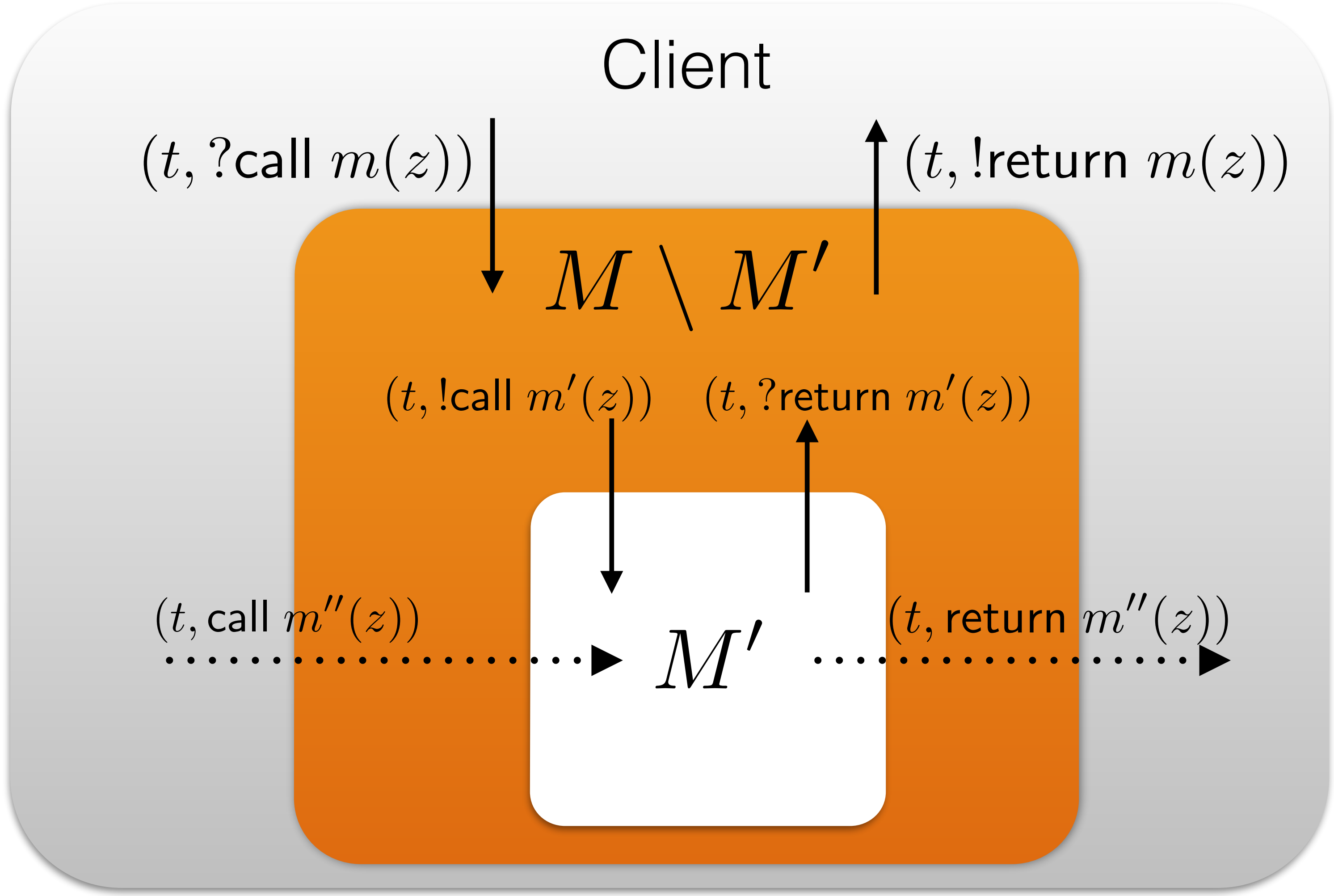
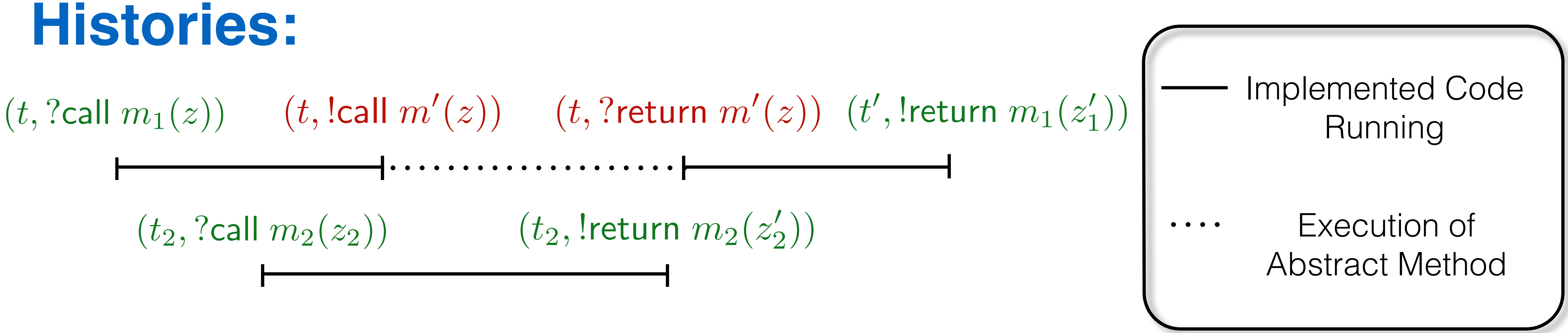
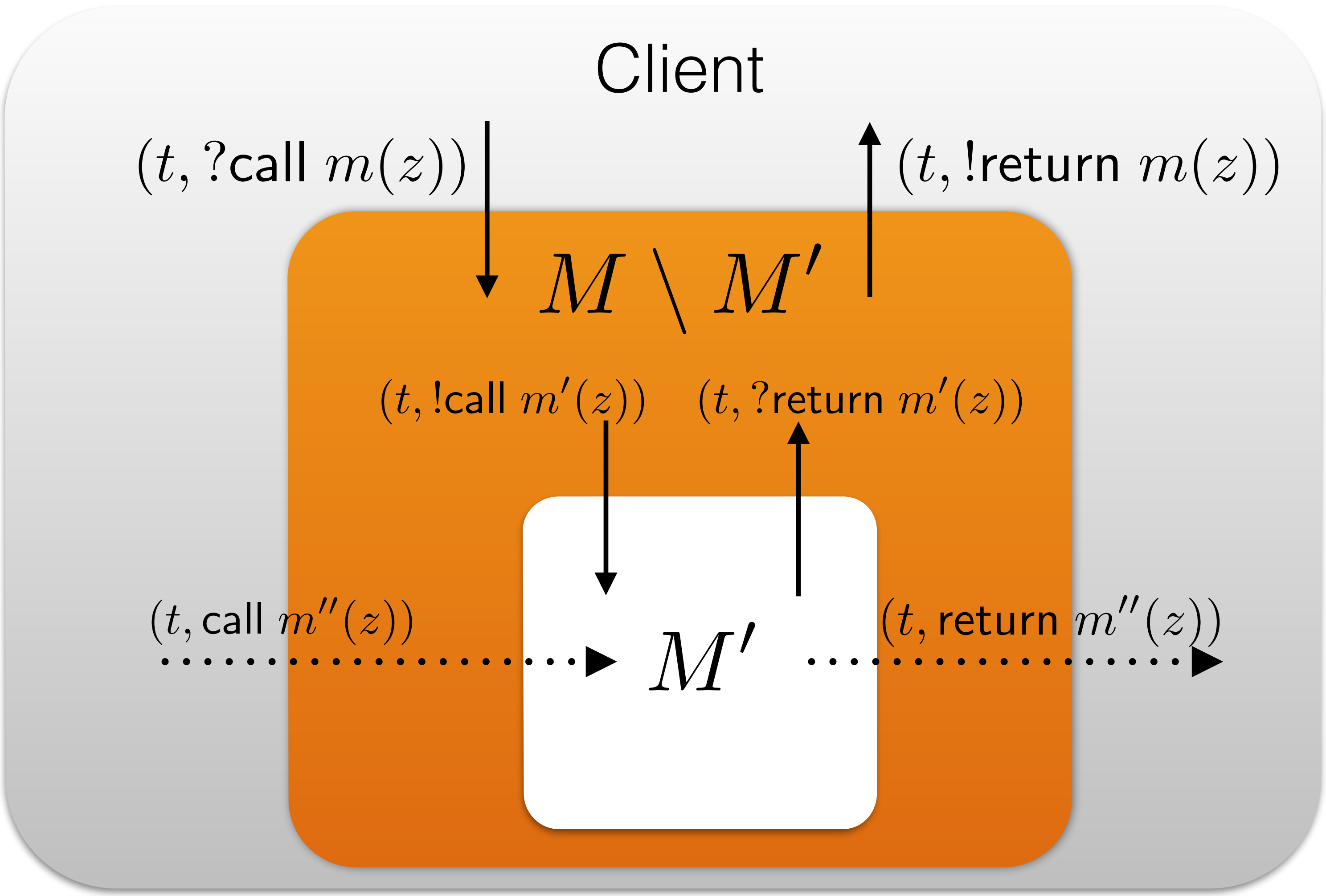


Histories:



Denotational Semantics of Libraries

$[\![L]\!]$: histories generated by a library
(arbitrary behaviour of client and library parameter)



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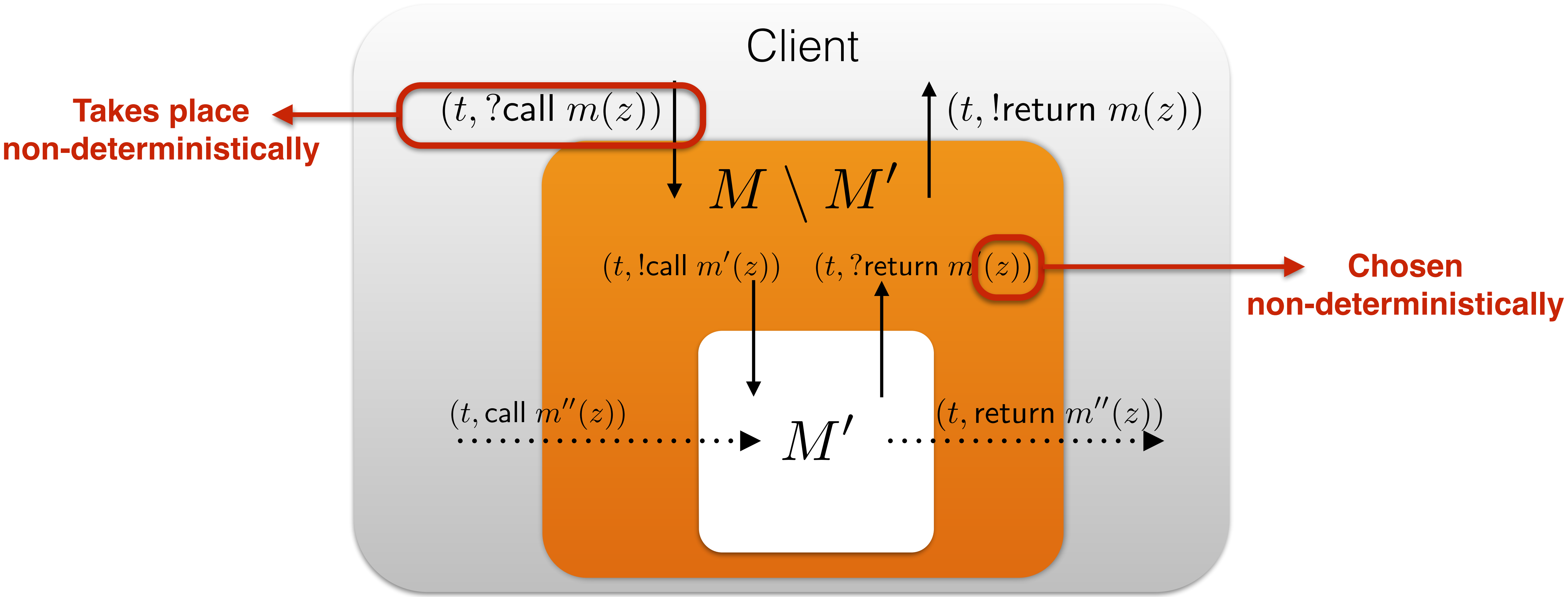
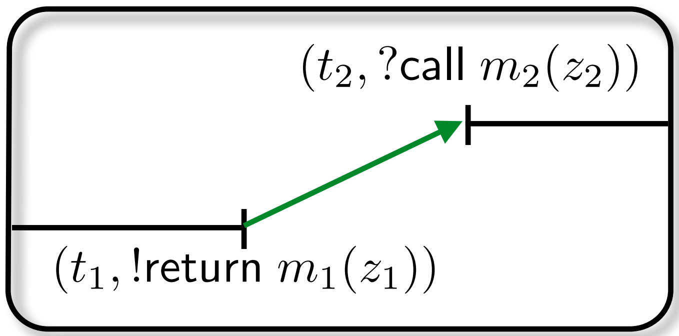
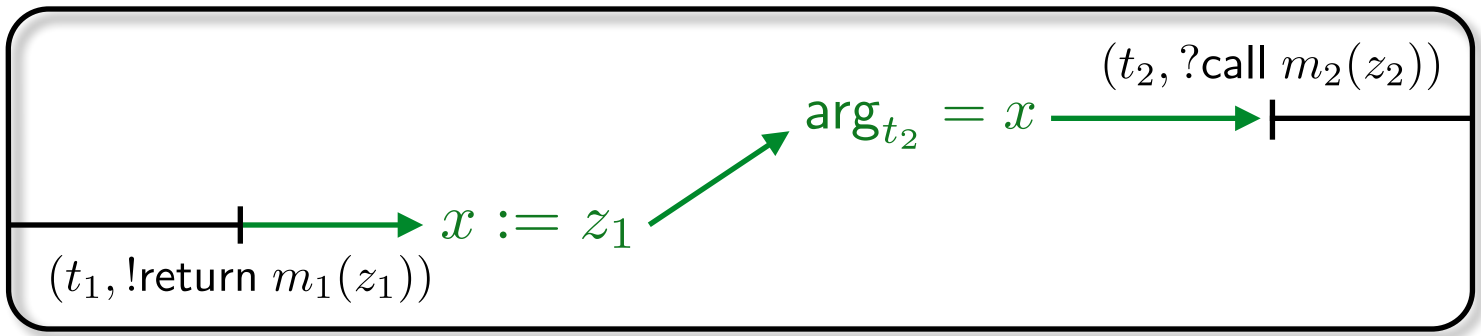


(General) Linearisability

$h_1 \sqsubseteq h_2$: • h_2 preserves thread-local subhistories of h_1
+ the following order of pairs of actions:

1 



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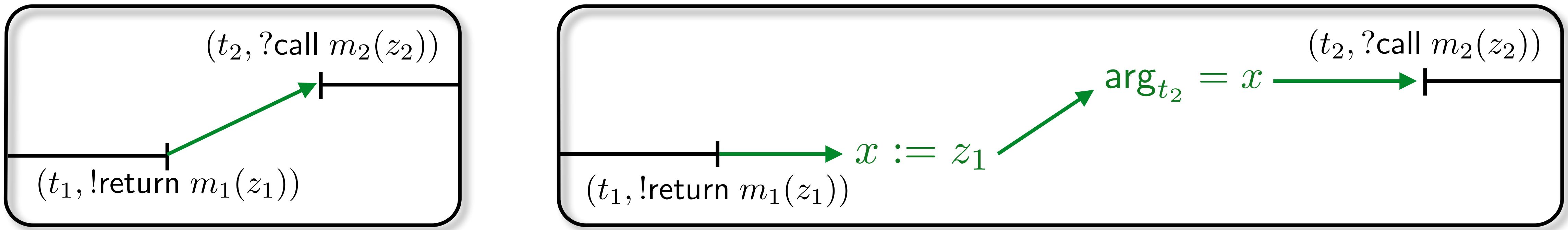

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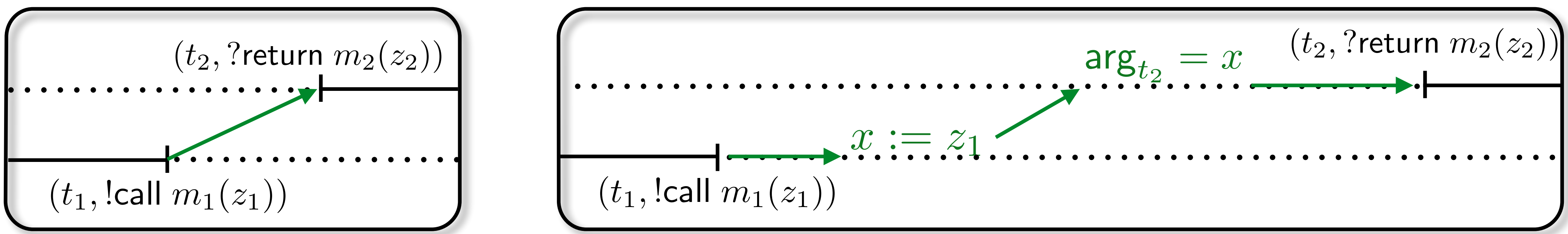

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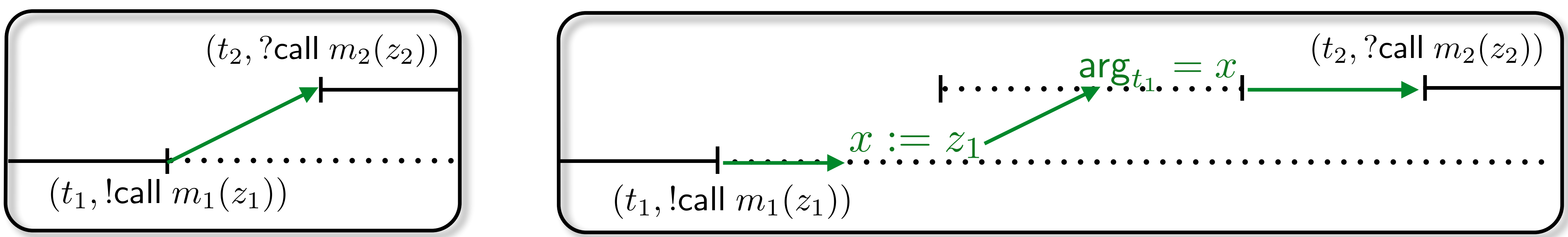

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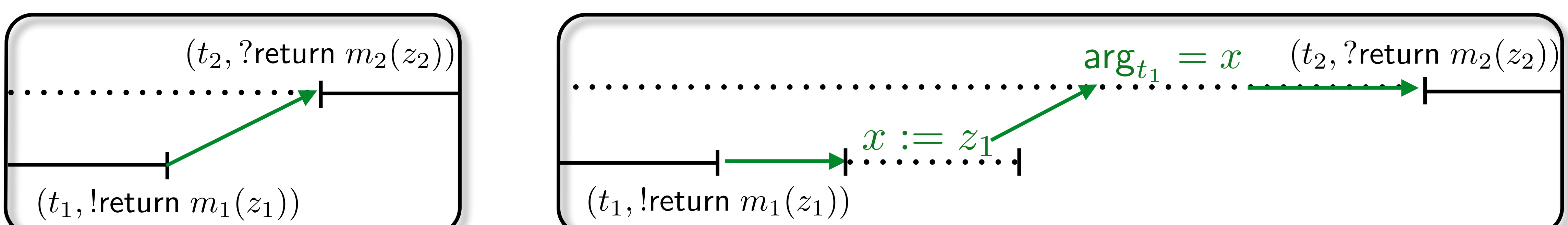

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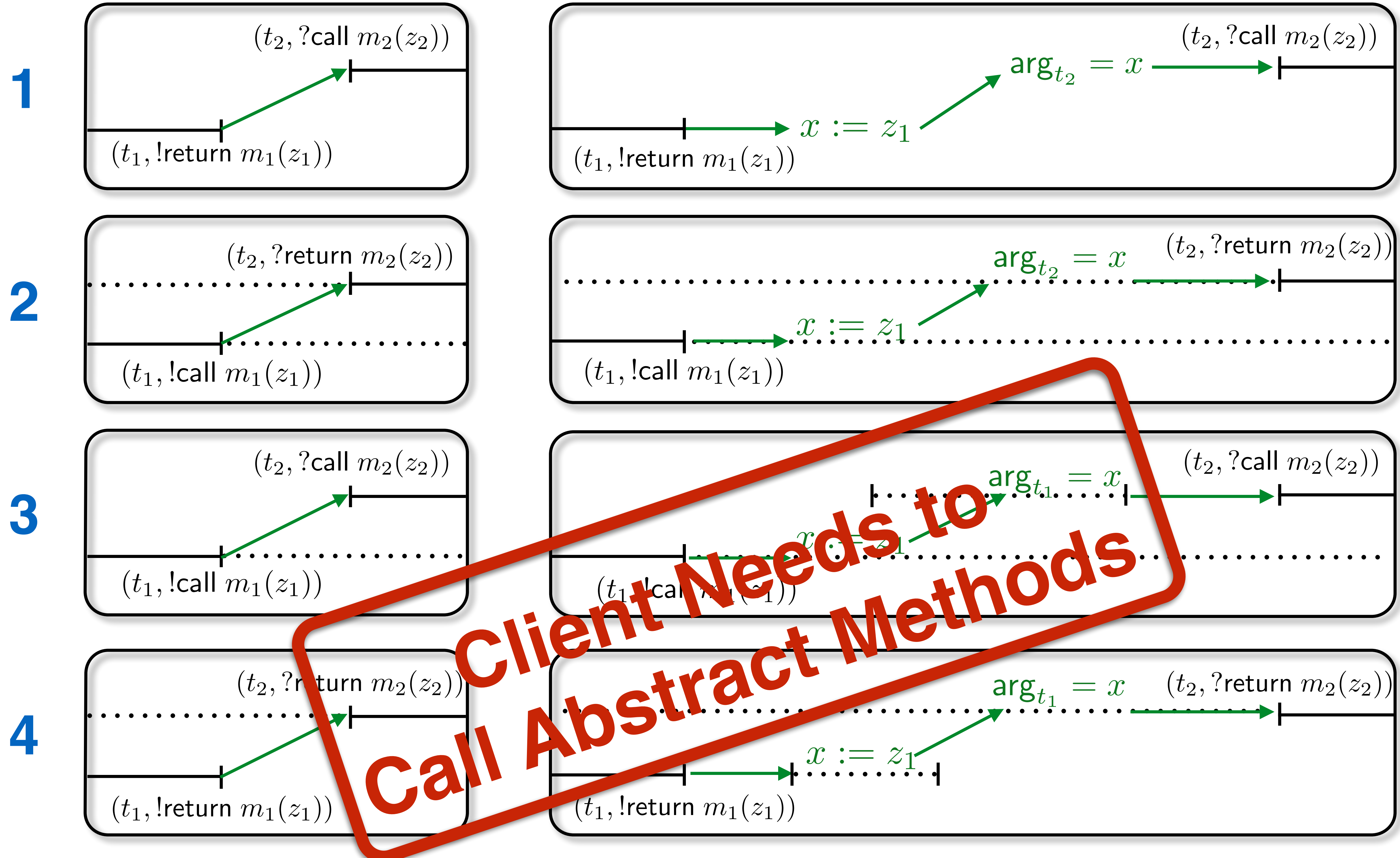

Encapsulated Linearisability

$$L : M' \to M \qquad M' \cap M = \emptyset$$

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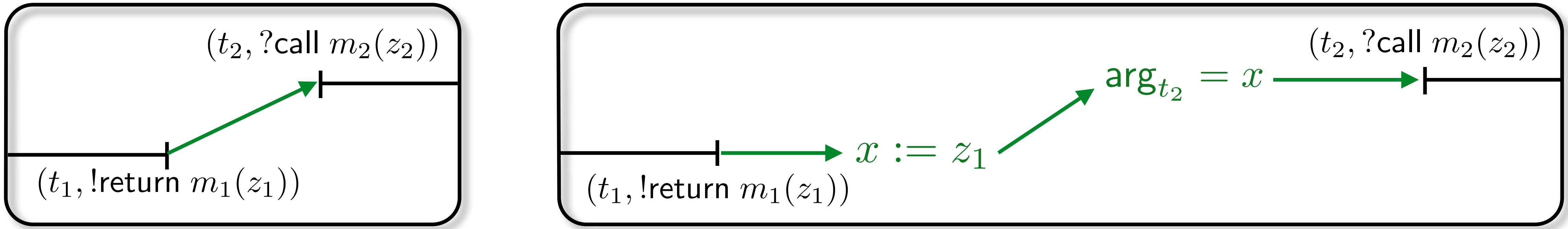

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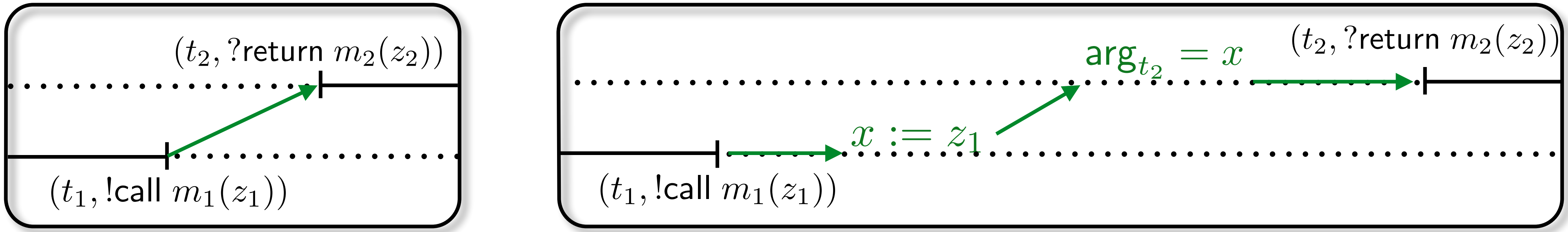
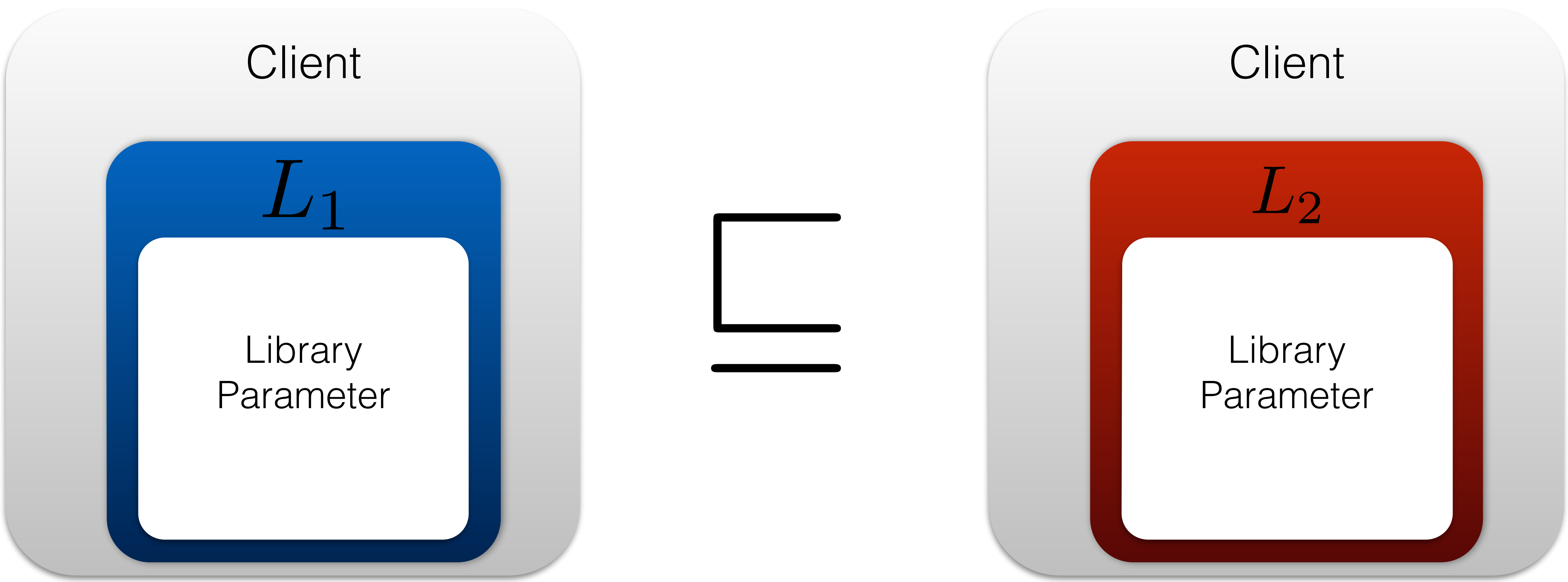


Contextuality of Linearisability

Theorem: $L_1, L_2 : M' \to M''$ $\qquad L_{\mathsf{in}} : M \to M'$

$$L_1 \sqsubseteq L_2 \implies (L_1 \circ L_{\mathsf{in}}) \sqsubseteq (L_2 \circ L_{\mathsf{in}})$$



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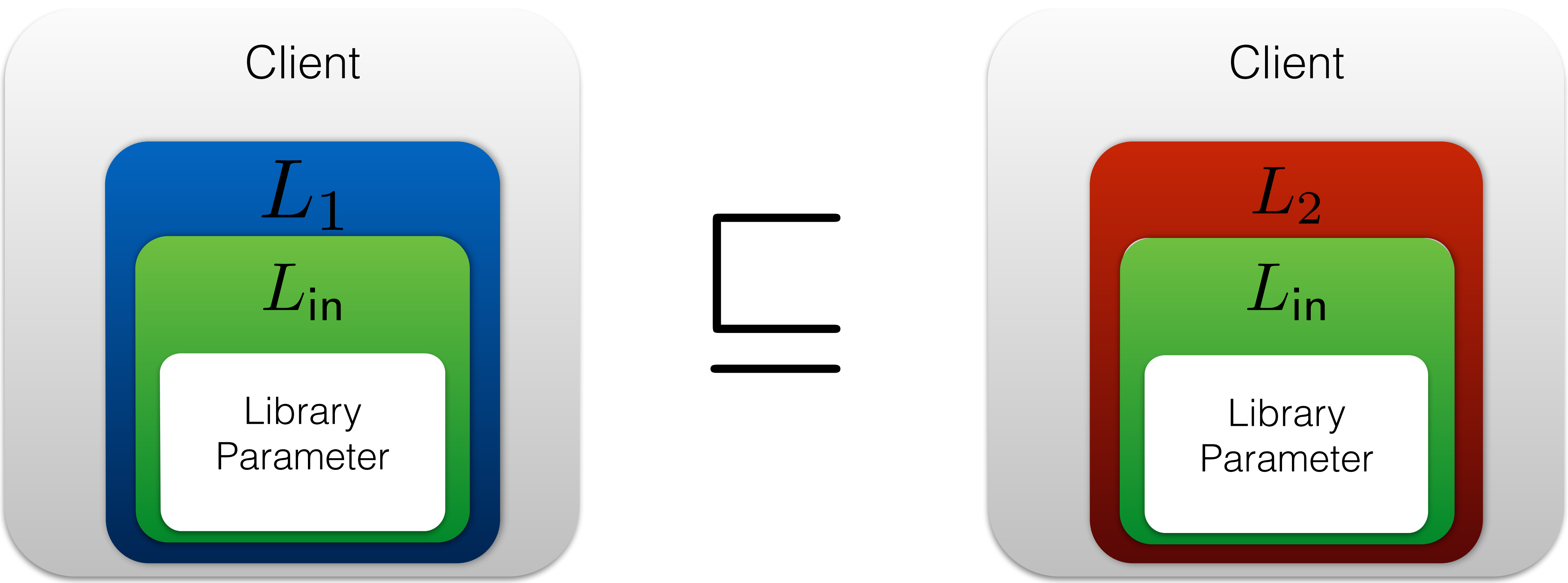

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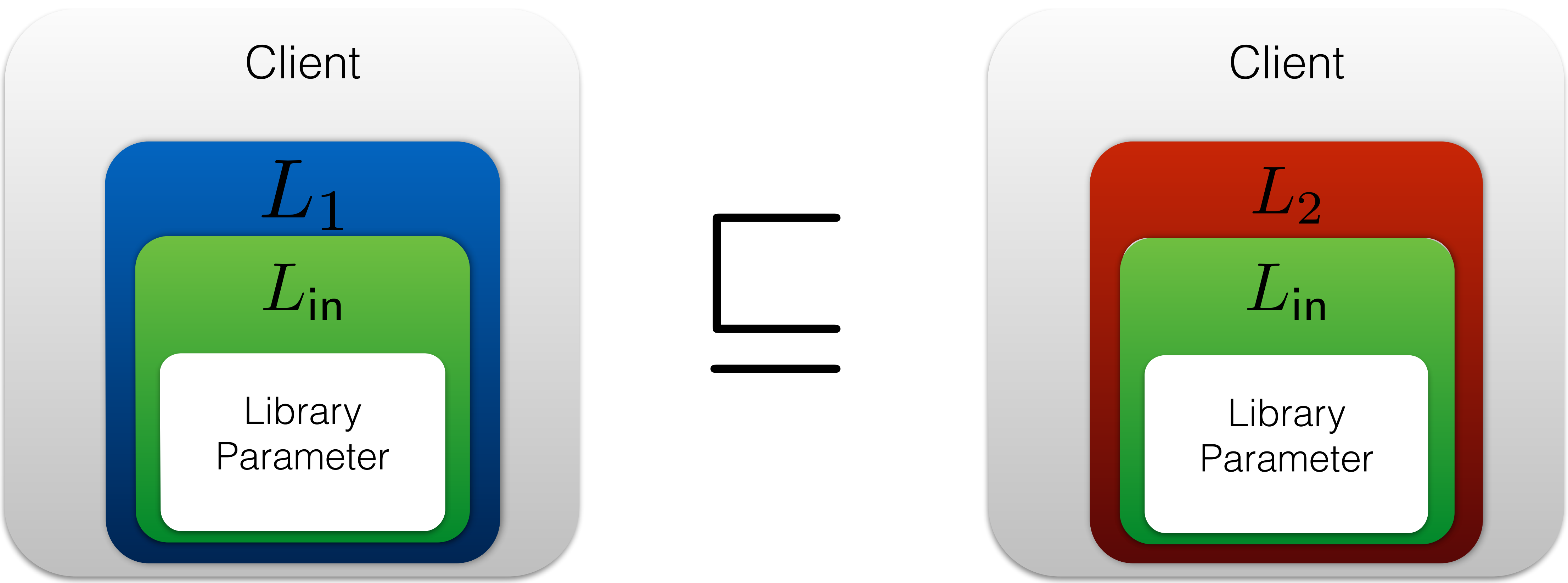

Corollary: $L_1 \sqsubseteq L_2 \implies L_1 \sqsubseteq_{\mathsf{obs}} L_2$

Contextuality of Encapsulated Linearisability

Theorem: for libraries with no public abstract methods encapsulated linearisability is preserved by instantiation of library parameters

Corollary: for libraries with no public abstract methods encapsulated linearisability implies observational refinement

Linearisability does not suffice for Flat Combiners

$$FC : \{m_i\}_{i \in I} \to \{\mathsf{do}_{m_i}\}_{i \in I}$$

a single thread handles all concurrent requests

$$FC_\# : \{m_i\}_{i \in I} \to \{\mathsf{do}_{m_i}\}_{i \in I}$$

calls to abstract methods regulated by a global lock

Instantiating the library parameter

$$int\ m_i()\{return\ \mathsf{getTid}();\}$$

Instantiating the Client

$$t_1 : \mathsf{do}_{m_i}(); \ \| \ t_2 : \mathsf{do}_{m_i}();$$

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using FC :

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$$FC \not\sqsubseteq_{\mathsf{Obs}} FC_{\#}$$

$$\Longrightarrow$$

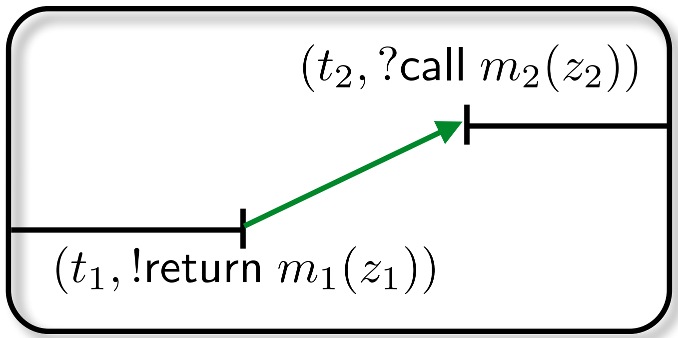
$$FC \not\sqsubseteq FC_{\#}$$

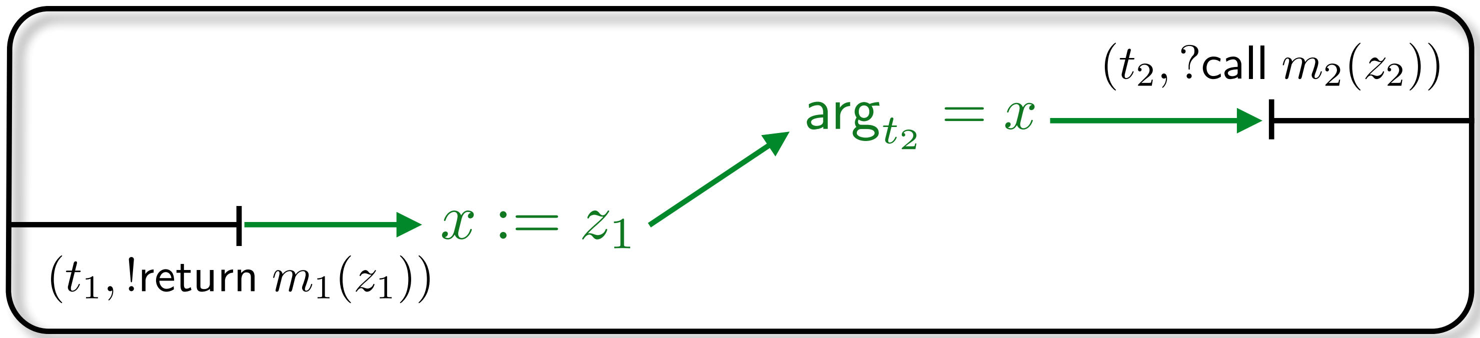
Properties Satisfied by Flat Combiners

$FC_{\#}$ linearises FC

if the library parameter does not use thread local information

1





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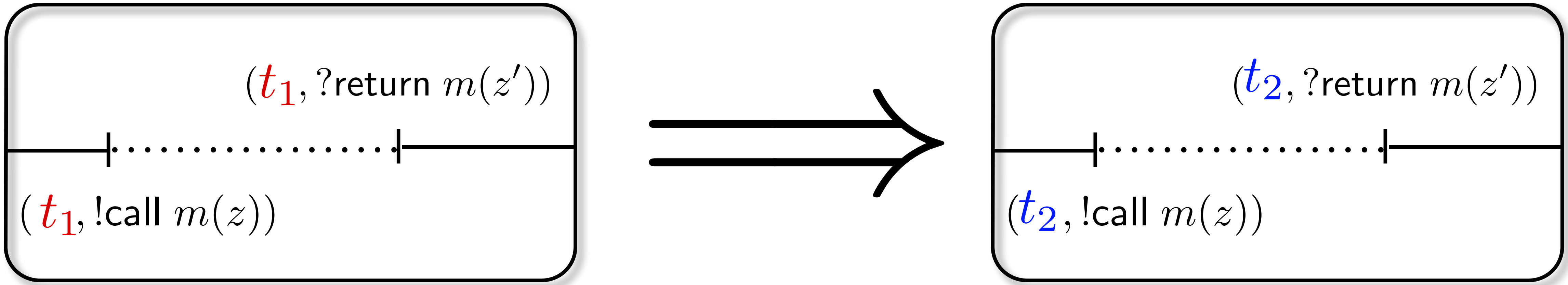

Up-to Linearisability and Observational Refinement

$M' \to M, M' \cap M = \emptyset$

$\mathcal{R}$: binary relation between sequences of calls and returns to methods in M'

$h_1 \sqsubseteq_{\mathcal{R}} h_2$:

- $h_1|_{\mathsf{ClAct}} \sqsubseteq h_2|_{\mathsf{ClAct}}$
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(a variant of contextuality holds for $\sqsubseteq_{\mathcal{R}}$)

Soundness: $L_1 \sqsubseteq_{\mathcal{R}} L_2 \implies L_1 \sqsubseteq^{\mathcal{R}}_{\mathsf{obs}} L_2$

only $\mathcal{R}$- closed library parameters are considered

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$$\mathsf{FC} \sqsubseteq_{\mathcal{R}_t} \mathsf{FC}^{\#} \implies \mathsf{FC} \sqsubseteq^{\mathcal{R}_t}_{\mathsf{obs}} \mathsf{FC}^{\#}$$

$\mathcal{R}_t$: equivalence of histories up-to thread identifiers

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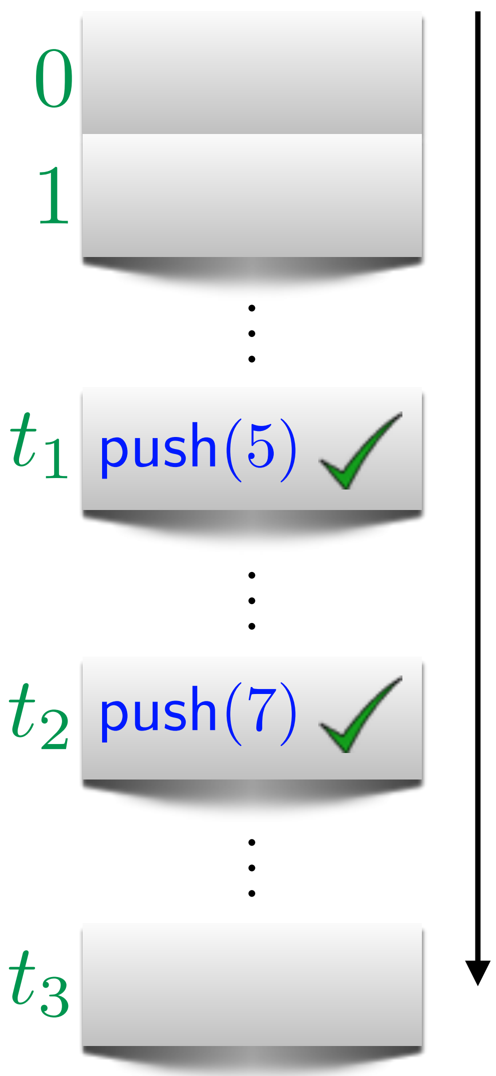

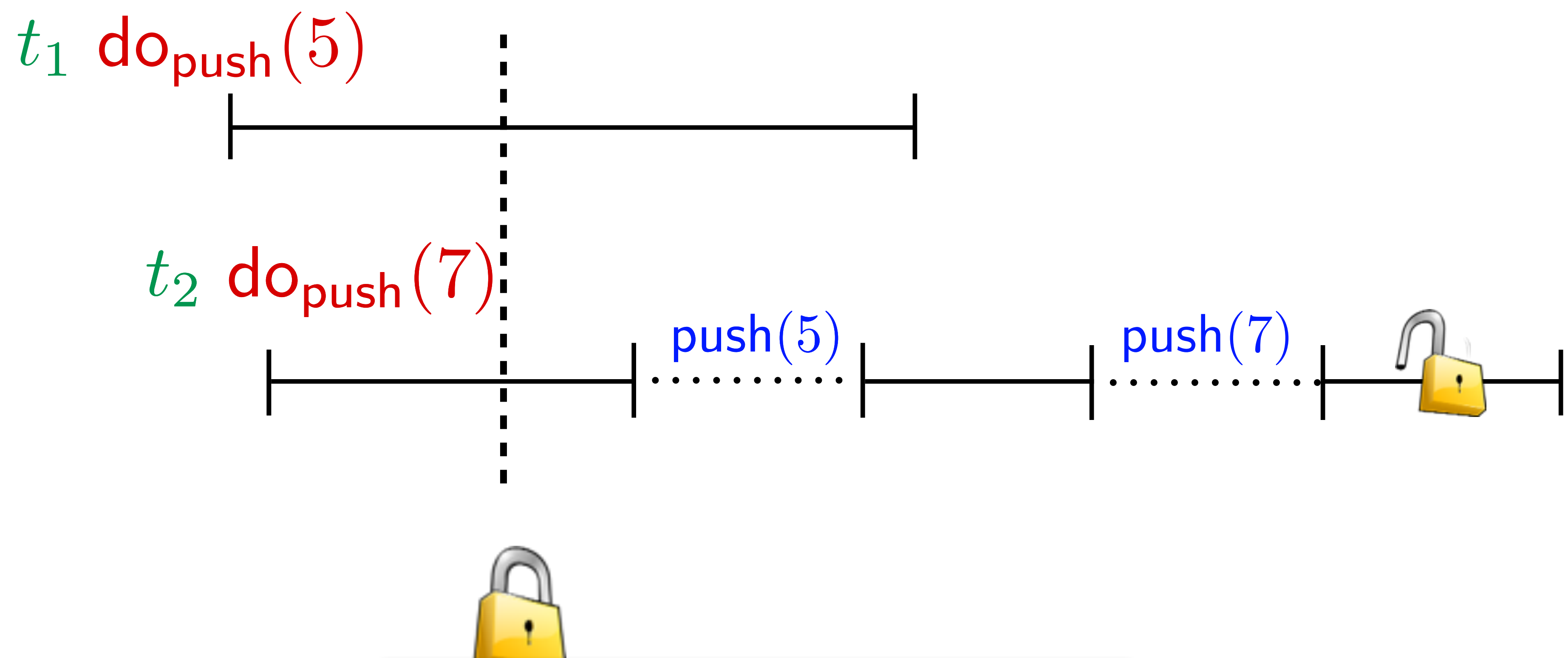

It appears to behave as the
naive
solution

- push , pop n... ct methods)
- Guaranteed to have correct behaviour for (**almost**) any implementation of abstract methods

Flat Combiners (Hendler et al, 2010)

Idea: let a single thread handle all requests

Stack provides methods push, pop to clients

$\mathsf{FC_{stack}}$: access to methods in Stack regulated by $\mathsf{do_{push}}$, $\mathsf{do_{pop}}$

0

1

⋮

t_1 push(5) ✓

⋮

t_2 push(7) ✓

⋮

t_3

t_1 $\mathsf{do_{push}}(5)$

t_2 $\mathsf{do_{push}}($

$\mathsf{h}(7)$

Theorem:

$$FC \sqsubseteq^{\mathcal{R}}_{\mathsf{obs}} FC_{\#}$$

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Conclusions

Future Research:

- Other applications (Joins, Iterators, …)
- STM and Transactional Boosting
- Linearisability for Higher Order Objects
- Connections with other forms of HO-reasoning (e.g. CaReSL)

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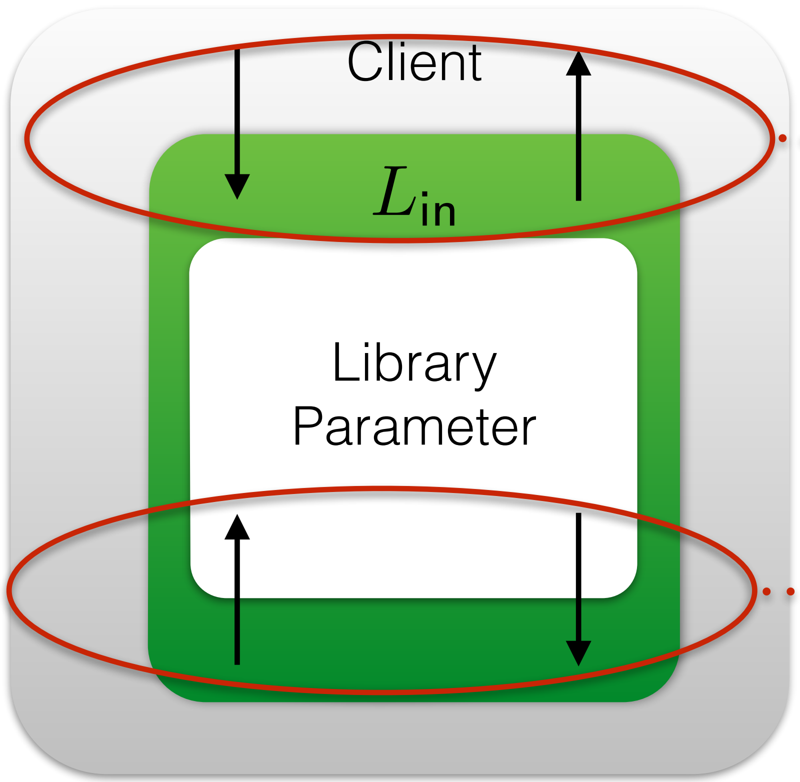
- Show that $h'' \in [\![FC_{\#}]\!]$

Contextuality of Up-to Linearisability

$$L_{\mathsf{in}} : M \rightarrow M'$$



rearranging this part of a history according to $\mathcal{R}$

causes this part of the history to be rearranged according to $\mathcal{G}$

$\mathcal{R}$ relates sequences of actions belonging to $M' \setminus M$

$\mathcal{G}$ relates sequences of actions belonging to M

$\binom{\mathcal{R}}{\mathcal{G}}$-closure: defines how closure properties of library parameters have to be changed when a inner library is instantiated

Contextuality of Up-to Linearisability

Theorem

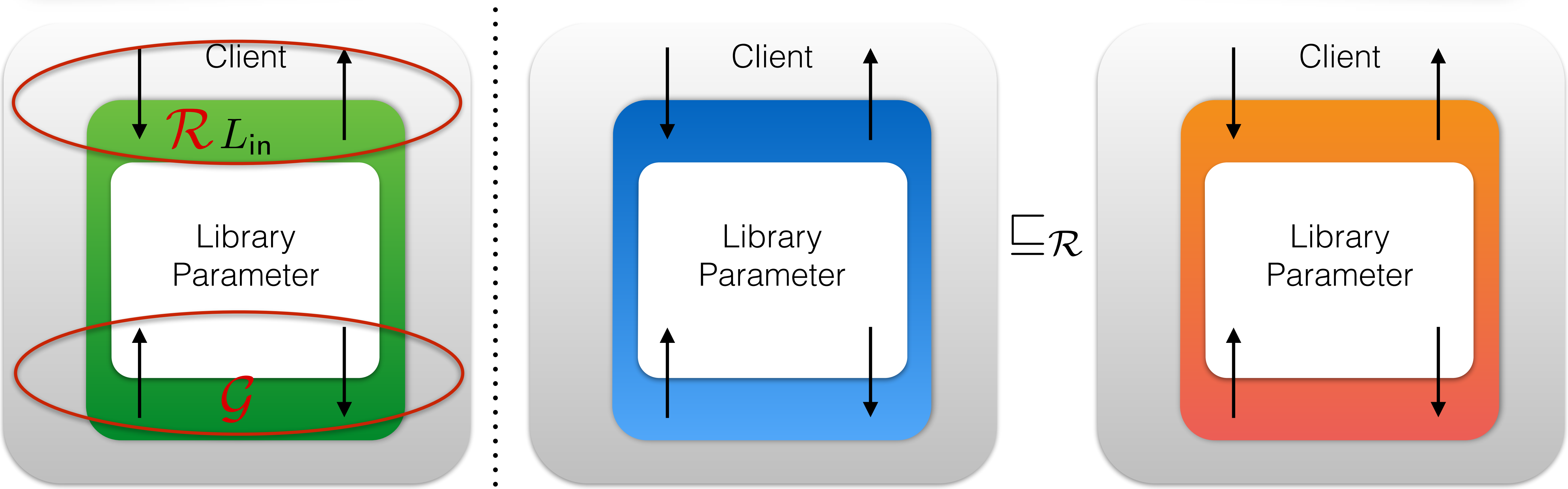
$L_1, L_2 : M' \to M''$

$L_1 \sqsubseteq_{\mathcal{R}} L_2$

$L_{\mathsf{in}} : M \to M'$

L_{in} is $\binom{\mathcal{R}}{\mathcal{G}}$-closed

$$\Longrightarrow$$



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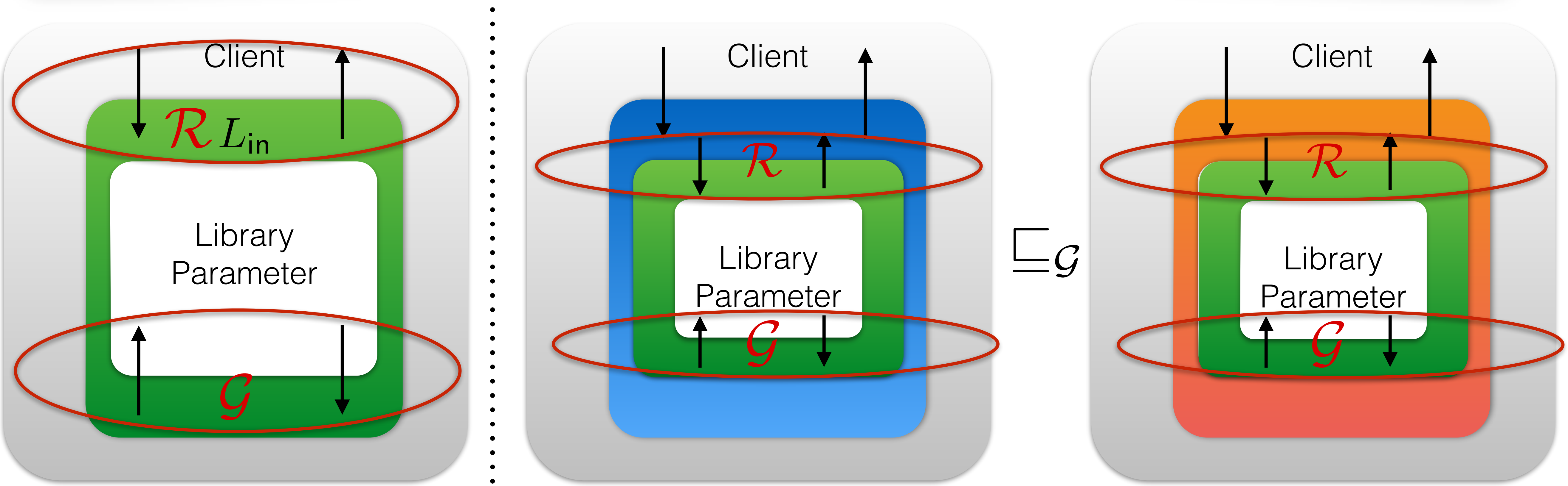
$$M' \cap M'' = \emptyset \;\wedge\; M \cap M' = \emptyset \implies (L_1 \circ L_{\text{in}}) \sqsubseteq_{\mathcal{G}} (L_2 \circ L_{\text{in}})$$



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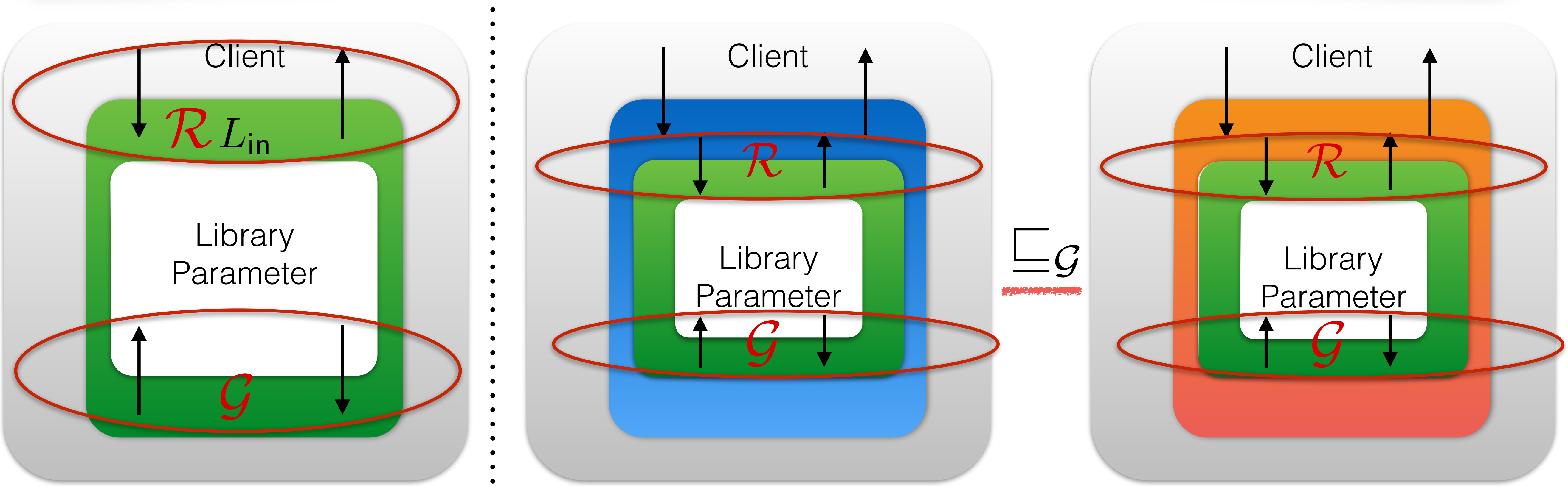

Remarks on Observational Refinement

Observational Refinement
=
May-Testing

General Linearisability
=
Histories Inclusion

Bonus Slide!

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Observational Refinement

=

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=

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So What About Must Testing?

- Gotsman et al. 2012: Liveness Preserving Atomicity Abstraction
  - Observational Refinement for Liveness Properties (1st order)
  - Sound Proof Technique via a Variant of Linearisability
    Main Observables: calls/returns, divergence, deadlocks
- It remains to be seen whether the results scale to 2nd order