Proving Linearizability Using Partial Orders

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In this talk

- Algorithms posing challenges for the linearization points method:
  - the Herlihy-Wing queue
  - the Optimistic Set / Lazy List
  - the Timestamp Queue

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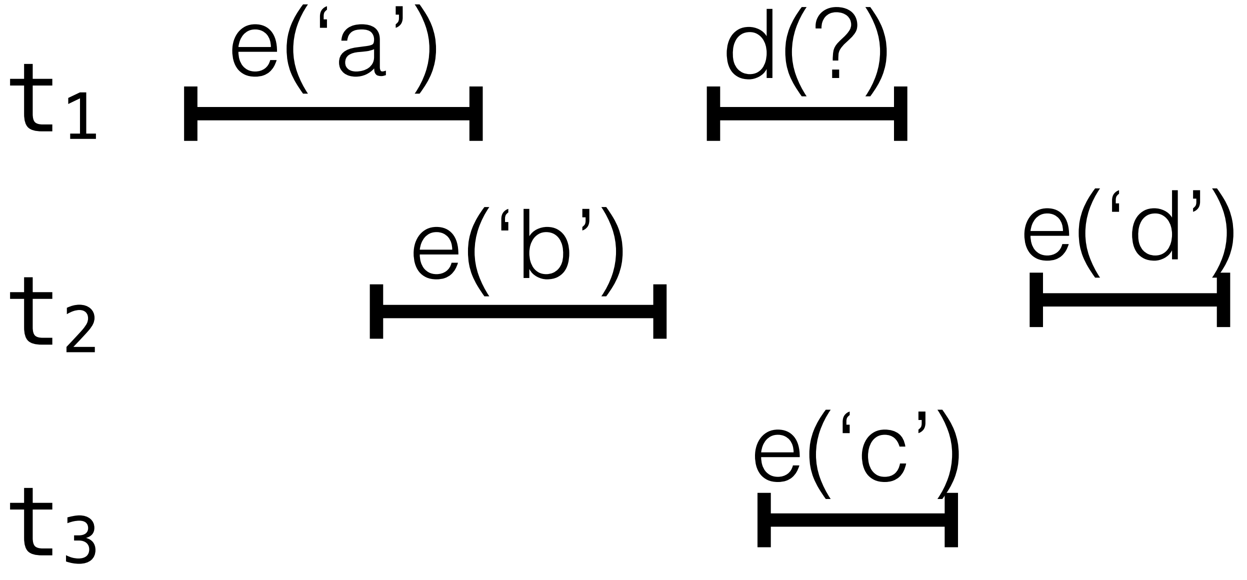
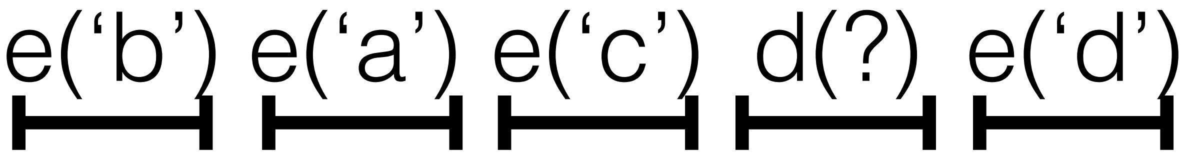
  - the Herlihy-Wing queue
  - the Optimistic Set / Lazy List
  - **the Timestamp Queue**

Linearizability

for every concrete history



find a linearization



Linearizability

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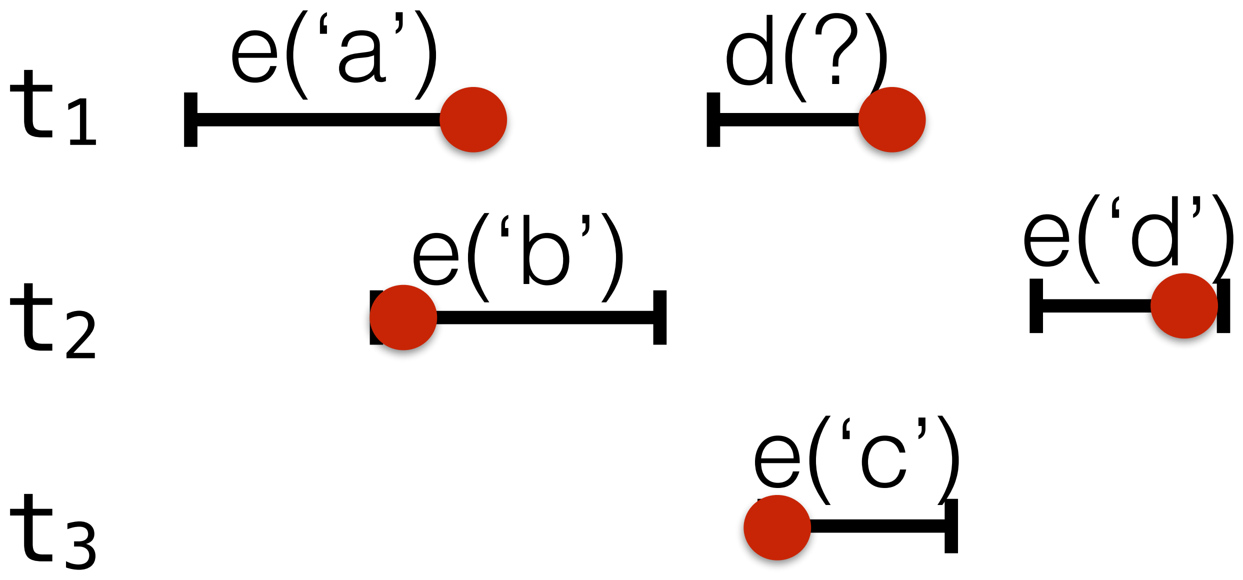

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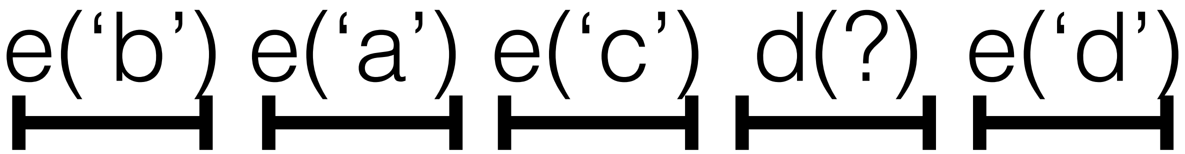
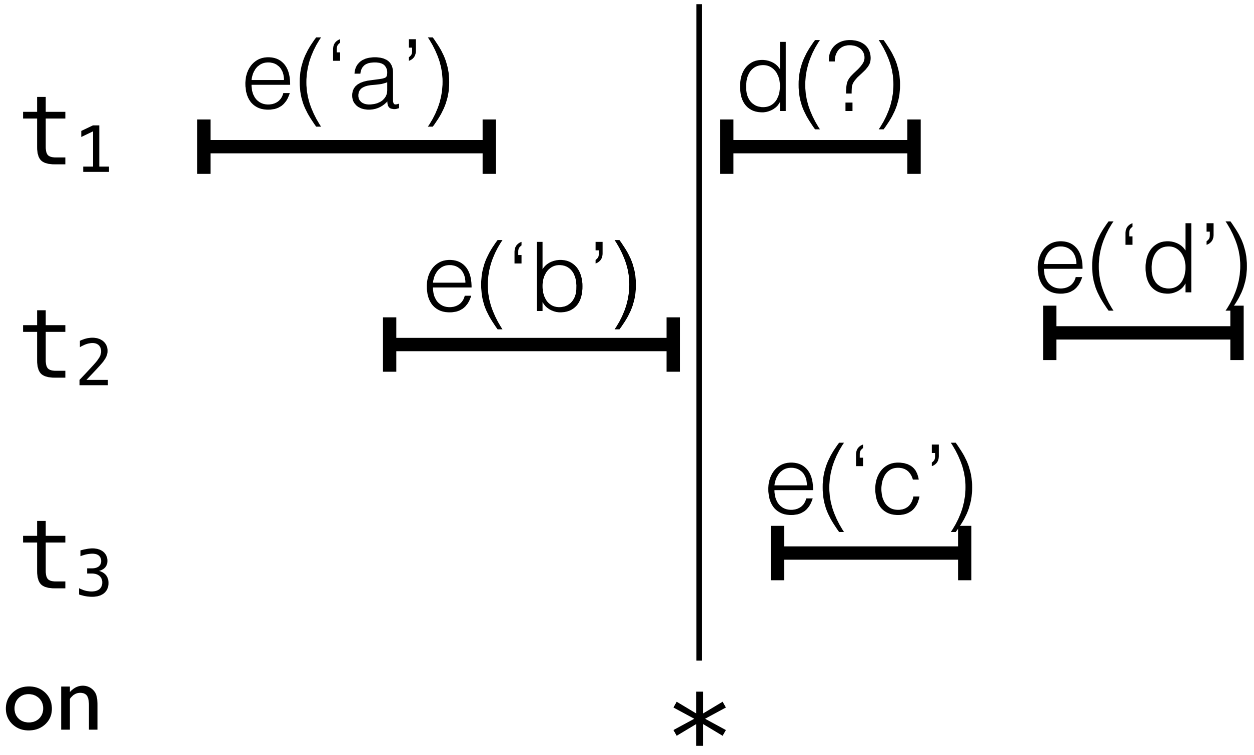


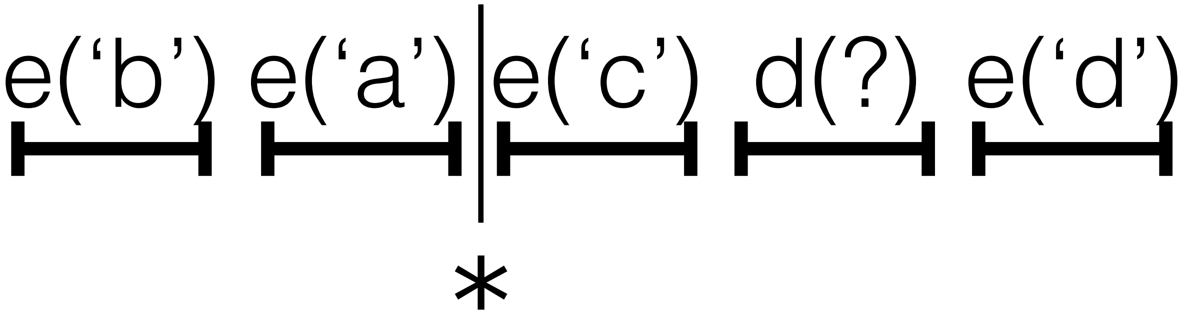
Standard proof technique: linearization points

Linearization points

for every concrete history

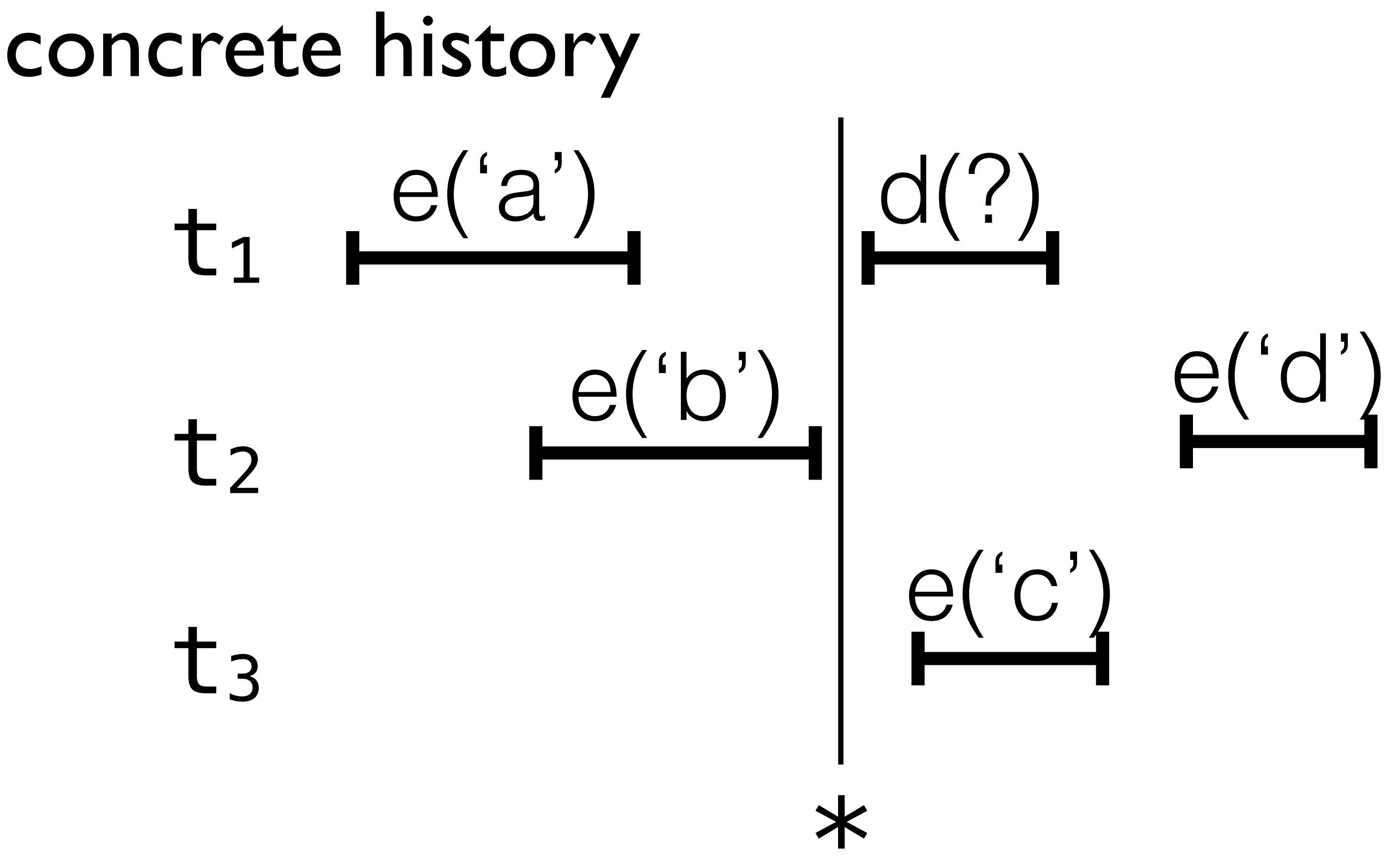


find a linearization



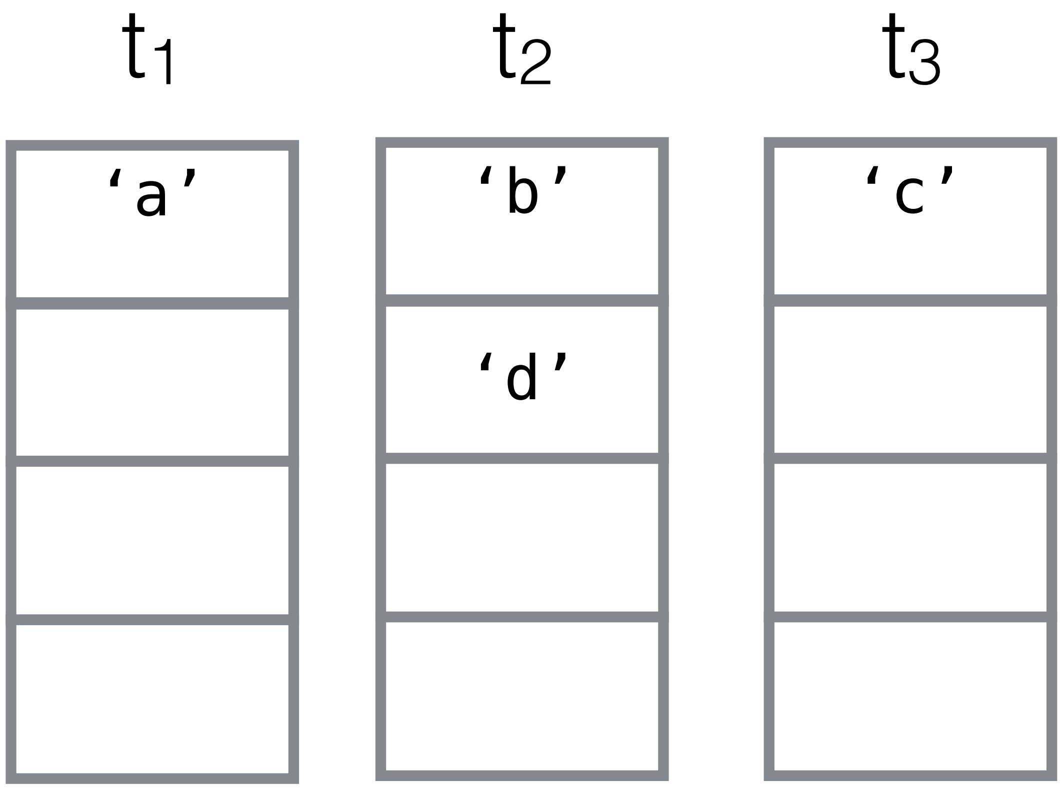
? = 'b'

Linearization points



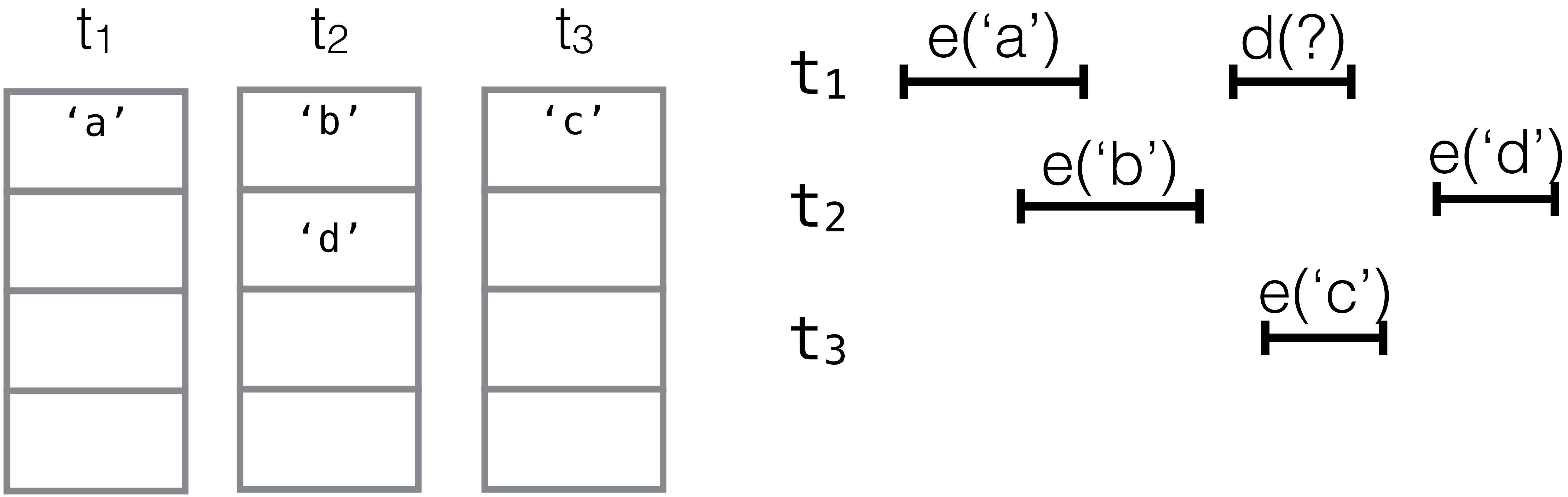
- by the moment (*) we still may not know where the linearization points of e('a') and e('b') are

The TS Queue



- each thread has its own pool
- pool is an abstract sequence
- (single producer multiple consumers)

The TS Queue



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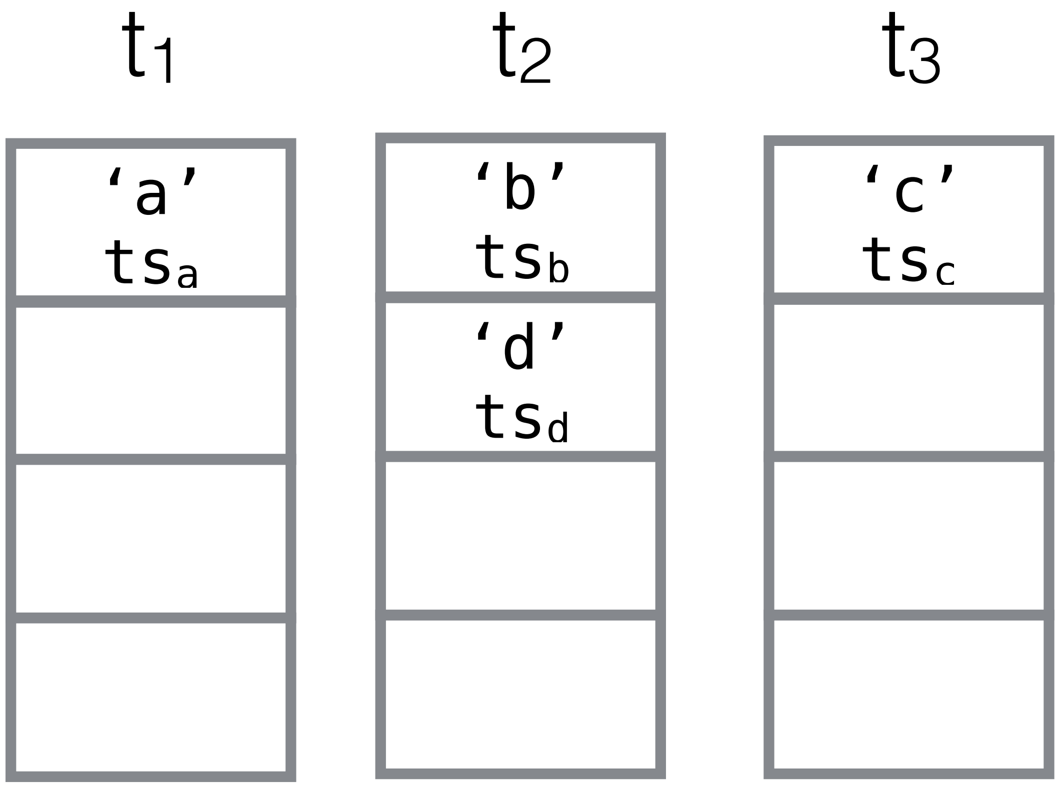
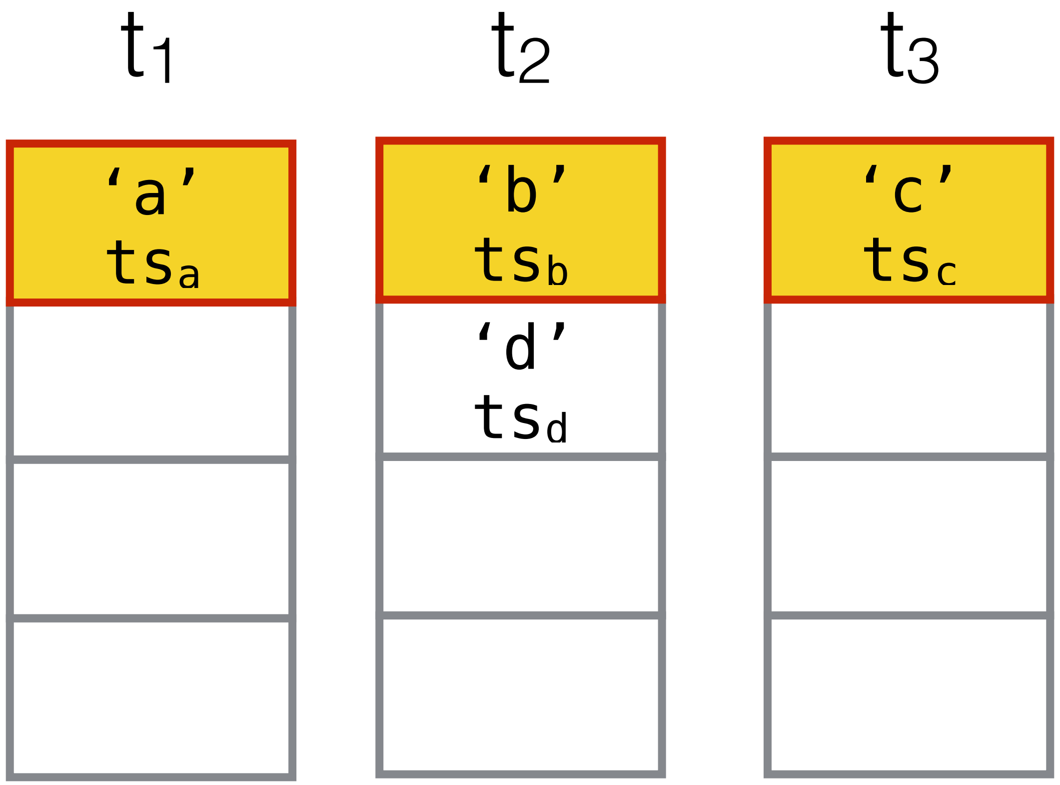


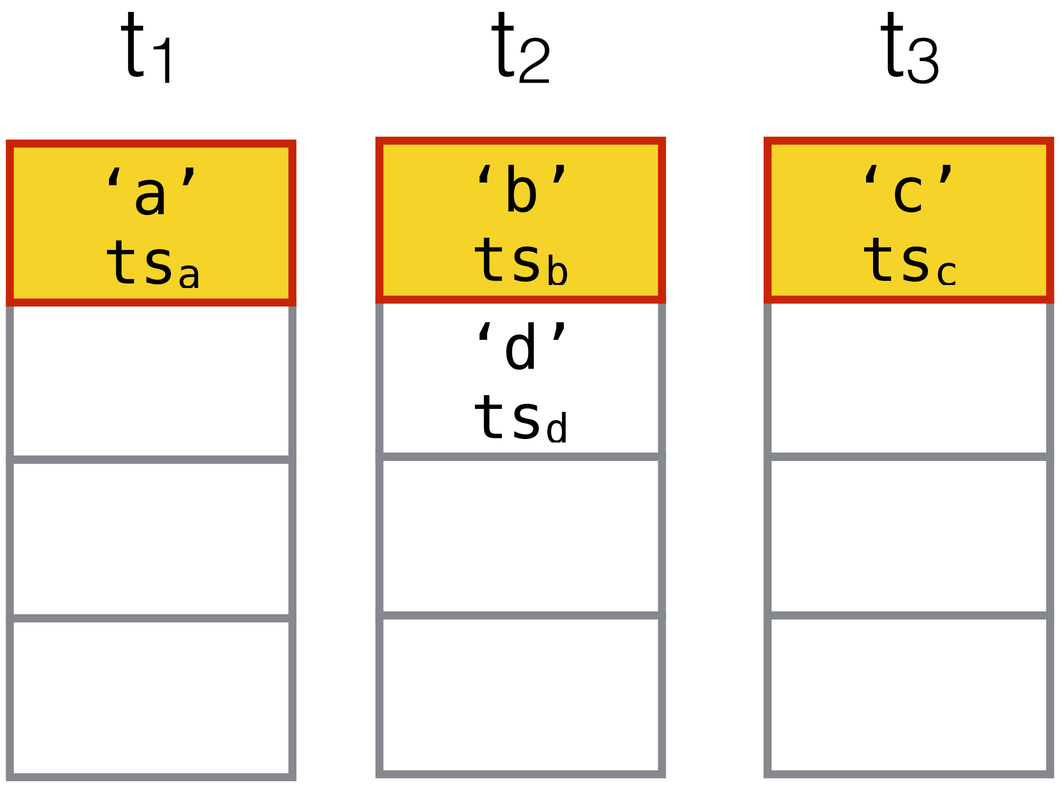
```
enqueue(Val v) {
  ts := newTimestamp();
  insert(this_thread, v, ts);
}
```

The TS Queue



```
Val dequeue() {
  do {
    for each pool do {
      get the front node;
      update the candidate
                for removal;
    }
    if (there is a candidate)
      try removing it and
      returning its value;
  } while (true);
}
```

The TS Queue



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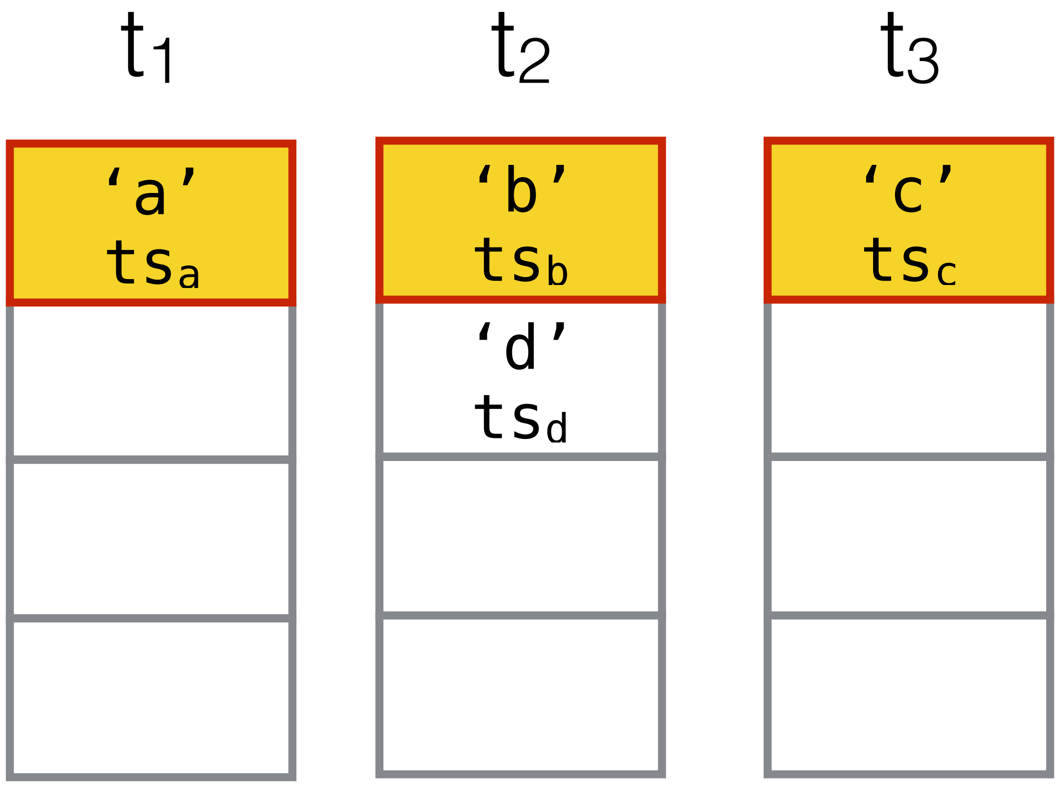

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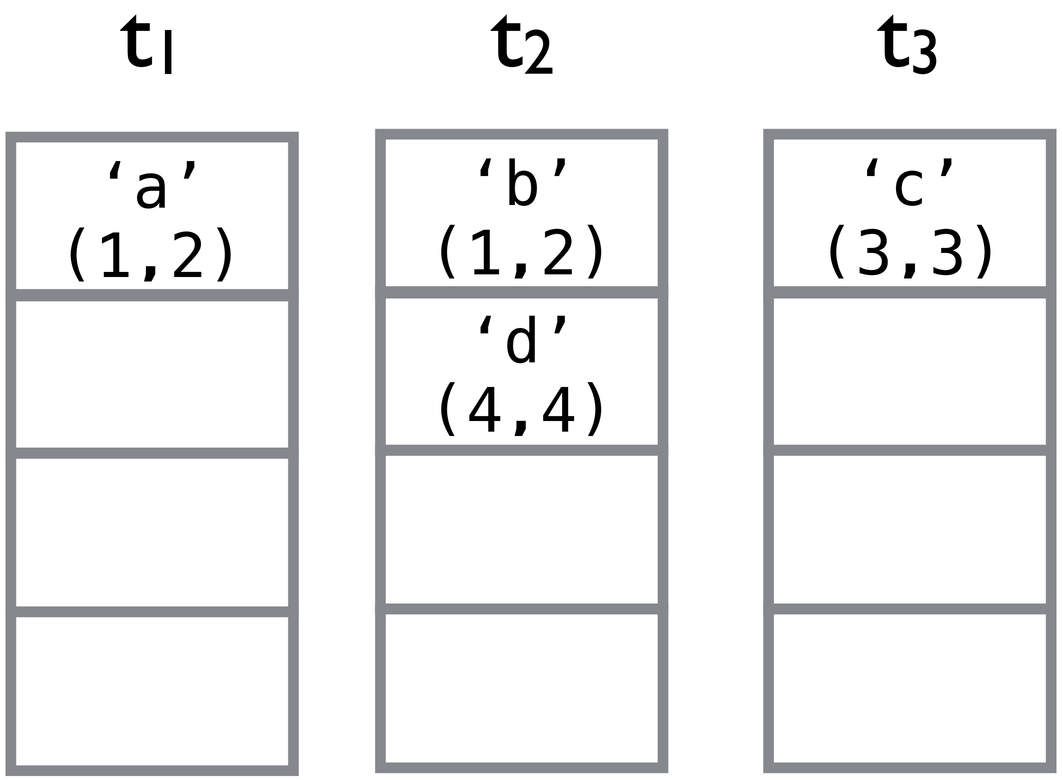
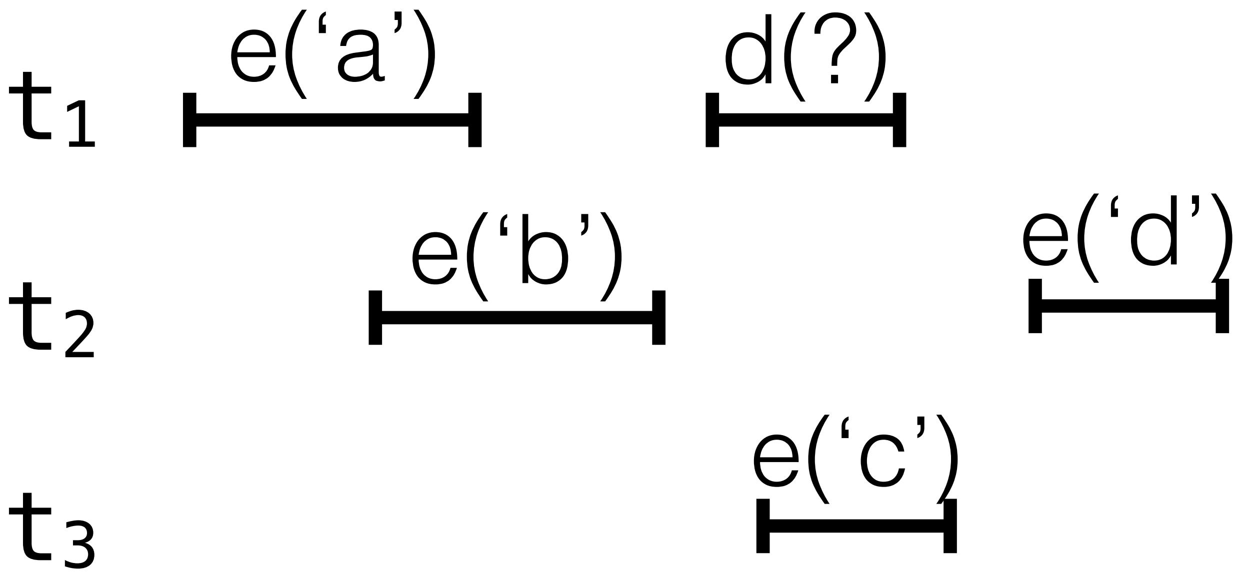
```

by choosing the smallest timestamp

Timestamps

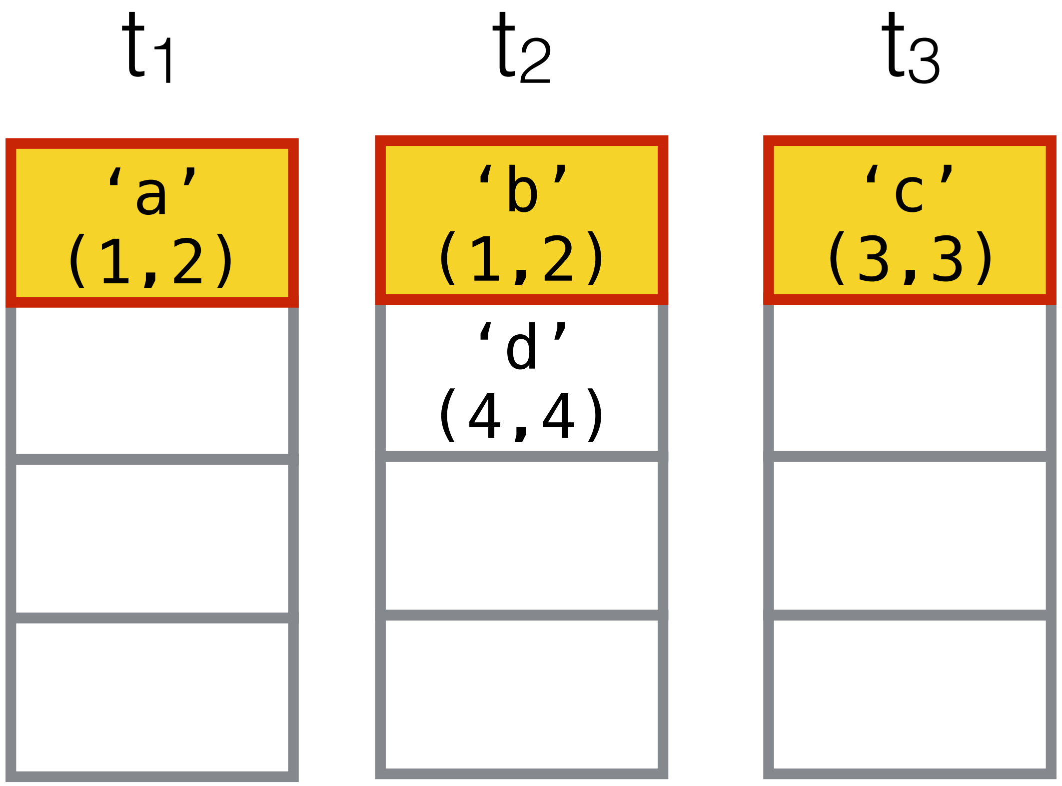


t_1	t_2	t_3
‘a’ (1,2)	‘b’ (1,2)	‘c’ (3,3)
	‘d’ (4,4)	



- the connection with the real-time order
- e1 —rt—> e2   ==>   timestamp(e1) < timestamp(e2)

Timestamps



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}
```
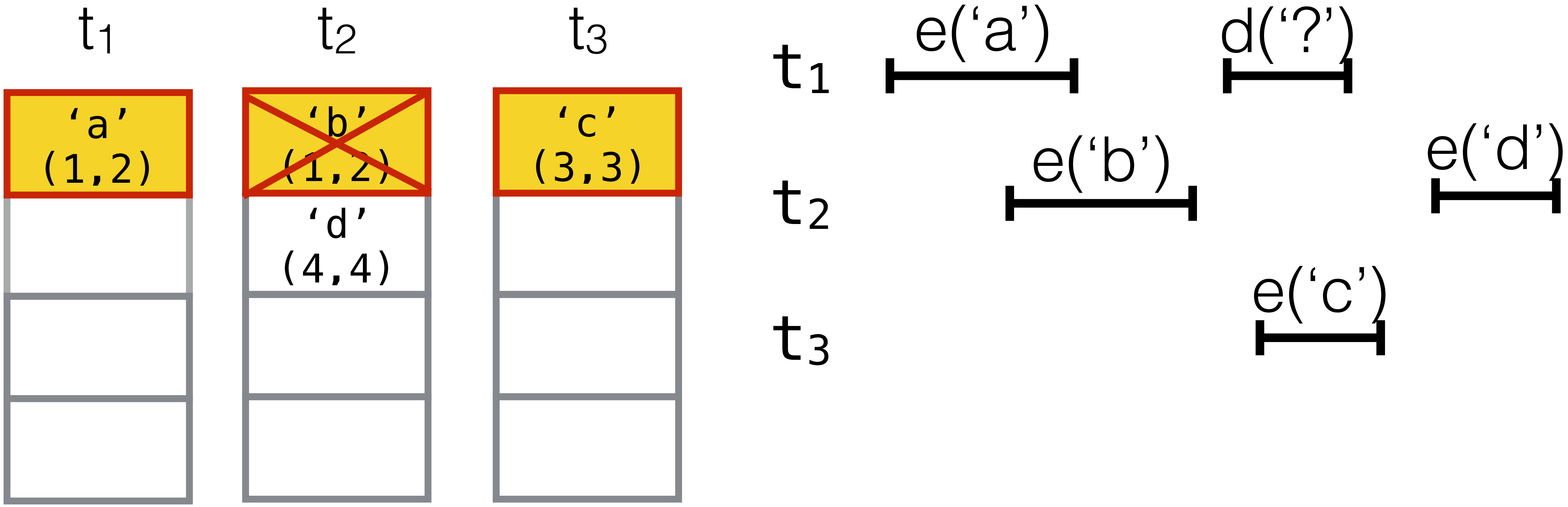

the smallest timestamp belongs to the “oldest” element

Timestamps



- problem: the order on enqueues may be impossible to determine till they are dequeued

Our technique

- We alter the standard proof technique to support late choice via partial orders
- Our proof technique is implemented in a logic, should be sound
- Examples: the TS queue, the Herlihy-Wing queue, the Optimistic Set

Abstract histories

- For each history of a data structure we construct a matching abstract history (instead of a linearization)
- An “abstract” history (E, R):
  - E — a set of events [eid: (tid, op, arg, rval)]
  - R — a partial order

Abstract histories

- For each history of a data structure we construct a **matching** abstract history (instead of a linearization)

1. the abstract history extends the real-time order
2. all linearizations meet the sequential spec

Abstract histories

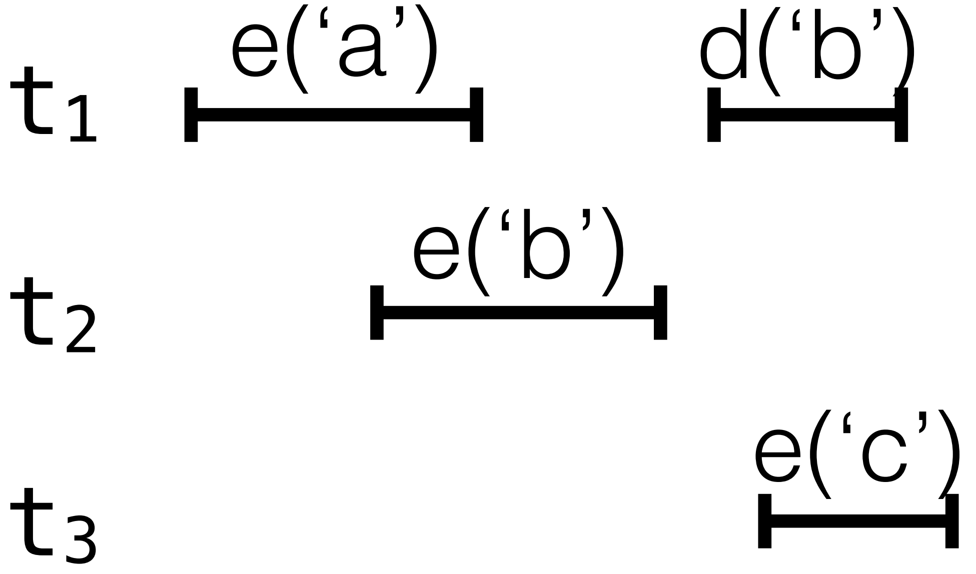
- We prove a number of invariant properties including:
  - all linearizations meet the sequential spec
  - for all enqueues e, e’ with values in the data structure:
    - e —> e’ ==> timestamp(e) < timestamp(e’)
  - …

Commitment points

- Abstract histories are constructed:
  - by adding new events
  - by adding more edges into the partial order
  - by assigning a return value to an event

Commitment points

- At the beginning of each operation, a fresh event and real-time order edges are added to the abstract history



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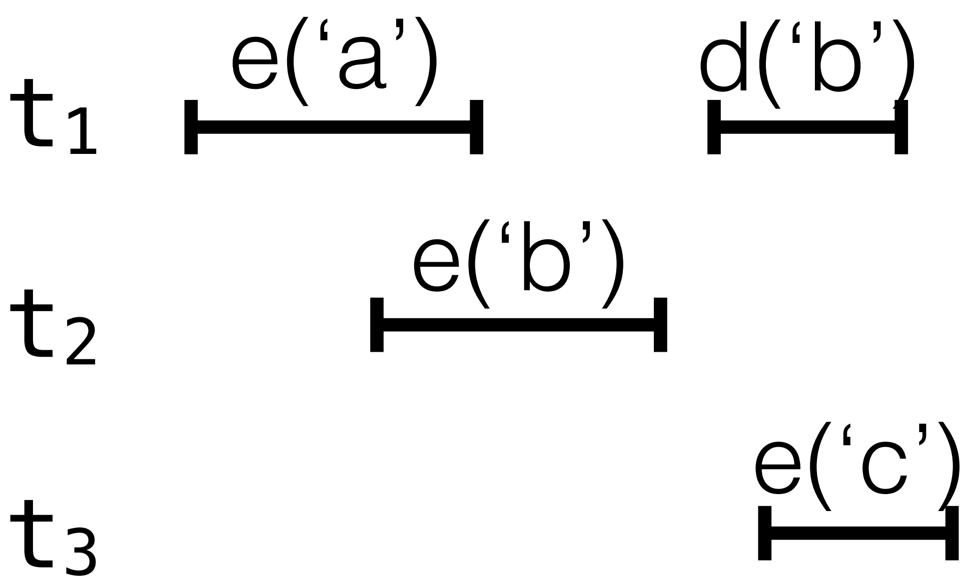
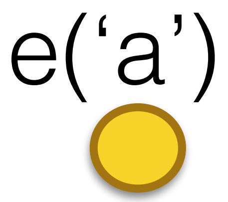
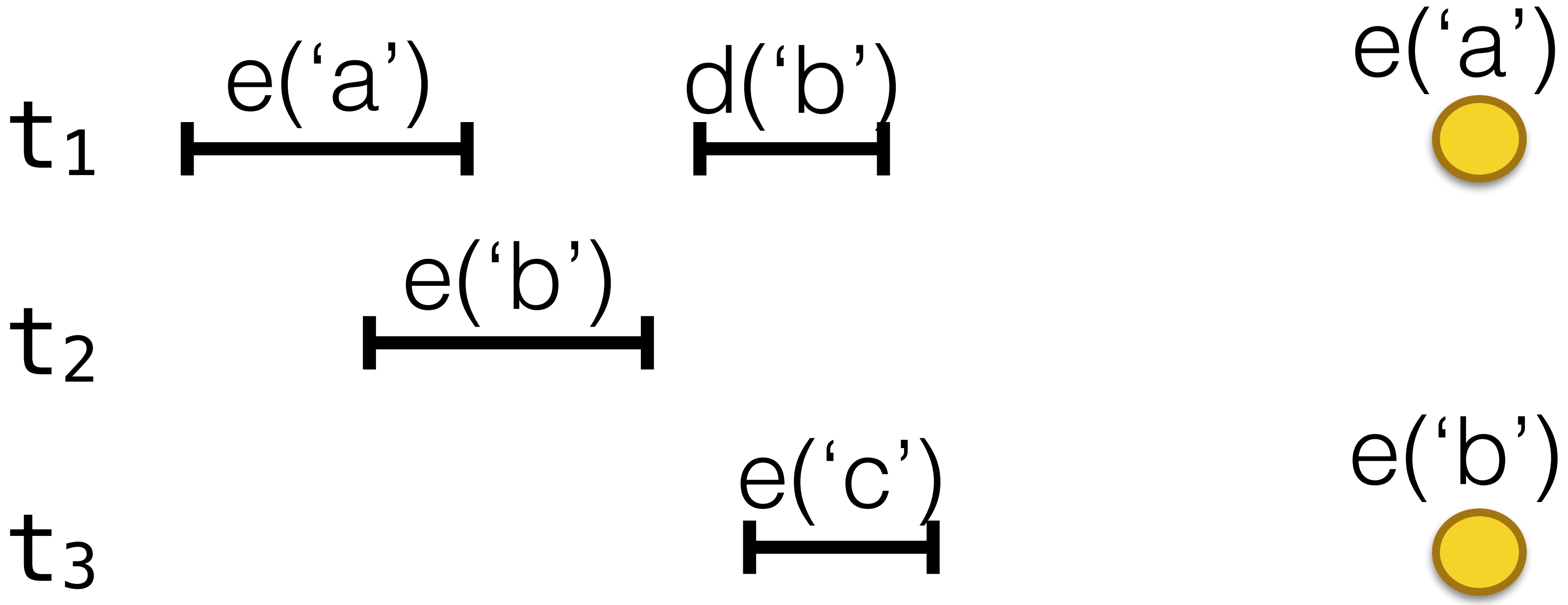




["e('a')" : t1, enq, 'a', TODO]

Commitment points

- At the beginning of each operation, a fresh event and real-time order edges are added to the abstract history



t_1 e(‘a’) d(‘b’)

t_2 e(‘b’)

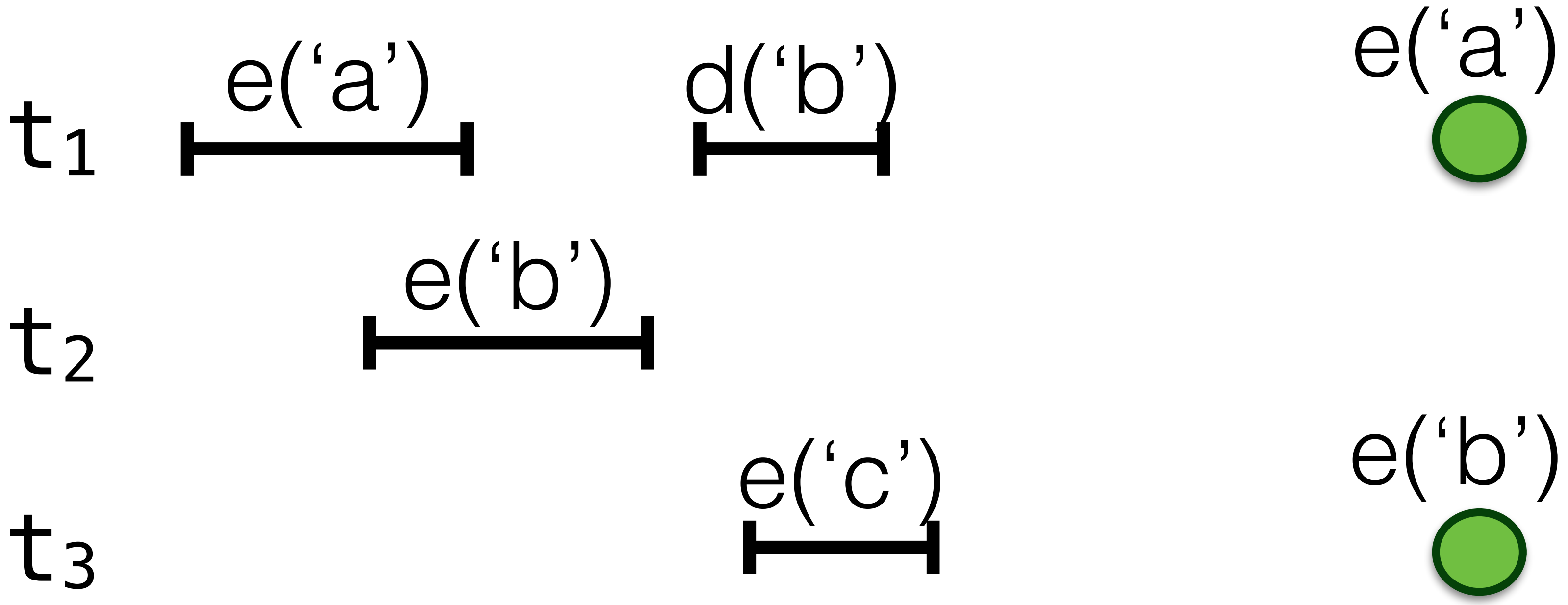
t_3 e(‘c’)

e(‘a’)

e(‘b’)

Commitment points

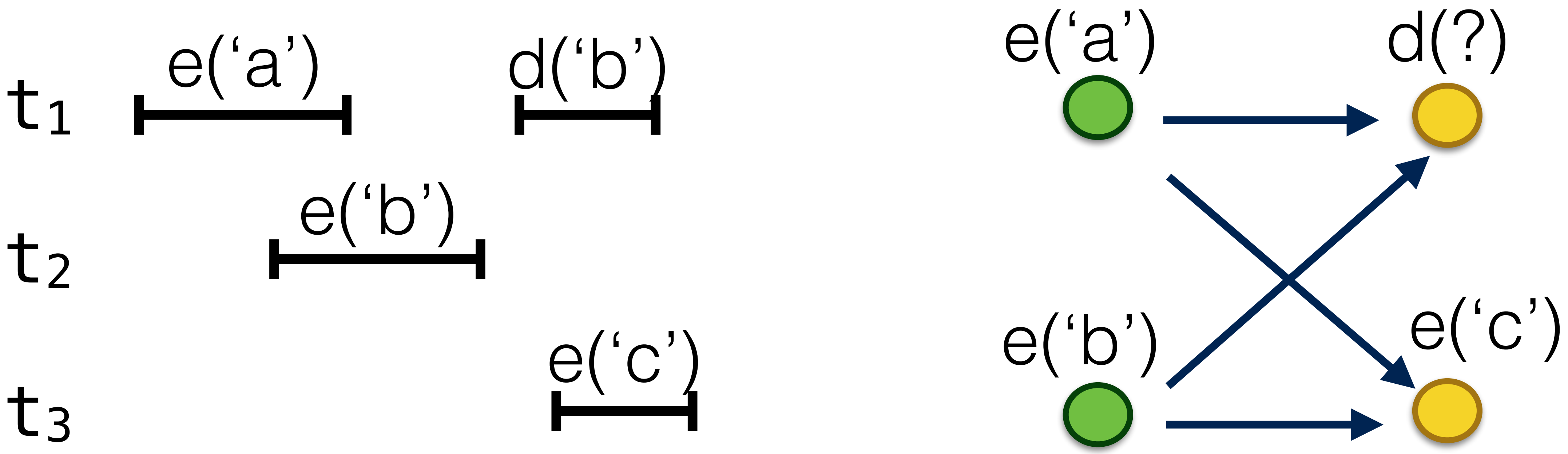
- At the beginning of each operation, a fresh event and real-time order edges are added to the abstract history



- events get completed by the end of the operations

Commitment points

- At the beginning of each operation, a fresh event and real-time order edges are added to the abstract history



- edges from the completed events are added

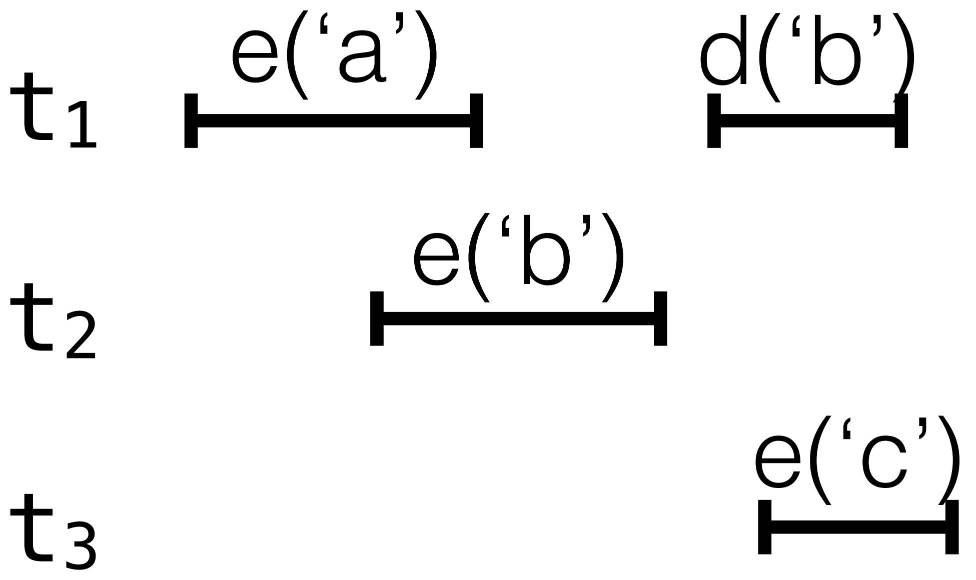
Enqueue’s commitment point

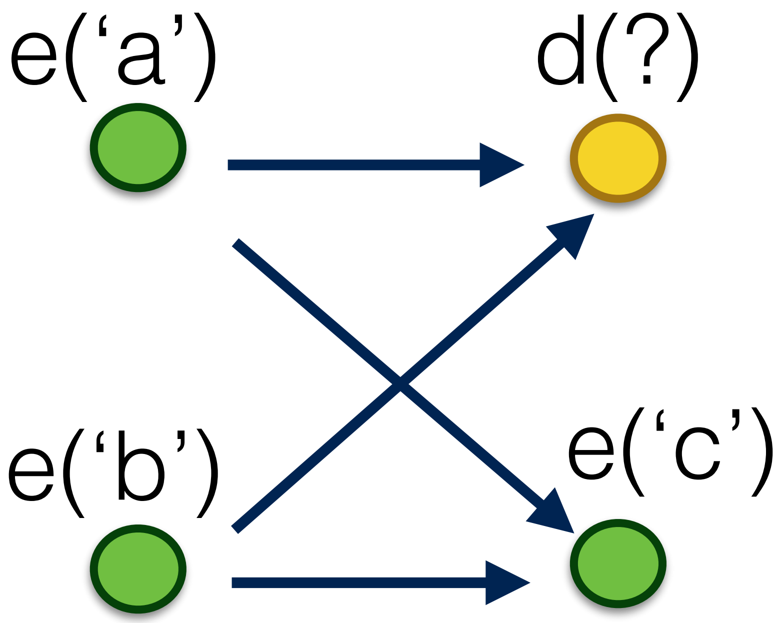
```
enqueue(Val v) {
  ts := newTimestamp();
  atomic {
    insert(this_thread, v, ts);
    E(this_event).rval := DONE;
    G[this_event] := ts;
  }
}
```

- Ghost state G is a map from events and timestamps
- Helps to establish a bijection between events and elements of the data structure

Commitment points

- The late choice is resolved by a dequeue with the FIFO policy in mind

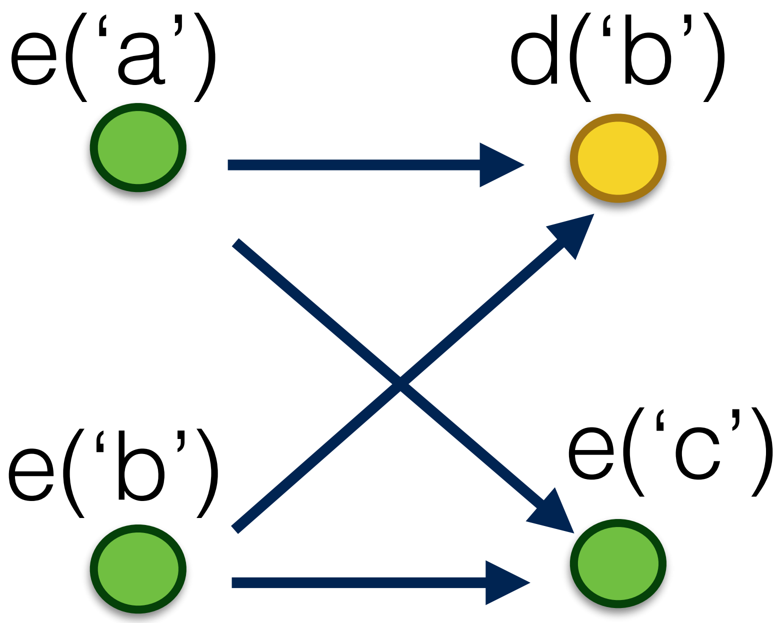




Commitment points

- The late choice is resolved by a dequeue with the FIFO policy in mind

1. e('b') e('a') d('b') e('c')
2. e('b') e('a') e('c') d('b')
3. e('a') e('b') d('b') e('c')
4. e('a') e('b') e('c') d('b')



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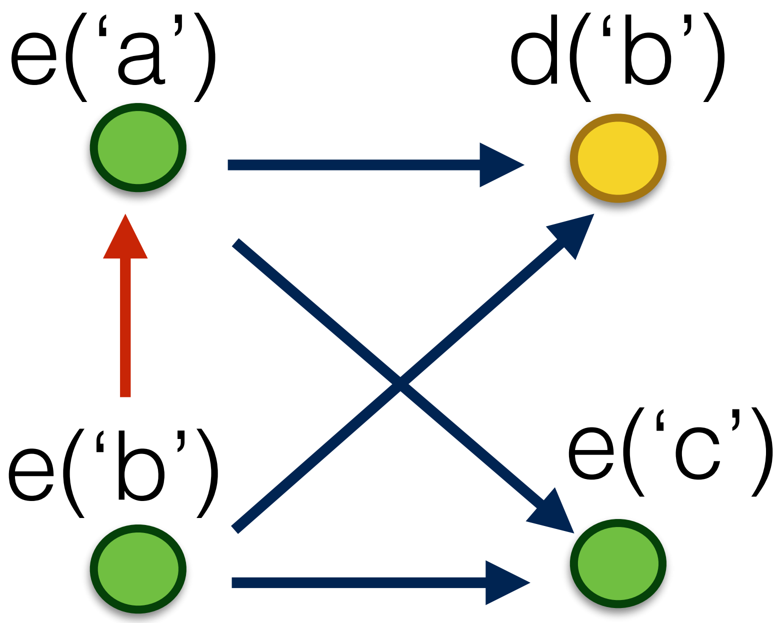
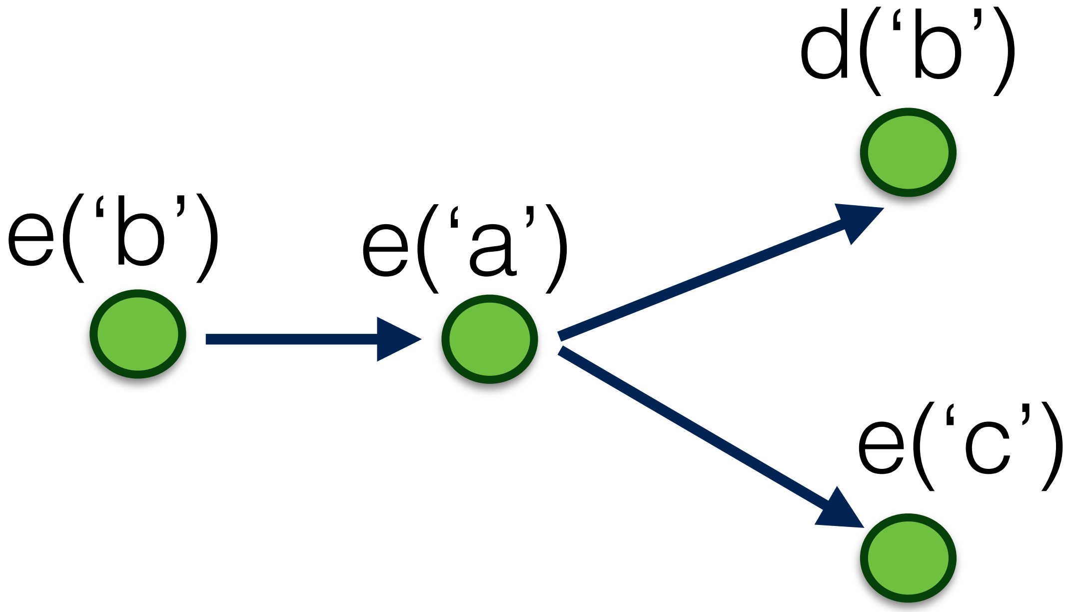


Commitment points

- The late choice is resolved by a dequeue with the FIFO policy in mind

1. e(‘b’) e(‘a’) d(‘b’) e(‘c’)
2. e(‘b’) e(‘a’) e(‘c’) d(‘b’)



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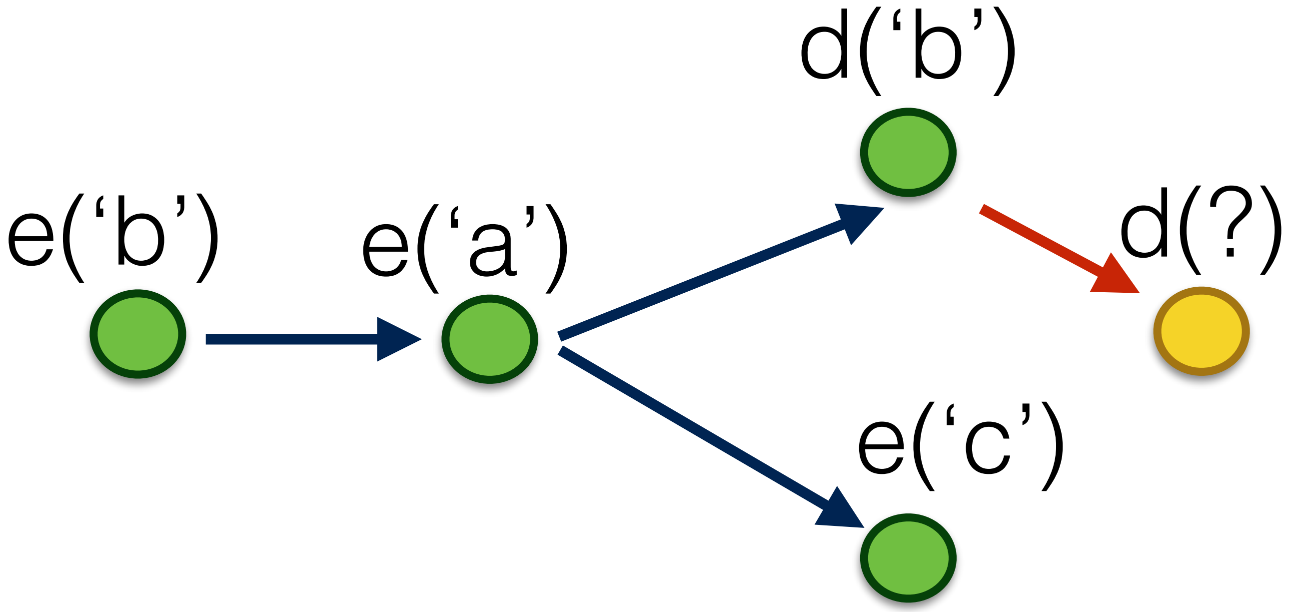

Dequeue’s commitment point

```
if (there is a candidate enq) {
  res := try removing enq;
  if (res != FAIL) {
    E(this_event).rval := res;
    R := (R
     U {(enq, this_event)}
     U {(enq, e’) | the value of e’ is in queue}
     U {(deq, d’} | d’ is an uncompleted dequeue})+;
 }
}
```

Acyclicity

- Need to prove that ordering by commitment points is possible without breaking acyclicity
    - Loop invariant — implies that the candidate for removal is the minimal in the partial order
    - Invariant properties (what holds by construction of the partial order)

The proof

- For each history of a data structure we construct a matching abstract history (instead of a linearization)
  - preserving the real-time order
  - ensuring that all linearizations meet the sequential specification
  - constructed with commitment points
- Partiality of the order enables delaying the choice of a linearization order

Conclusions

- The technique for proving linearizability of algorithms that are challenging with the linearization points method
- Examples: the TS queue, the Herlihy-Wing queue, the Optimistic Set
- Does not work for the TS stack