Towards formal verification of imperative concurrent data structures

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- Imperative programs

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data structures
(heap)

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$$P_1 \parallel \cdots \parallel P_n \models \varphi$$

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Regional Logic { data structures (heap)

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$\}$ LTL ($\square, \diamond, \bigcirc, \mathcal{U}$)

$P_1 \parallel \cdots \parallel P_n$ → Regional Logic { data structures (heap)

$\models$ → Verification Diagram

Reasoning about the heap

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- Separation Logic
    - Hoare logic extension to reason about shared mutable data structure

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Reasoning about the heap

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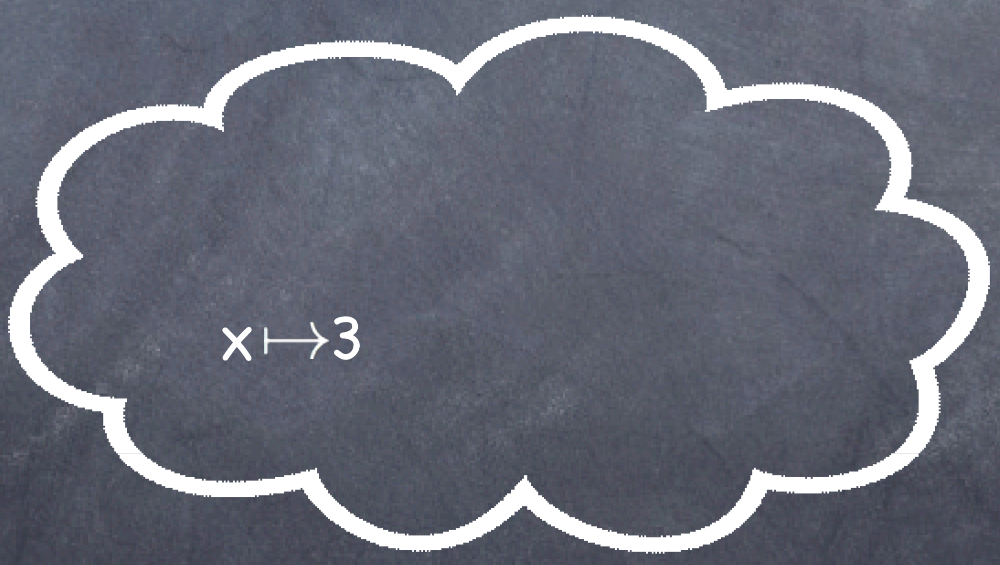

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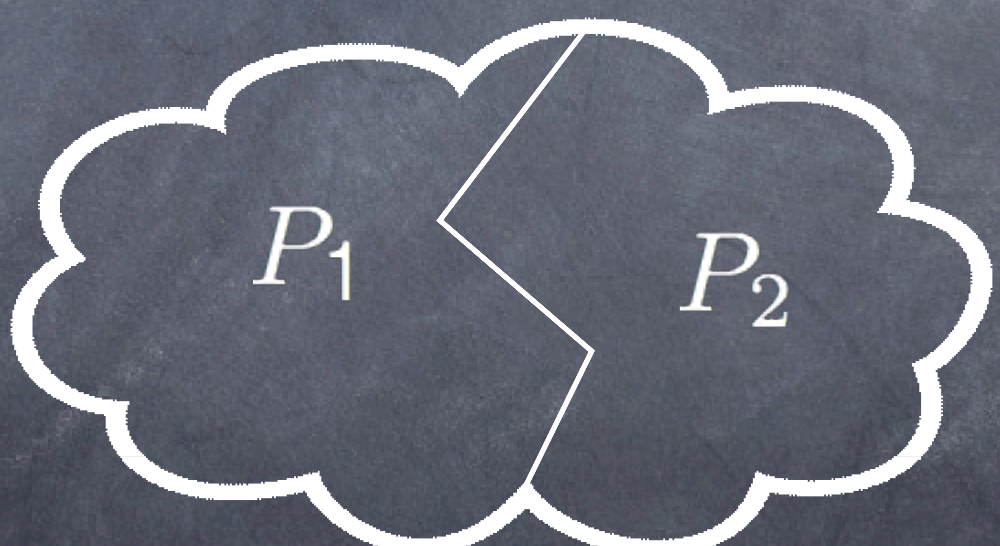

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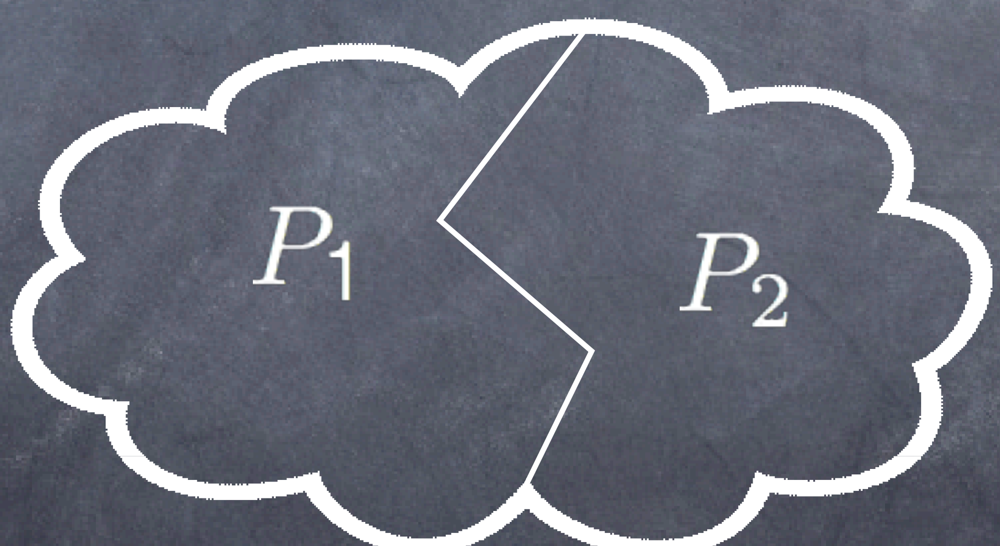

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    $\forall x : K \in R \mid P$

Verification Diagrams

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$$\begin{array}{c} P \models \varphi \\ \downarrow \\ \Psi \end{array}$$

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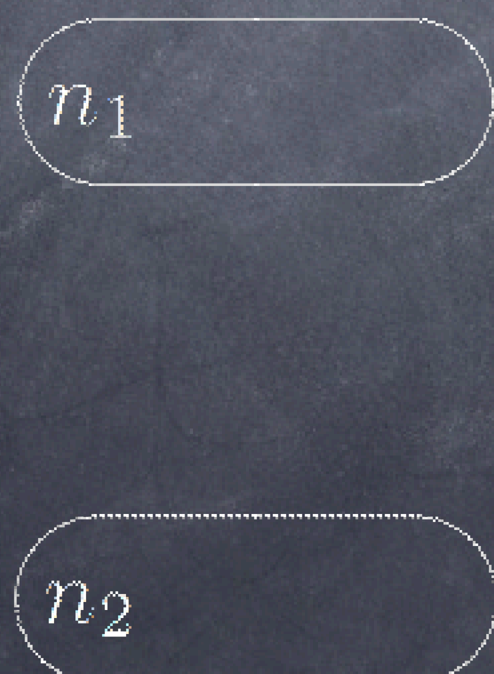

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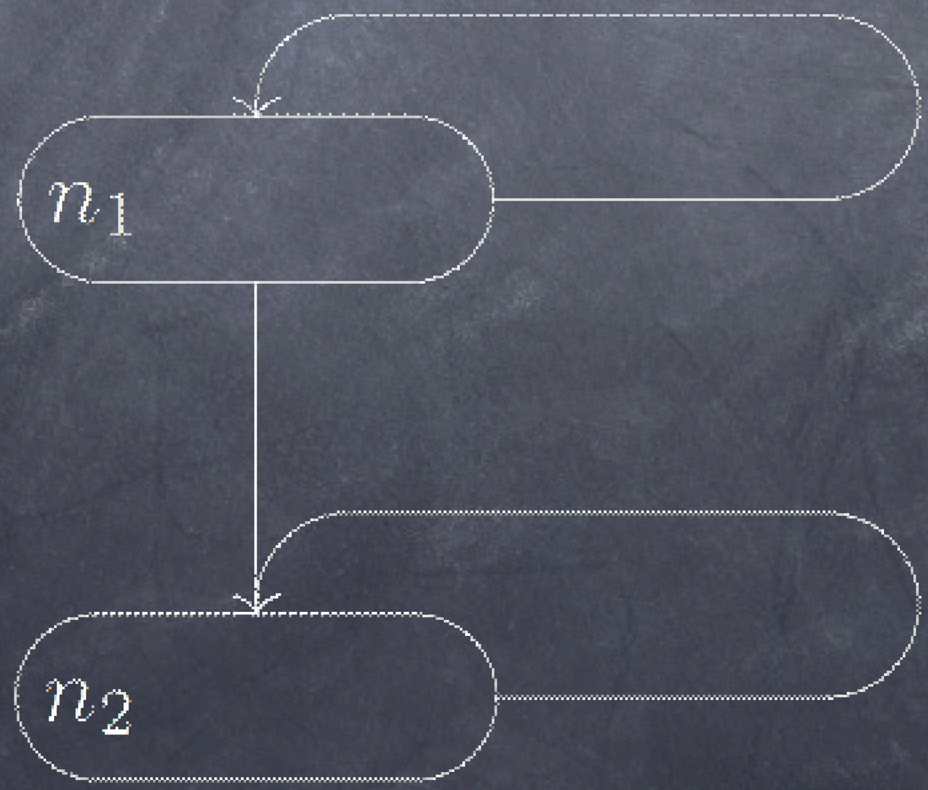

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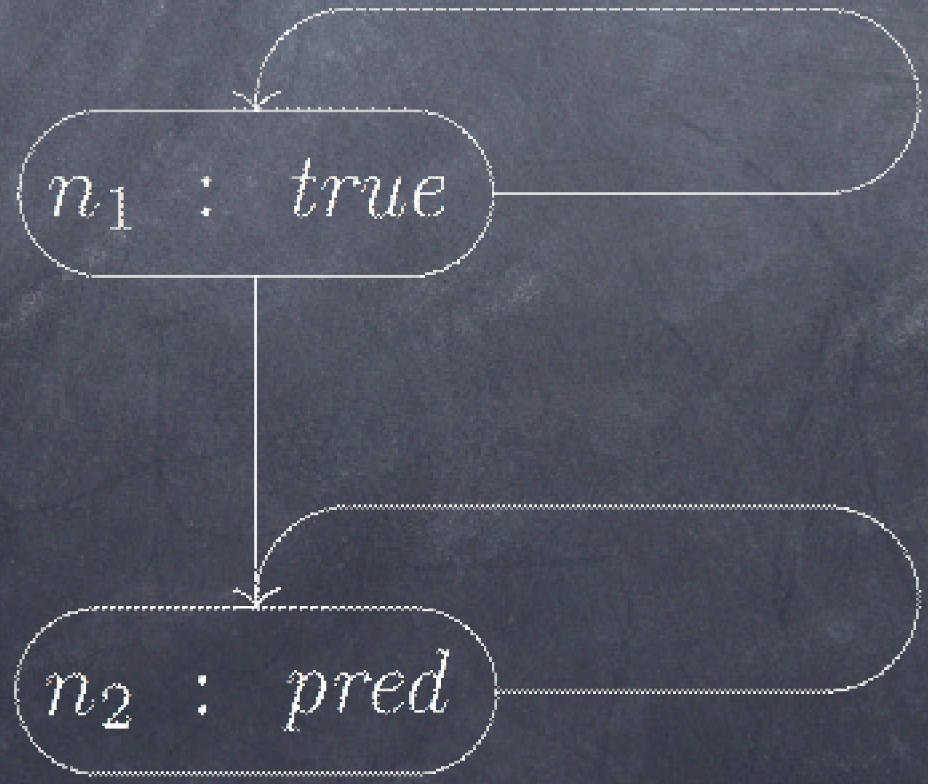

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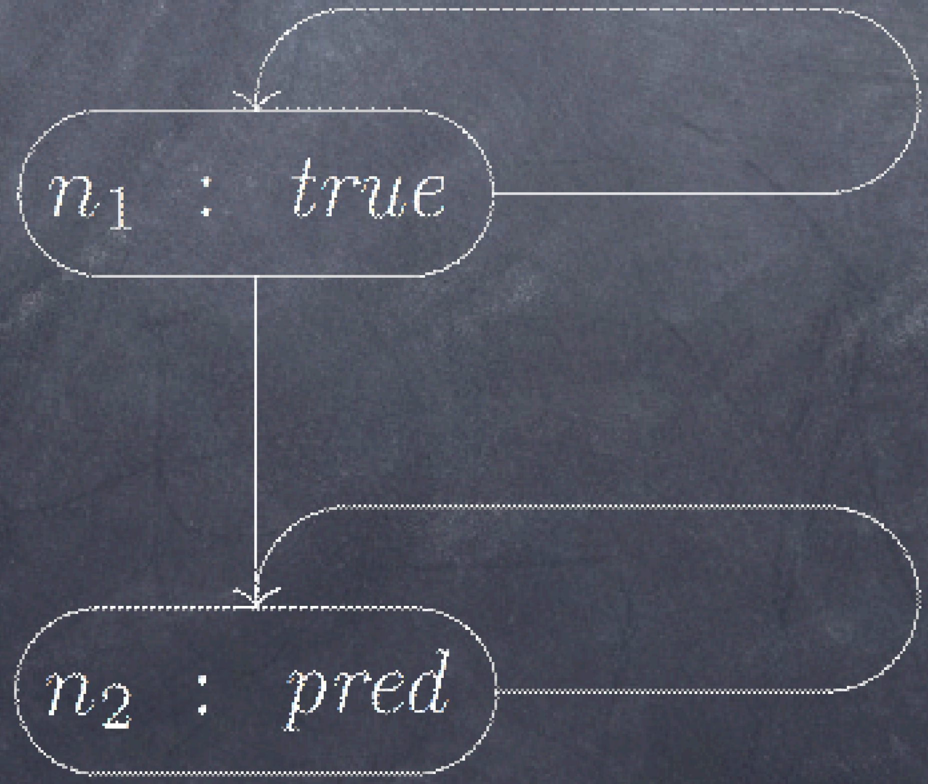

$$\Diamond \Box pred$$

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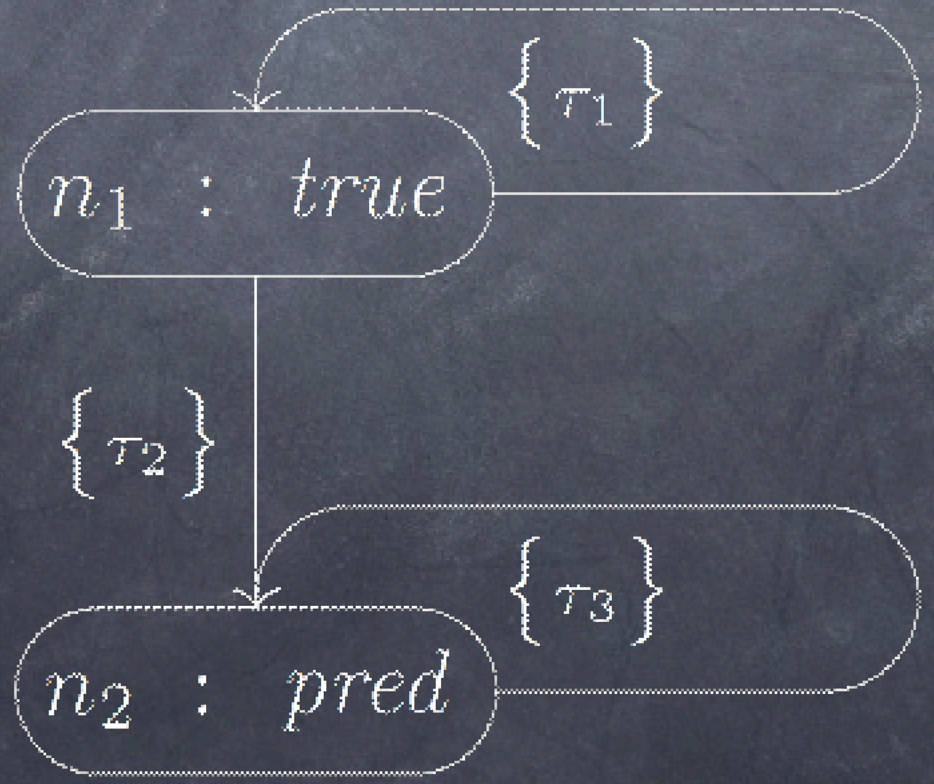

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φ

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Most General Client [N]

(extended with GV)

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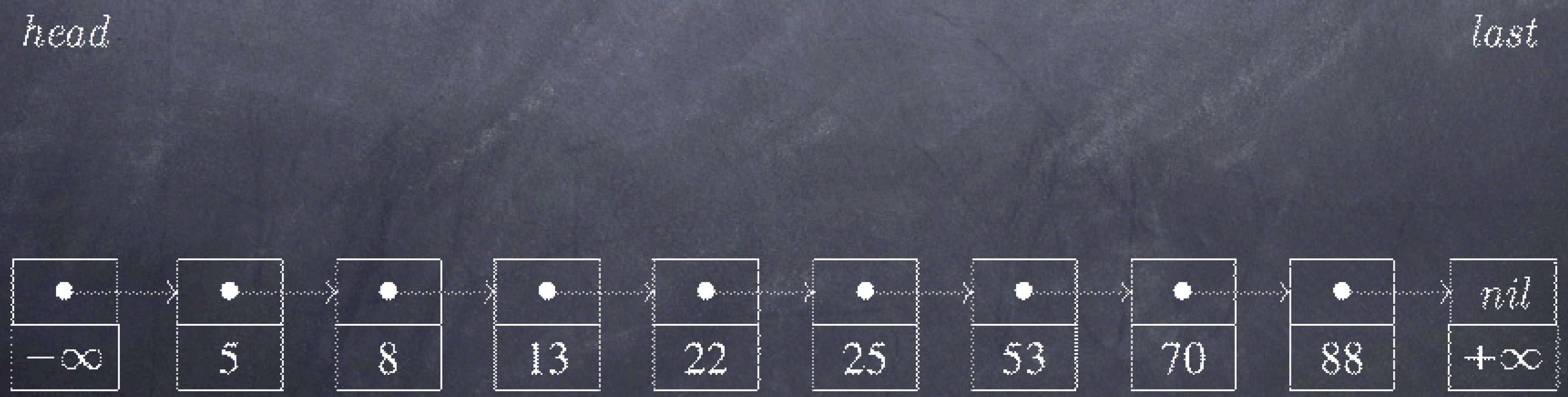

Skiplists

- Sorted list of elements
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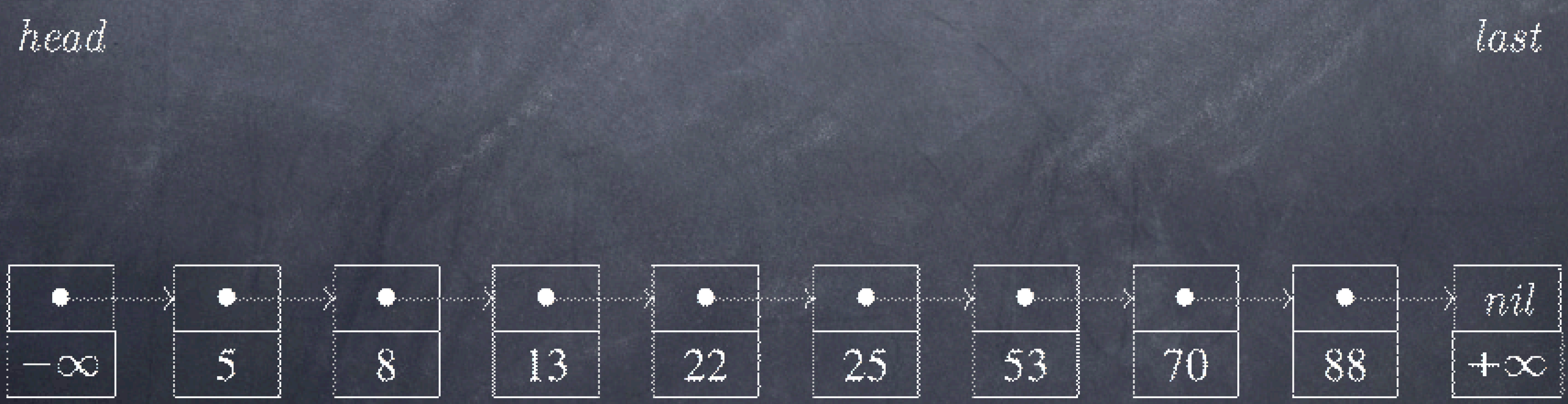

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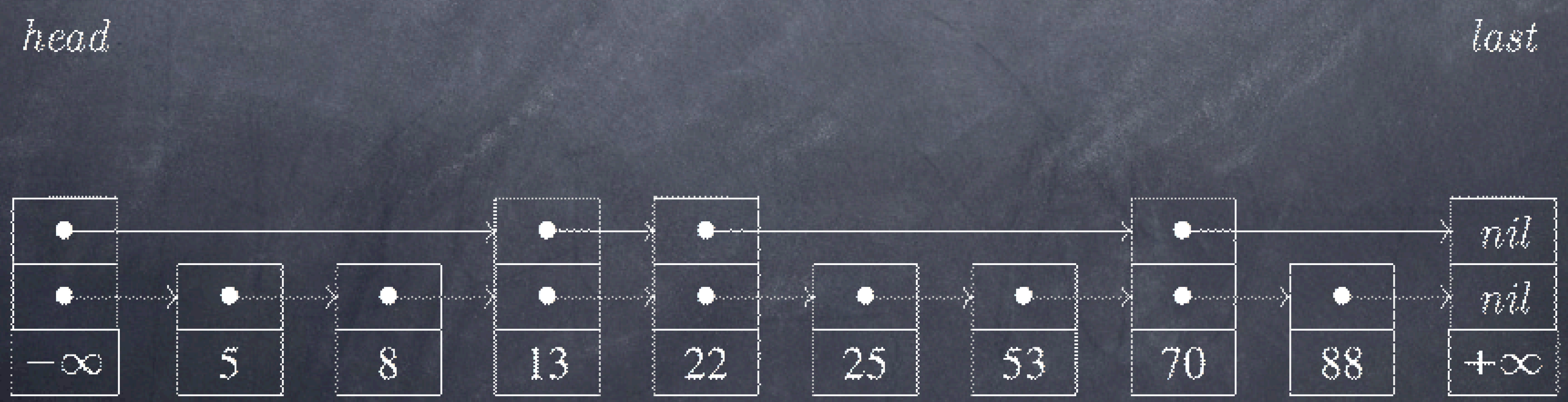

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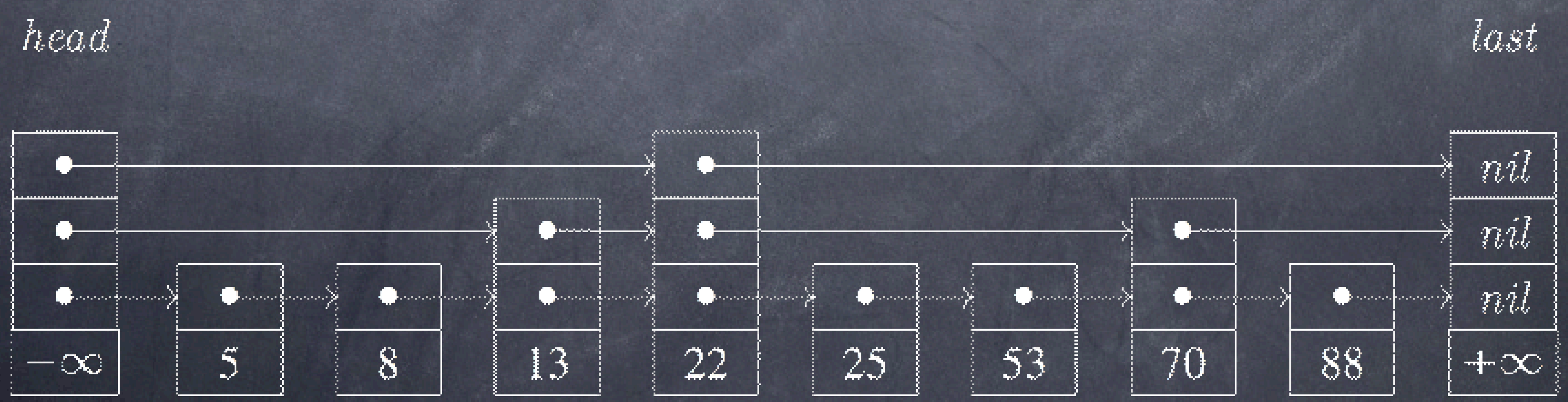

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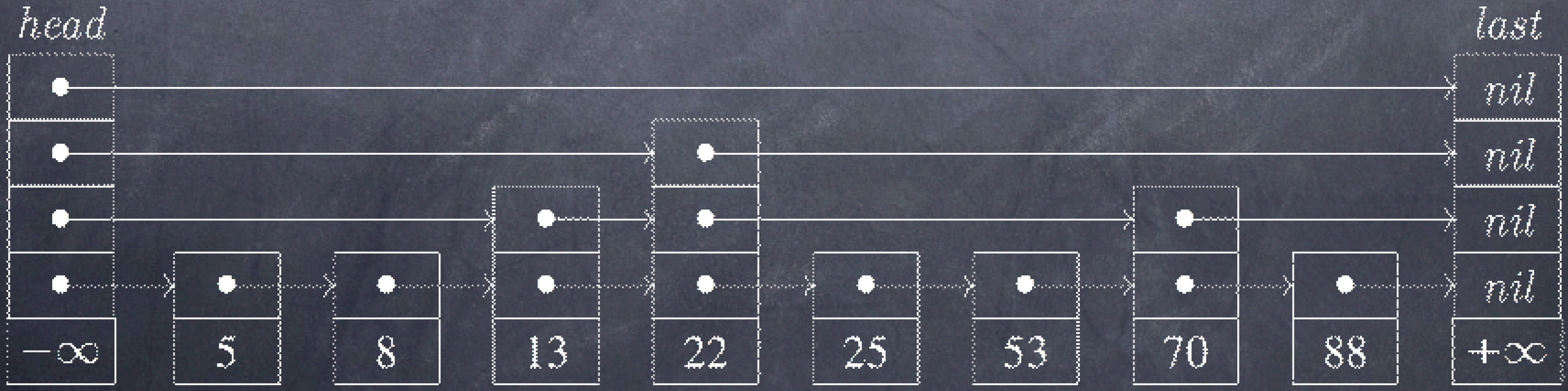

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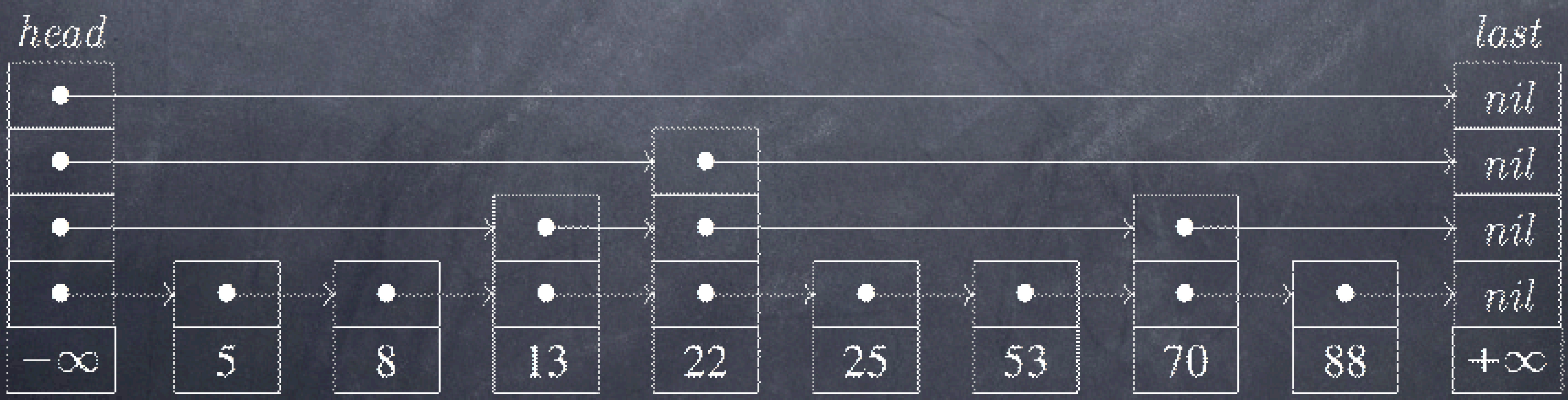

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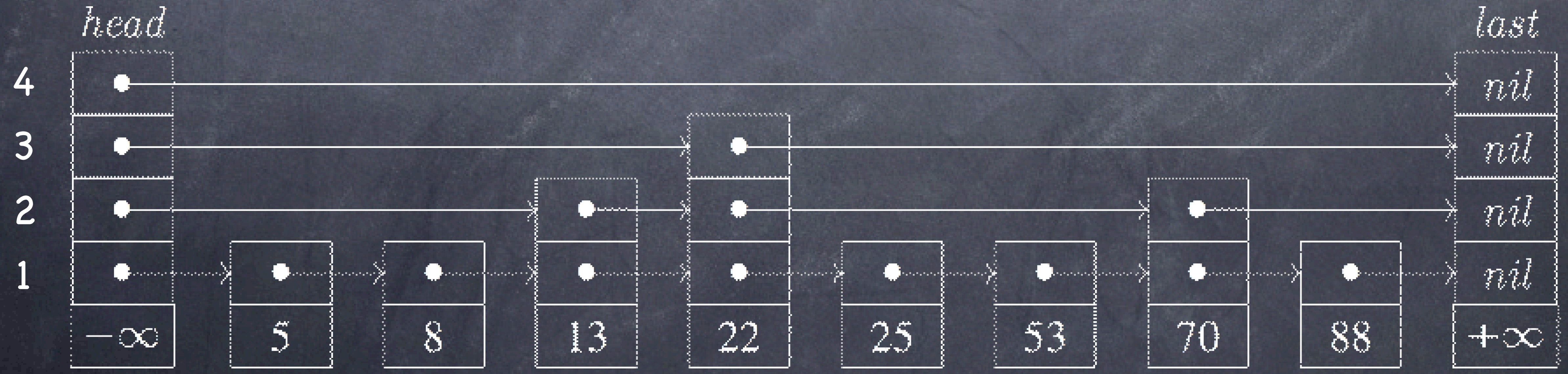

Fine-grained lock-coupling concurrent skiplists

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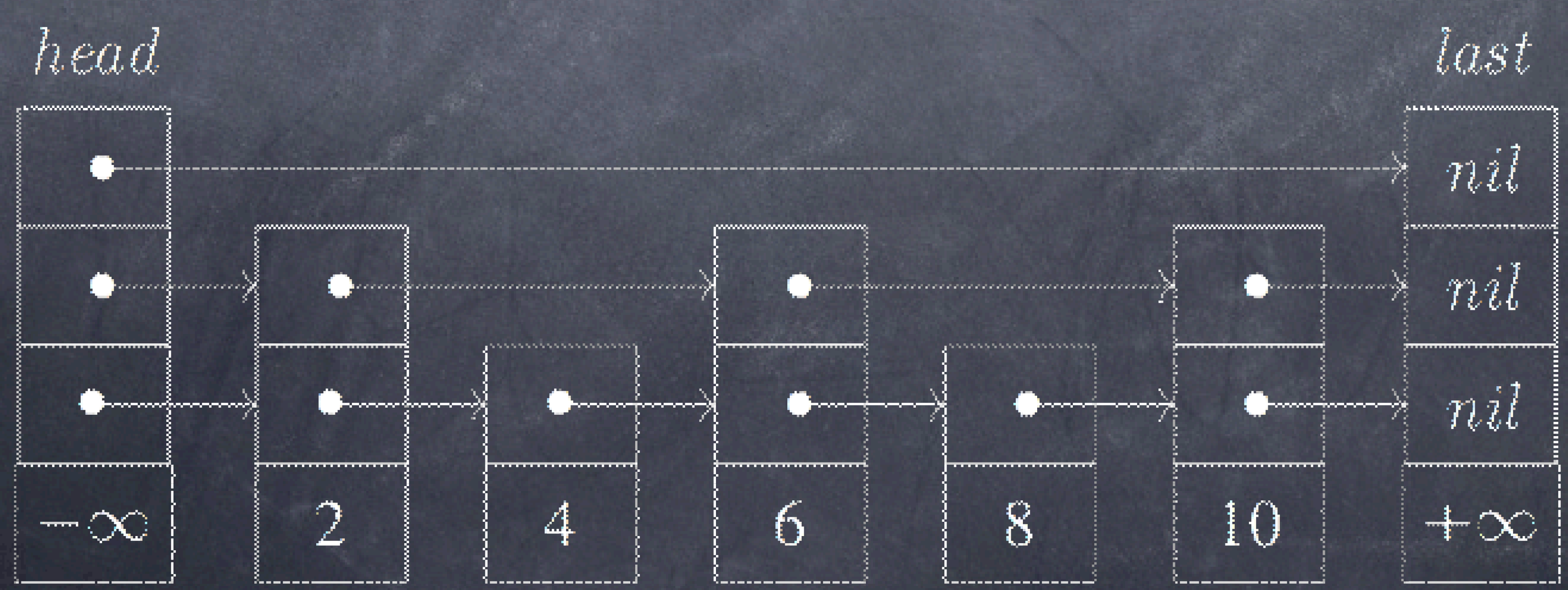

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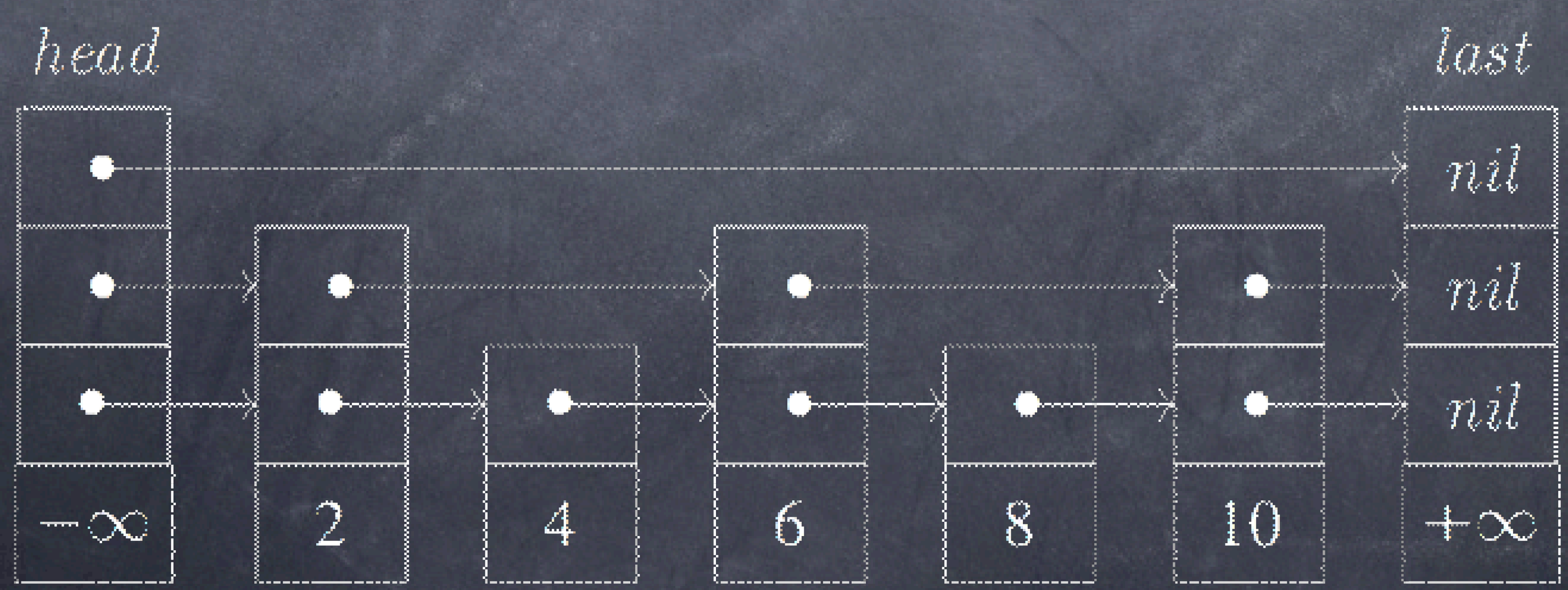

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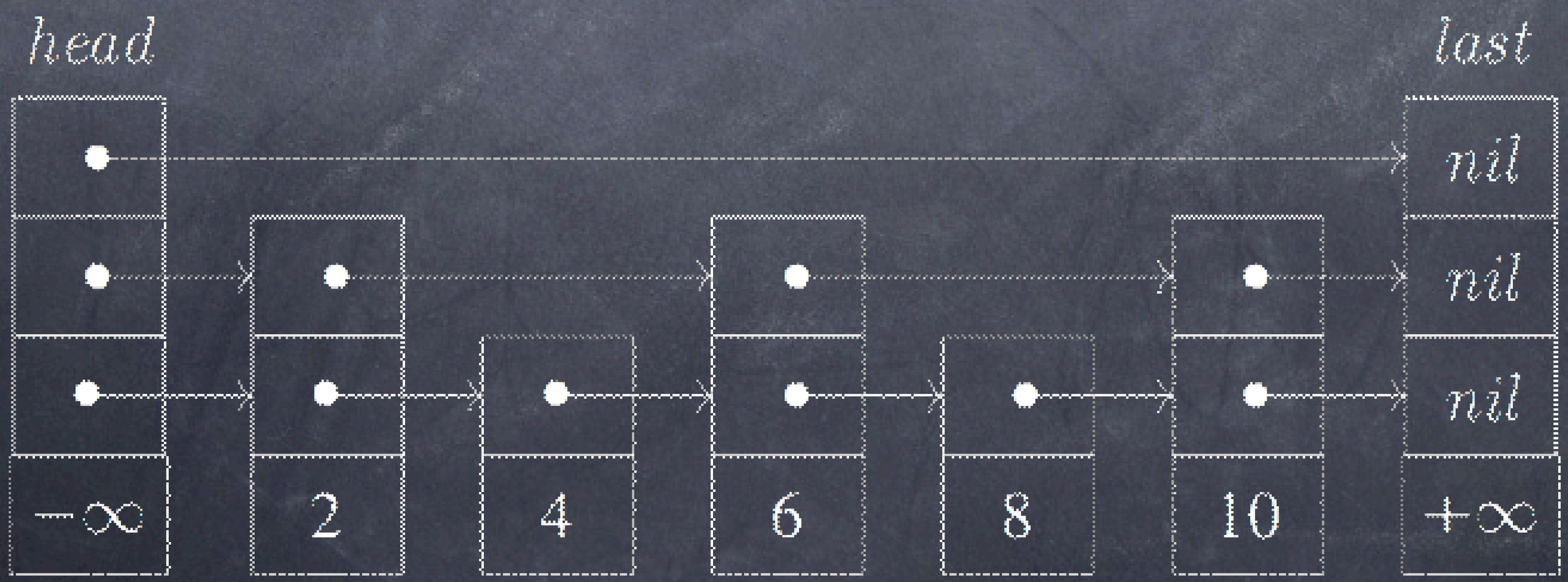

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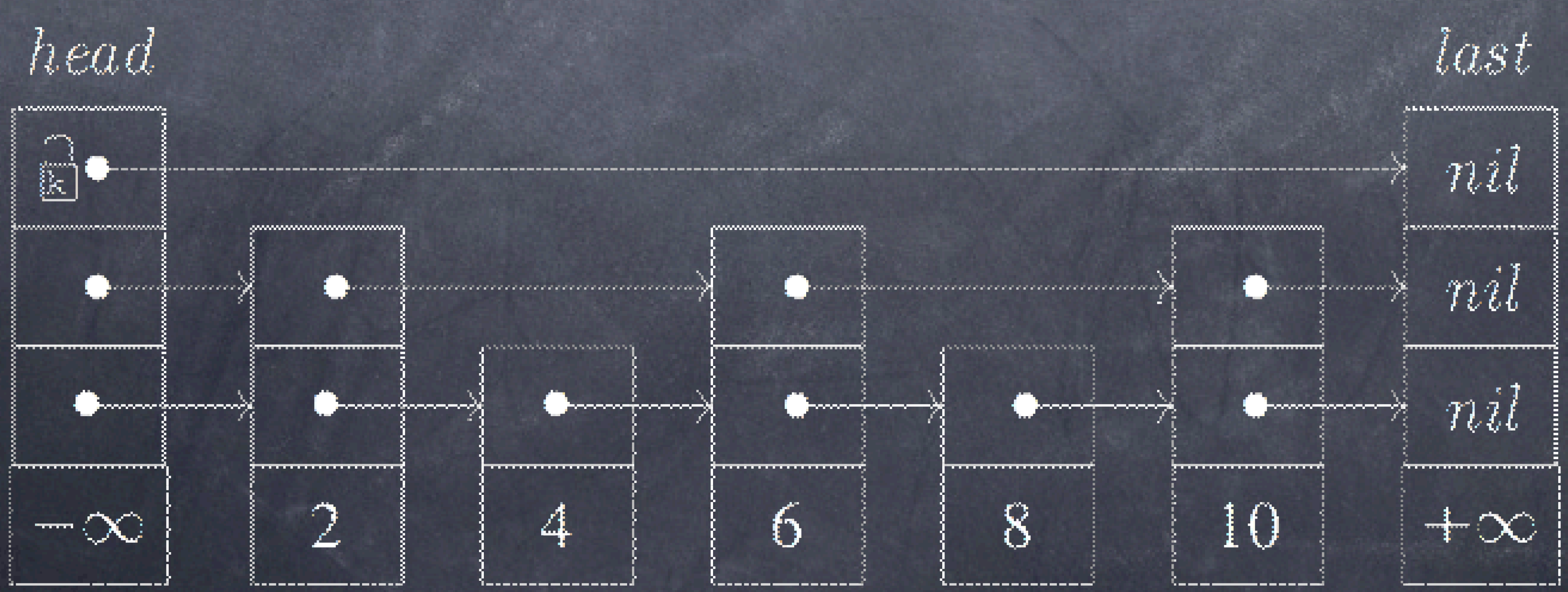

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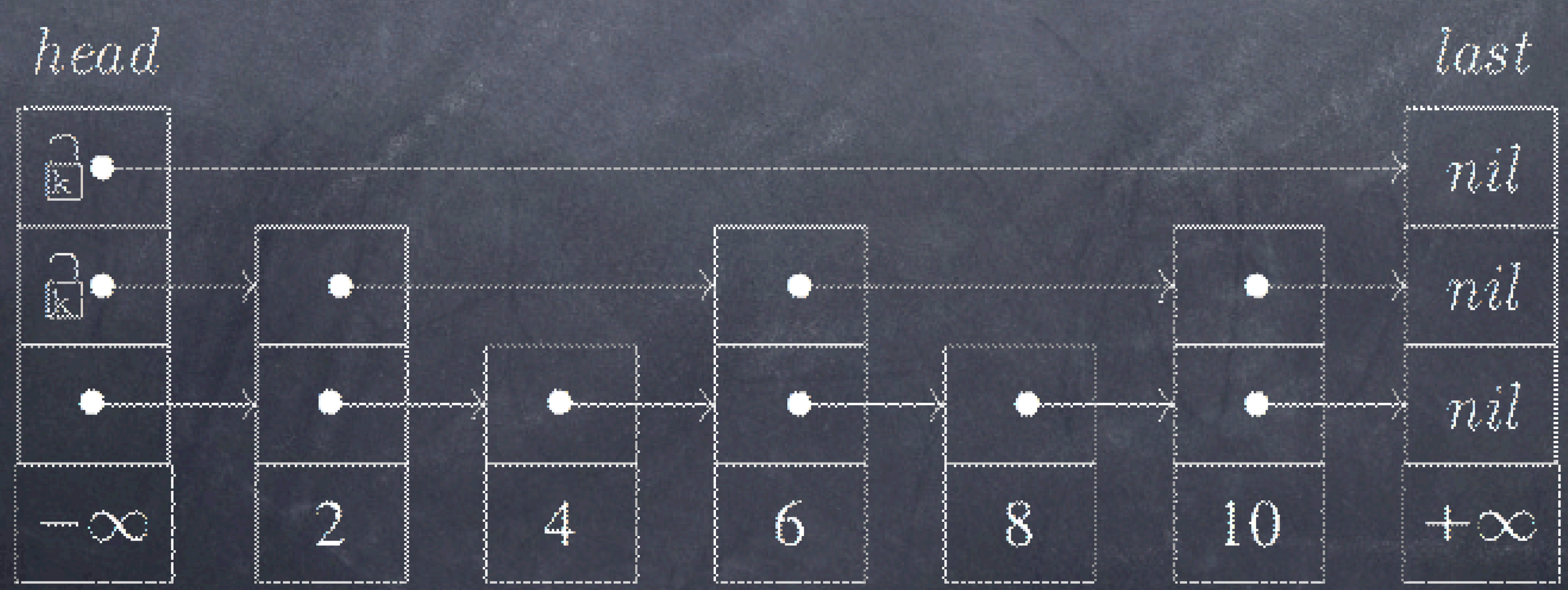

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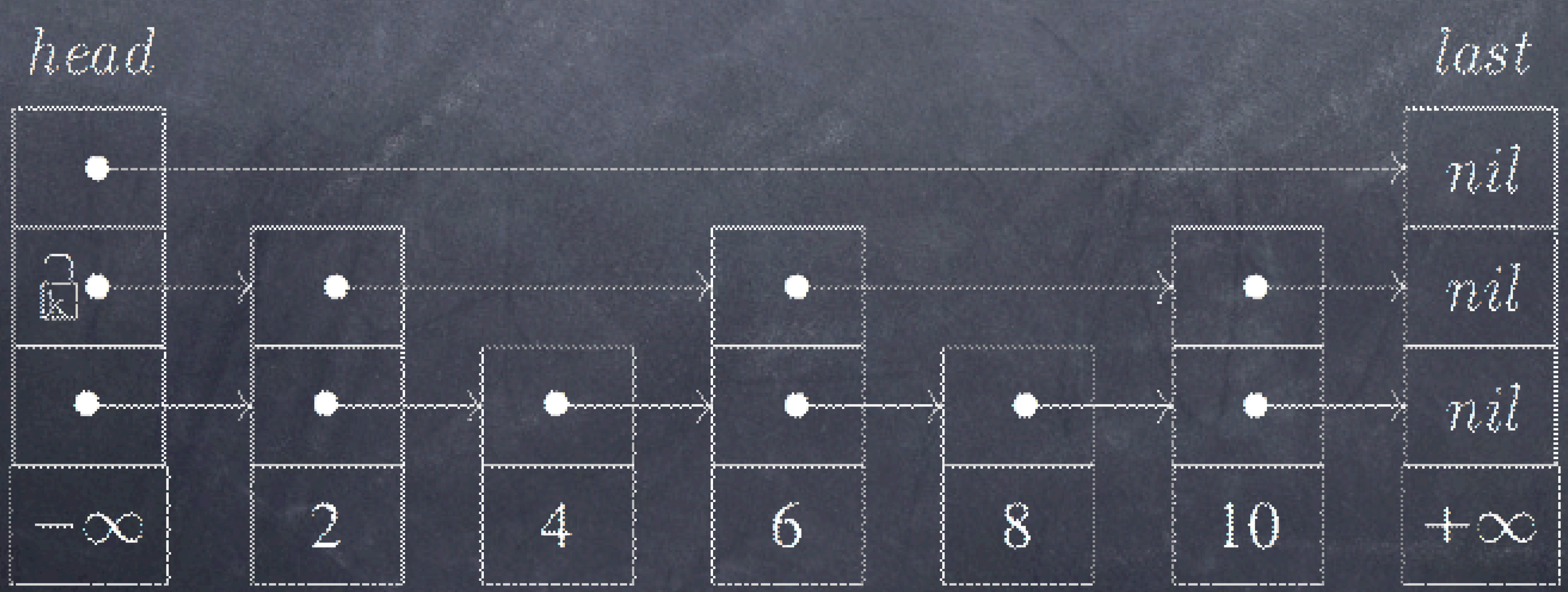

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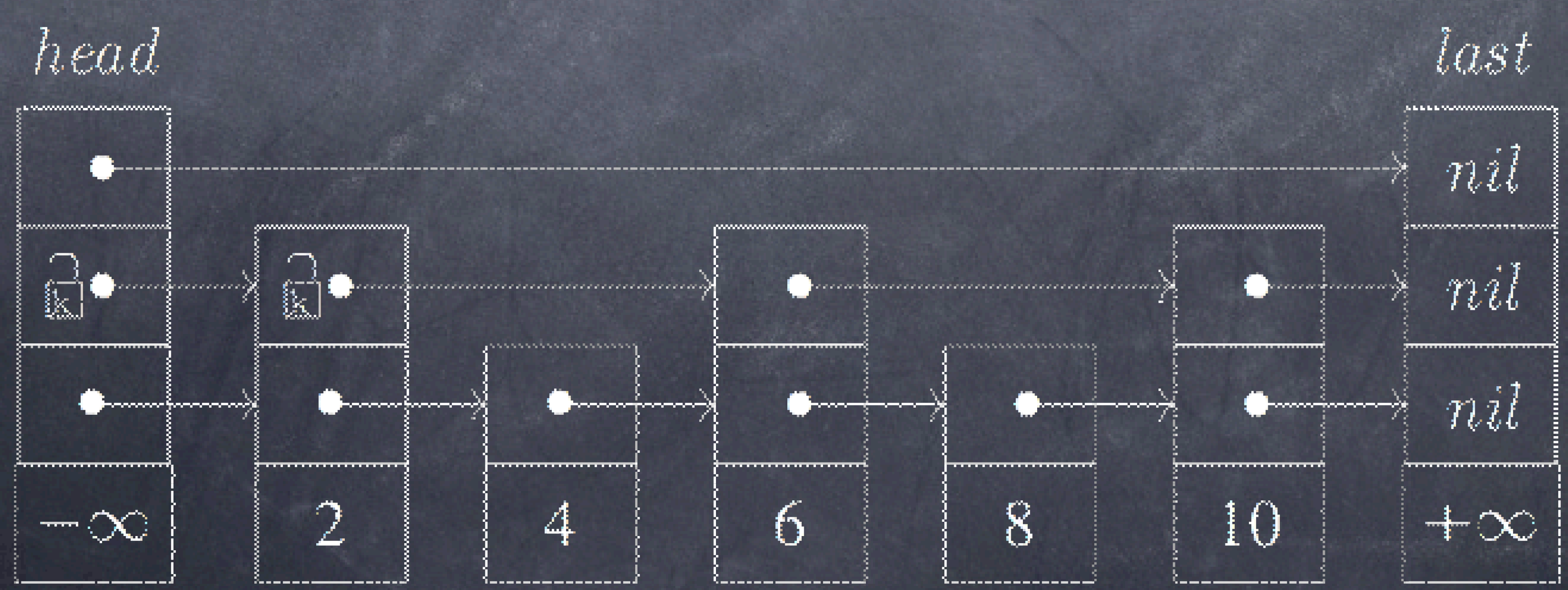

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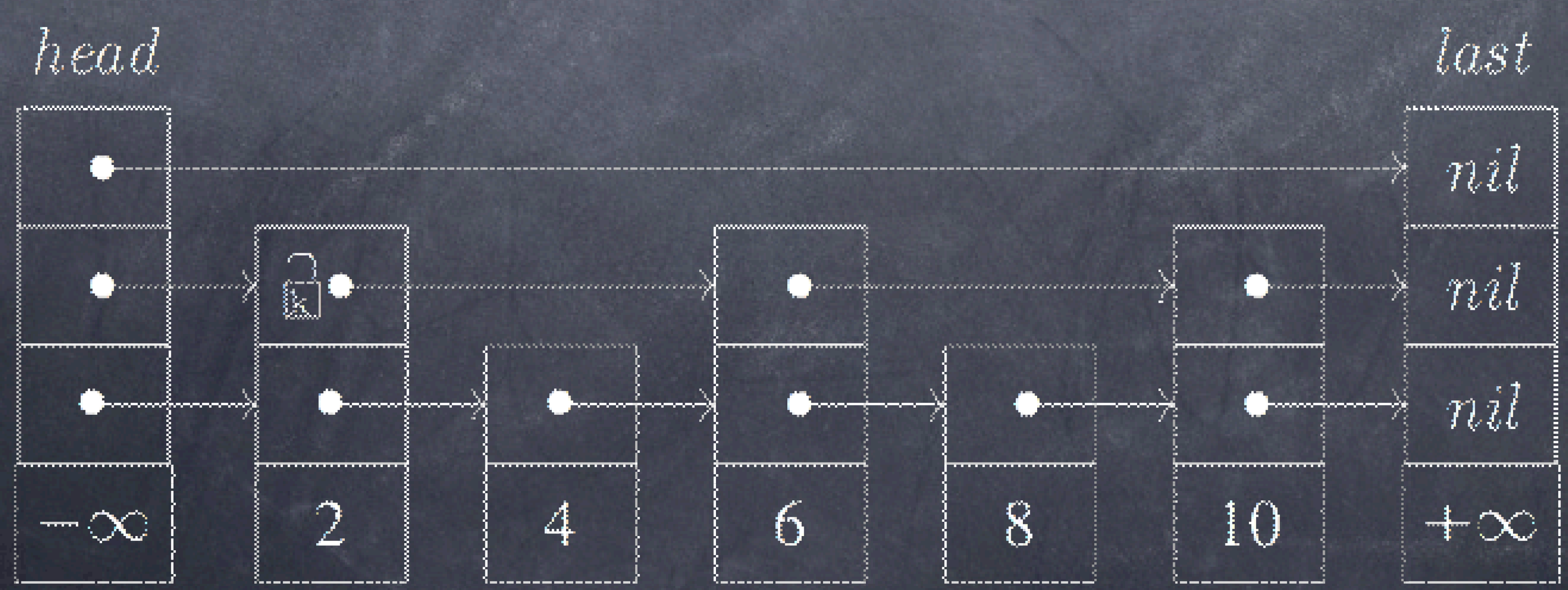

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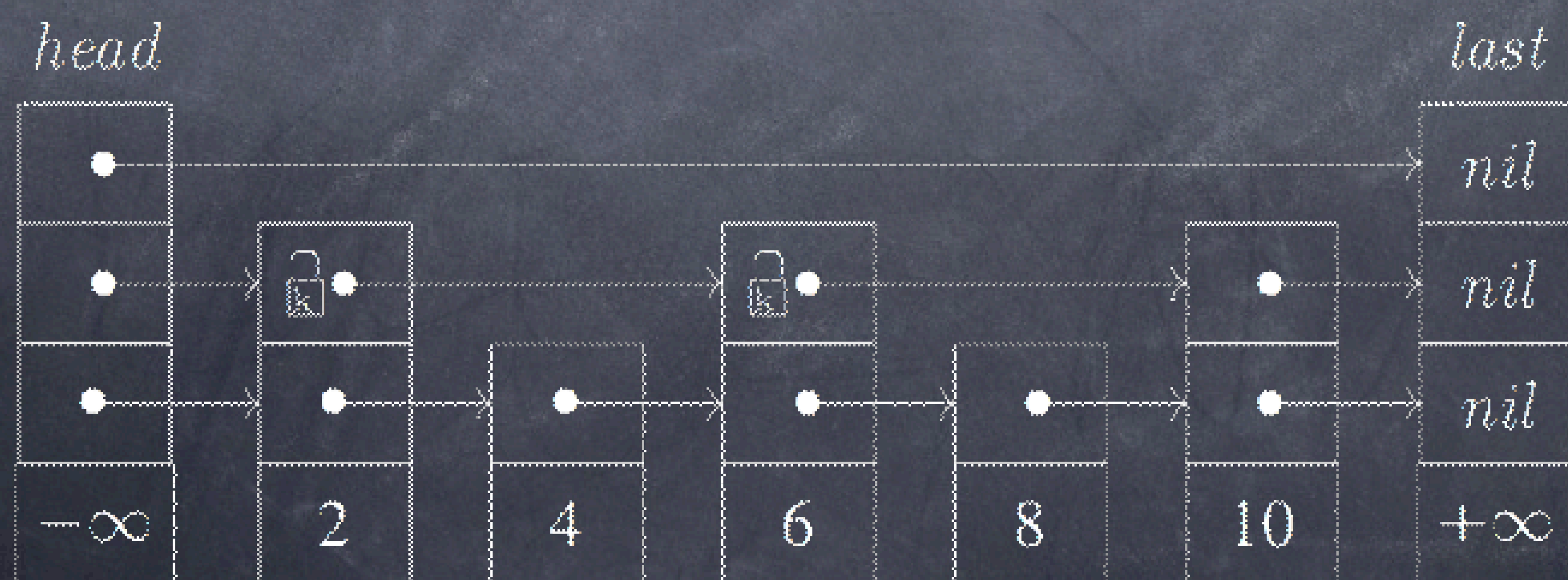

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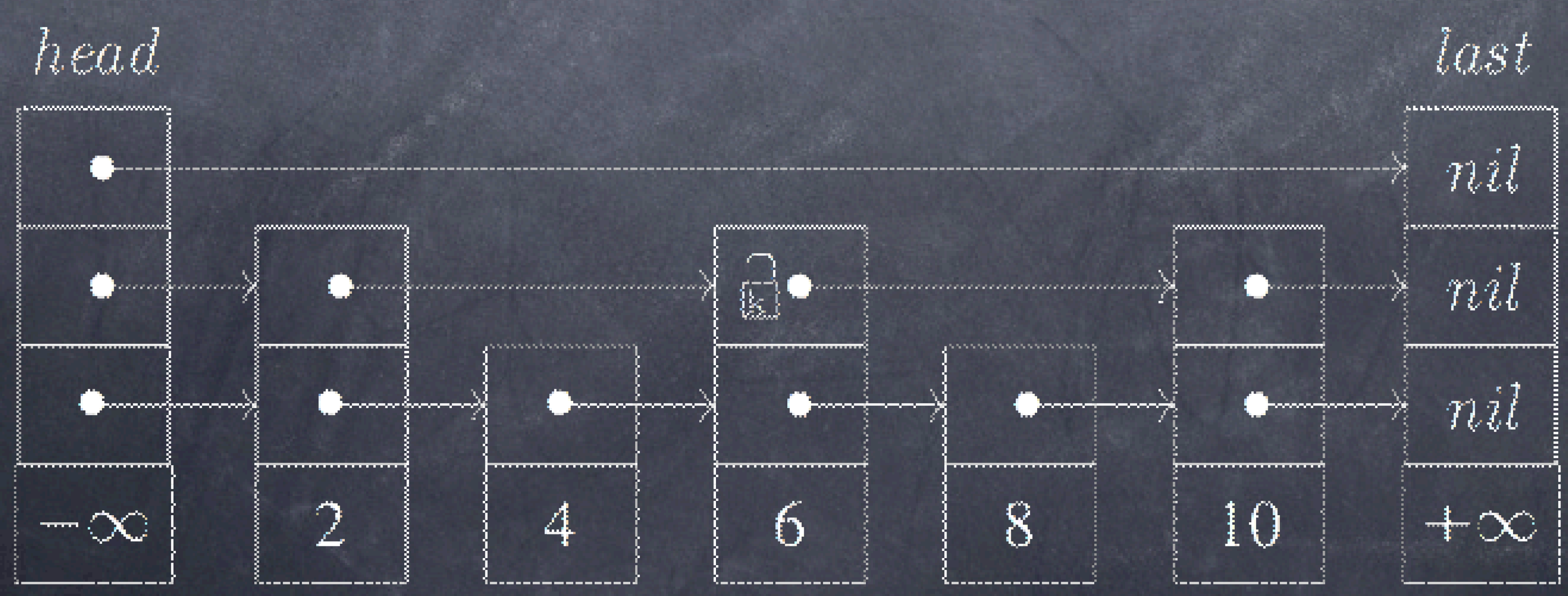

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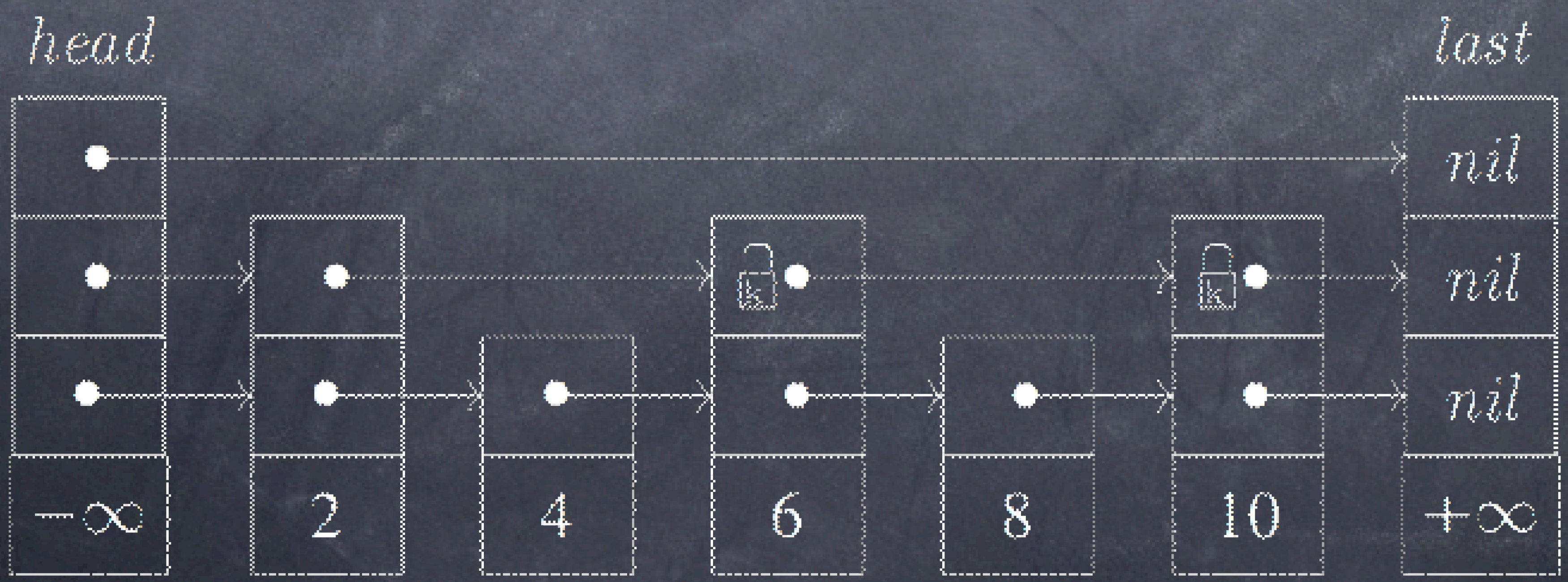

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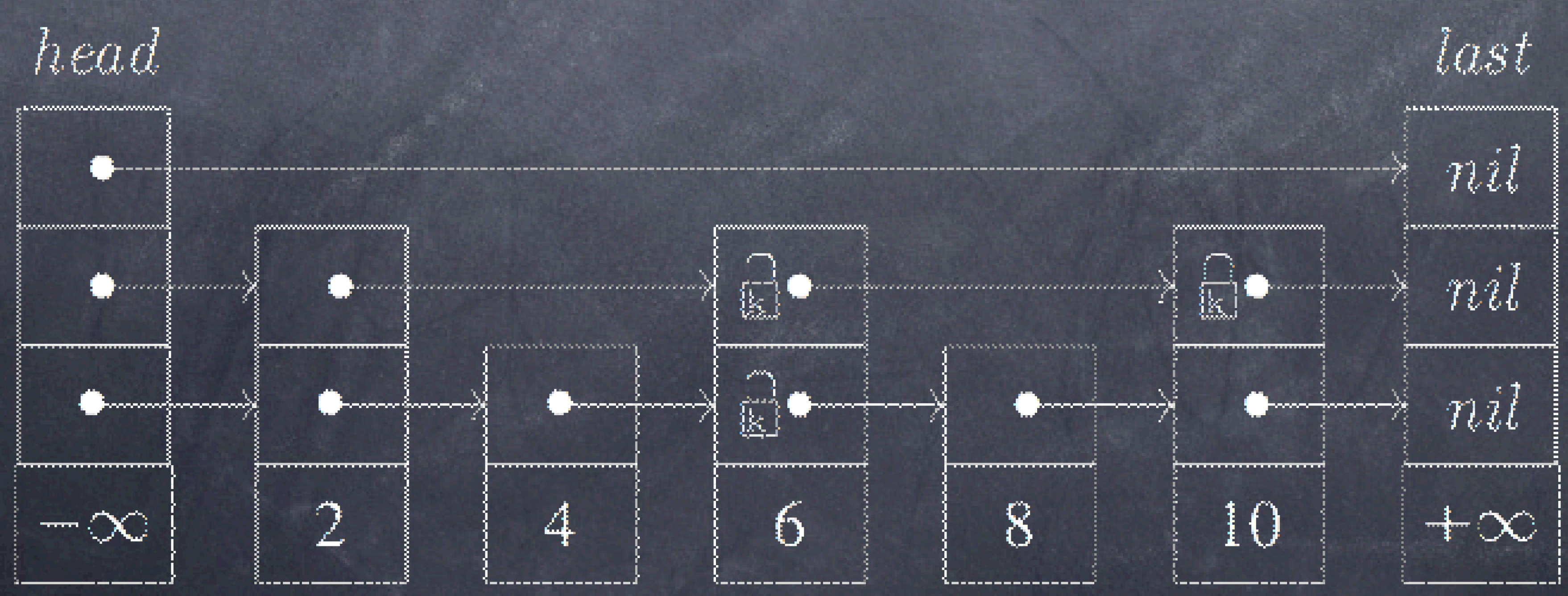

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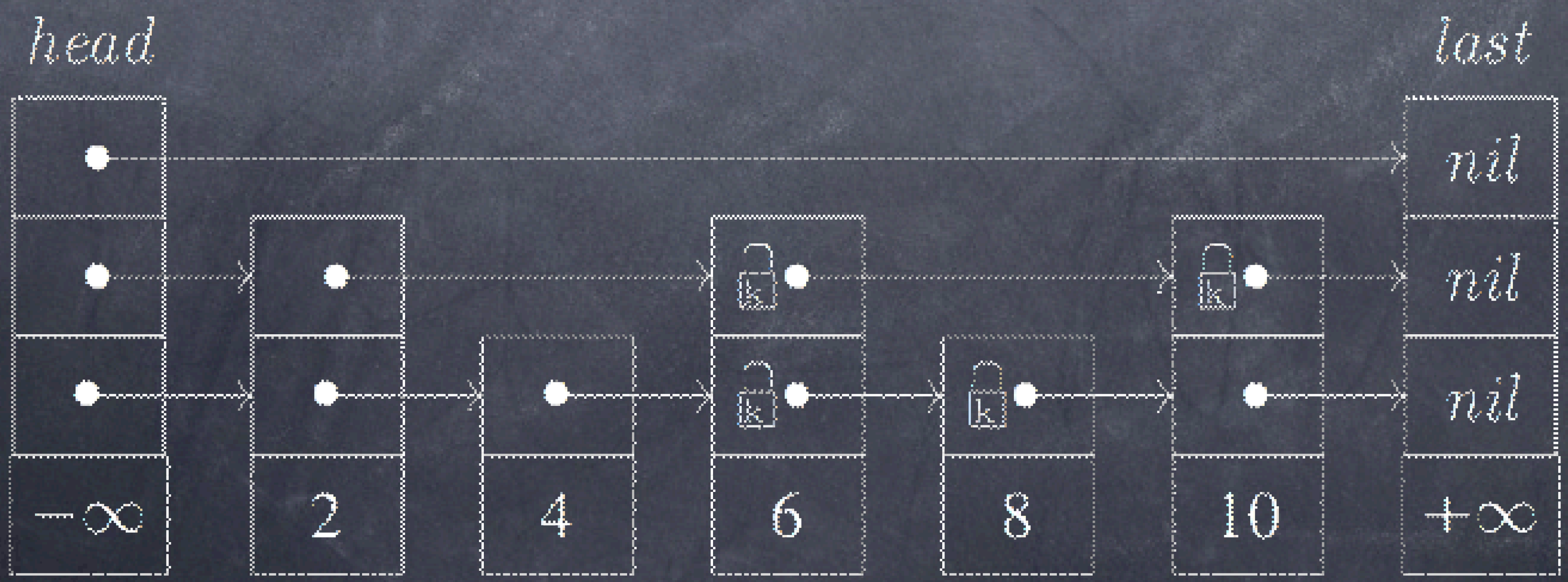

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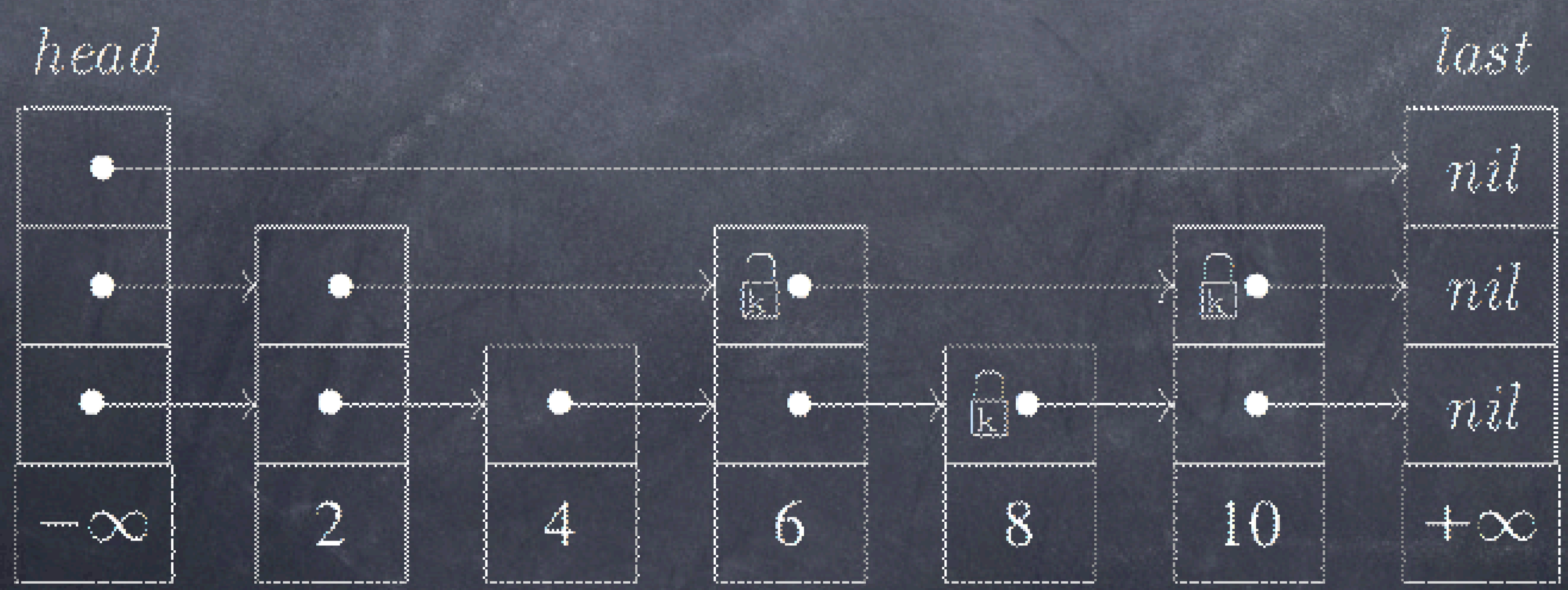

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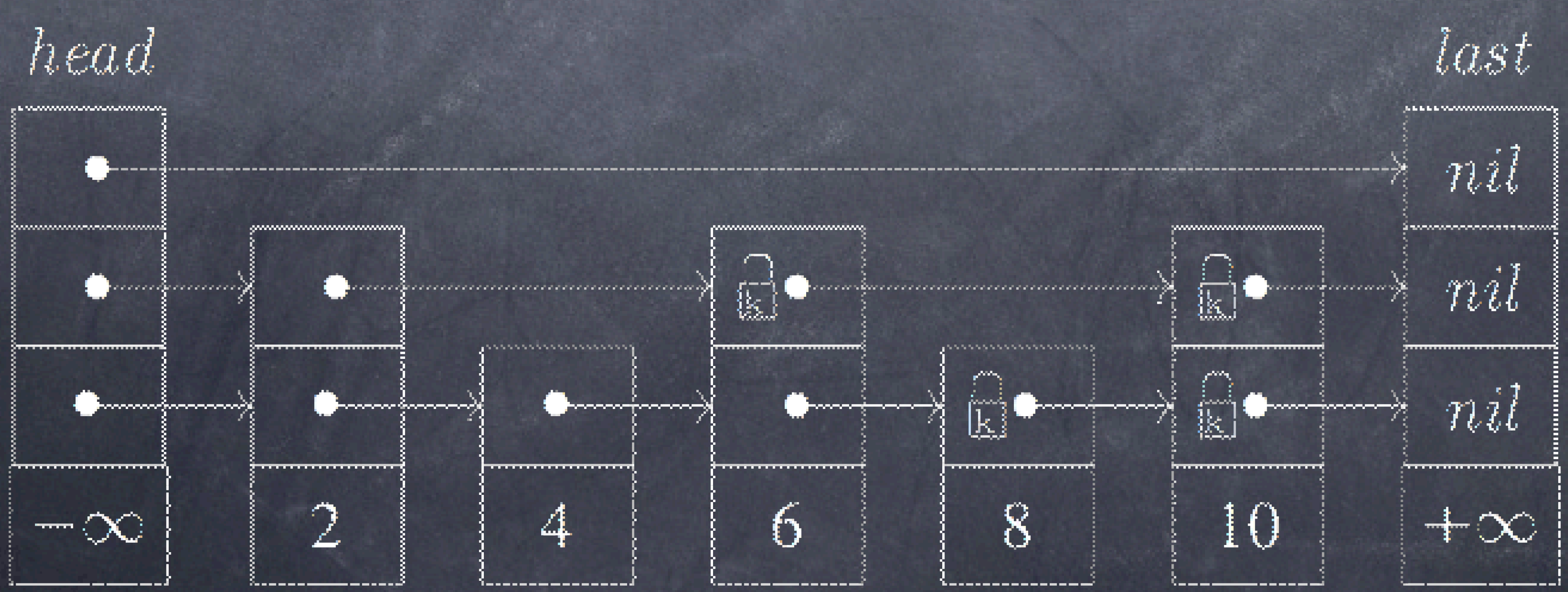

Fine-grained lock-coupling CSL

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$insert(sl, v)$

Algorithm 4 Insertion on a lock-coupling concurrent skiplist

```
procedure INSERT(SkipList sl, Key k, Value newval)
    Vector < Node* > update[1..sl.maxLevel]    //@ mrgn m_r := emp
    lvl := randomLevel()
    Node * pred := sl.head
    pred.locks[lvl].lock()                      //@ m_r := m_r ∪ (pred.forward[lvl])
    Node * curr := pred.forward[lvl]
    curr.locks[lvl].lock()                      //@ m_r := m_r ∪ (curr.forward[lvl])
    for i := lvl downto 1 do
        if i < lvl then
            pred.locks[i].lock()                //@ m_r := m_r ∪ (pred.forward[i])
            curr := pred.forward[i]
            curr.locks[i].lock()                //@ m_r := m_r ∪ (curr.forward[i])
        end if
        while curr.key < k do
            pred.locks[i].unlock()              //@ m_r := m_r − (pred.forward[i])
            pred := curr
            curr := pred.forward[i]
            curr.locks[i].lock()                //@ m_r := m_r ∪ (curr.forward[i])
        end while
        update[i] := pred
    end for
    if curr.key = k then
        curr.val = newval
        for i := 1 to lvl do
            update[i].forward[i].locks[i].unlock()
                              //@ m_r := m_r − (update[i].forward[i].forward[i])
            update[i].locks[i].unlock()         //@ m_r := m_r − (update[i].forward[i])
        end for
    else
        x := CreateNode(lvl, k, newval)
        for i := 1 to lvl do
            x.forward[i] := update[i].forward[i]
            update[i].forward[i] := x           //@ sl.r := sl.r ∪ (x)
            x.forward[i].locks[i].unlock() //@ m_r := m_r − (x.forward[i].forward[i])
            update[i].locks[i].unlock()         //@ m_r := m_r − (update[i].forward[i])
        end for
    end if
end procedure
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Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v)$

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$\quad pred.locks[i].unlock()$ $\qquad //@\ m_r := m_r - (pred, forward[i])$
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$\quad curr := pred.forward[i]$
$\quad curr.locks[i].lock()$ $\qquad //@\ m_r := m_r \cup (curr, forward[i])$
end while

Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$

while $curr.key < k$ **do**

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T_i

Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$

$$T_i \models \Box \varphi_{insert}(i)$$

$$\varphi_{insert}(i) \hat{=} at\_insert^{[i]}_{8..36} \rightarrow at\_insert^{[i]}_{8..36} \,\mathcal{U}\, at\_insert^{[i]}_{37}$$

Fine-grained lock-coupling CSL

$insert(sl, v) \quad search(sl, v) \quad remove(sl, v) \quad decide(sl)$

$$\|_{j \in T_{ID} - \{i\}} T_j \parallel T_i \models \Box \varphi_{insert}(i)$$

$$\varphi_{insert}(i) \hat{=} at\_insert^{[i]}_{8..36} \rightarrow at\_insert^{[i]}_{8..36} \, \mathcal{U} \, at\_insert^{[i]}_{37}$$

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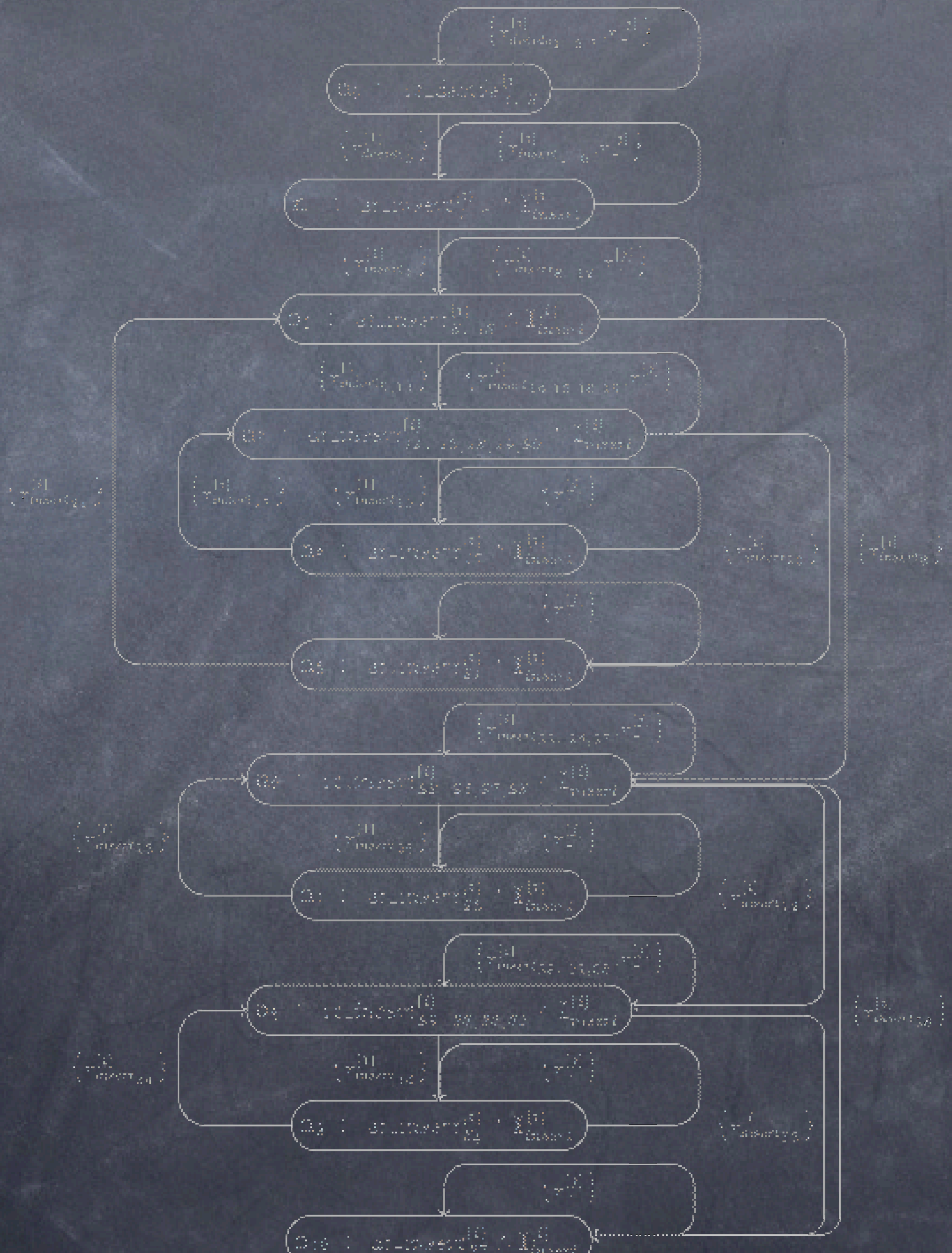

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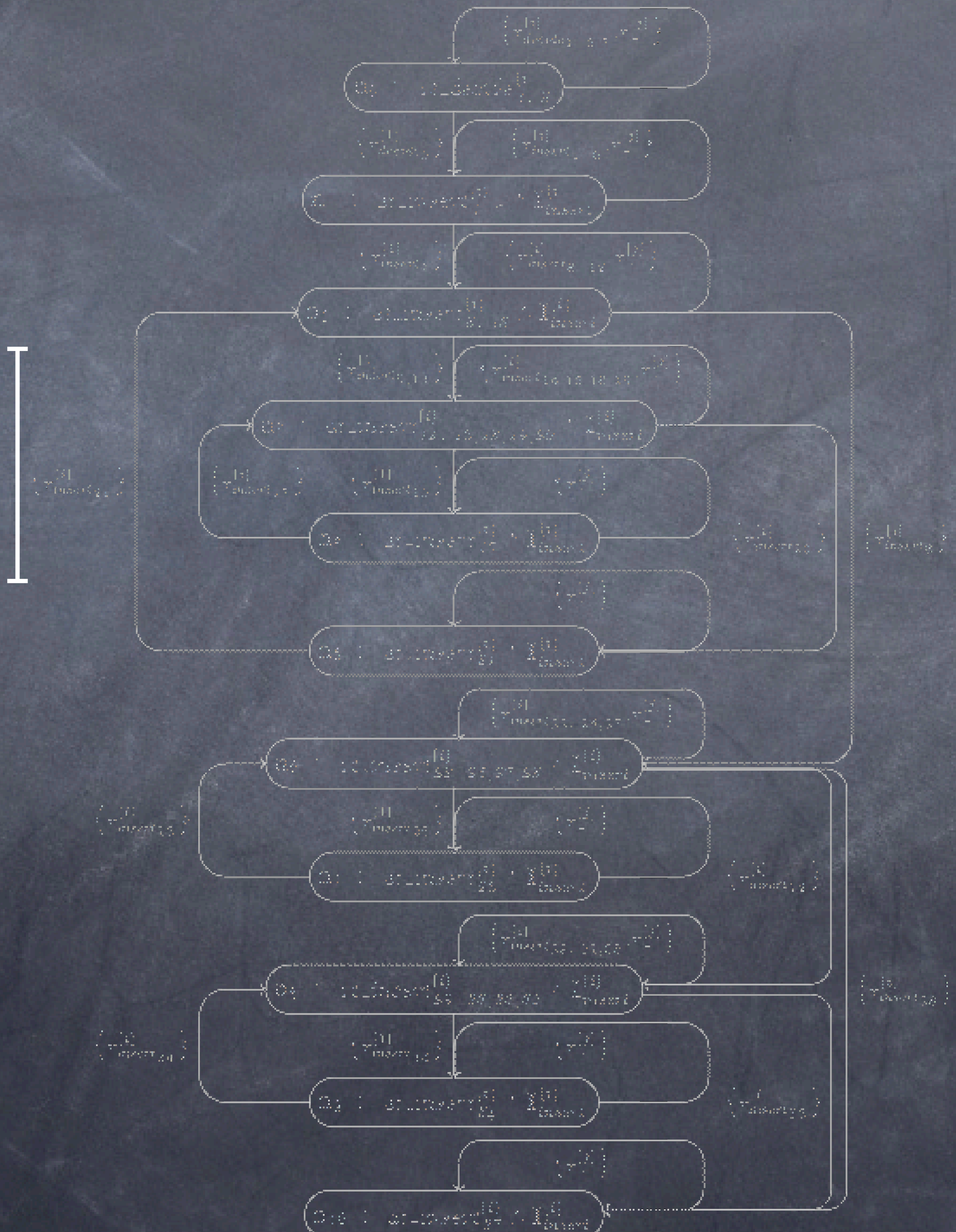

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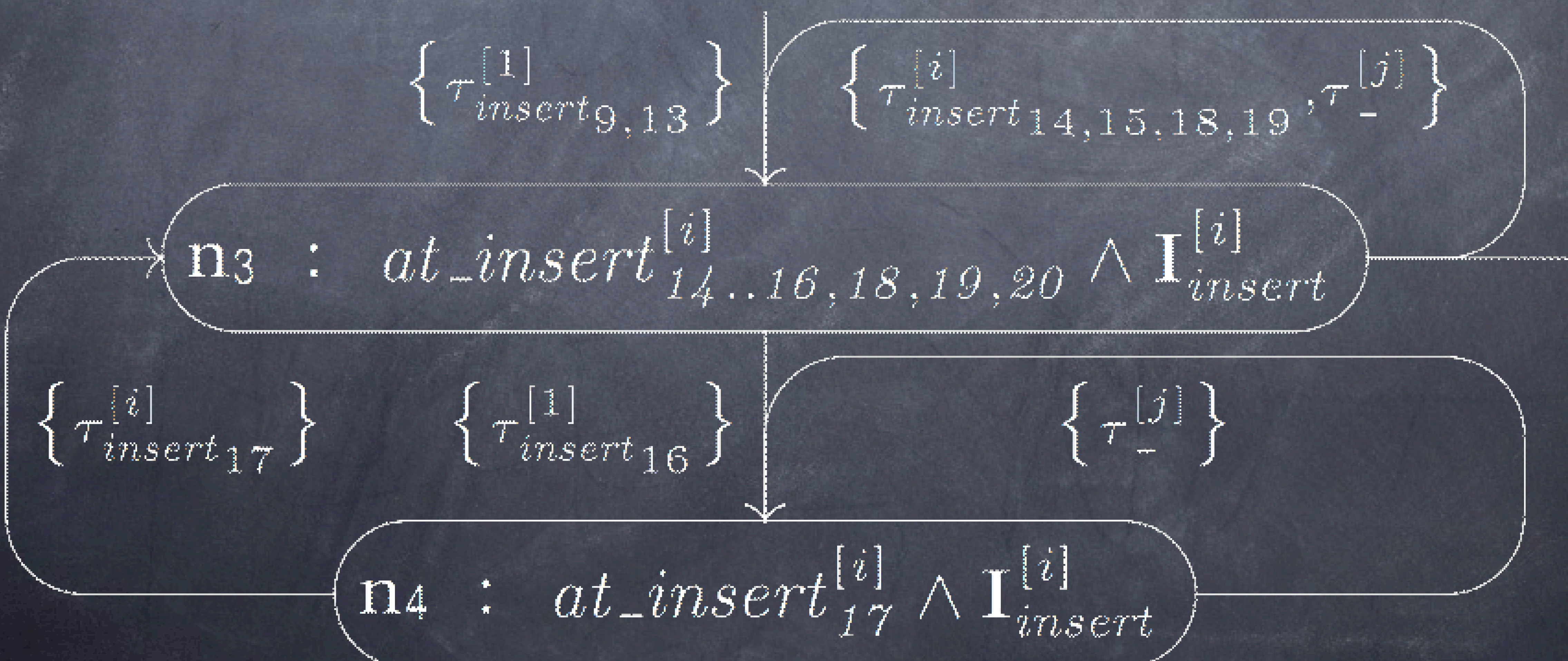
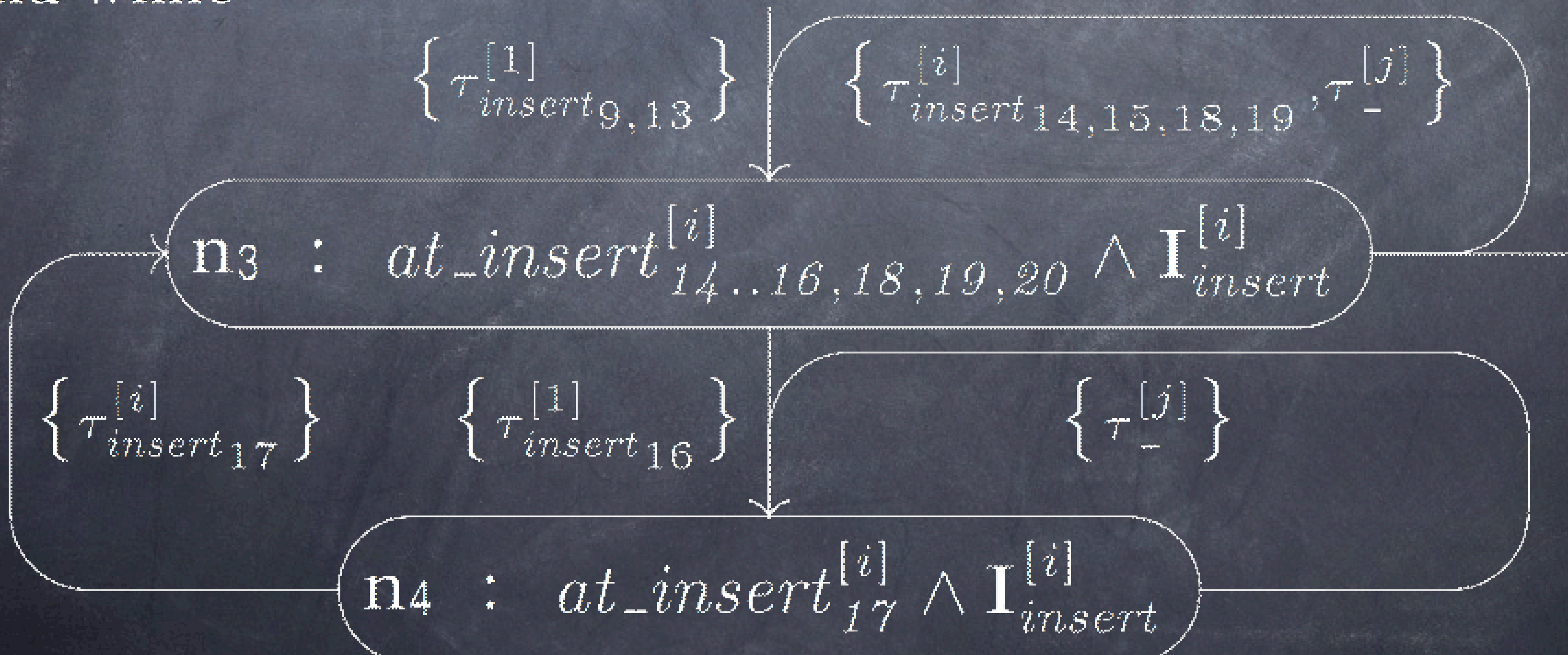


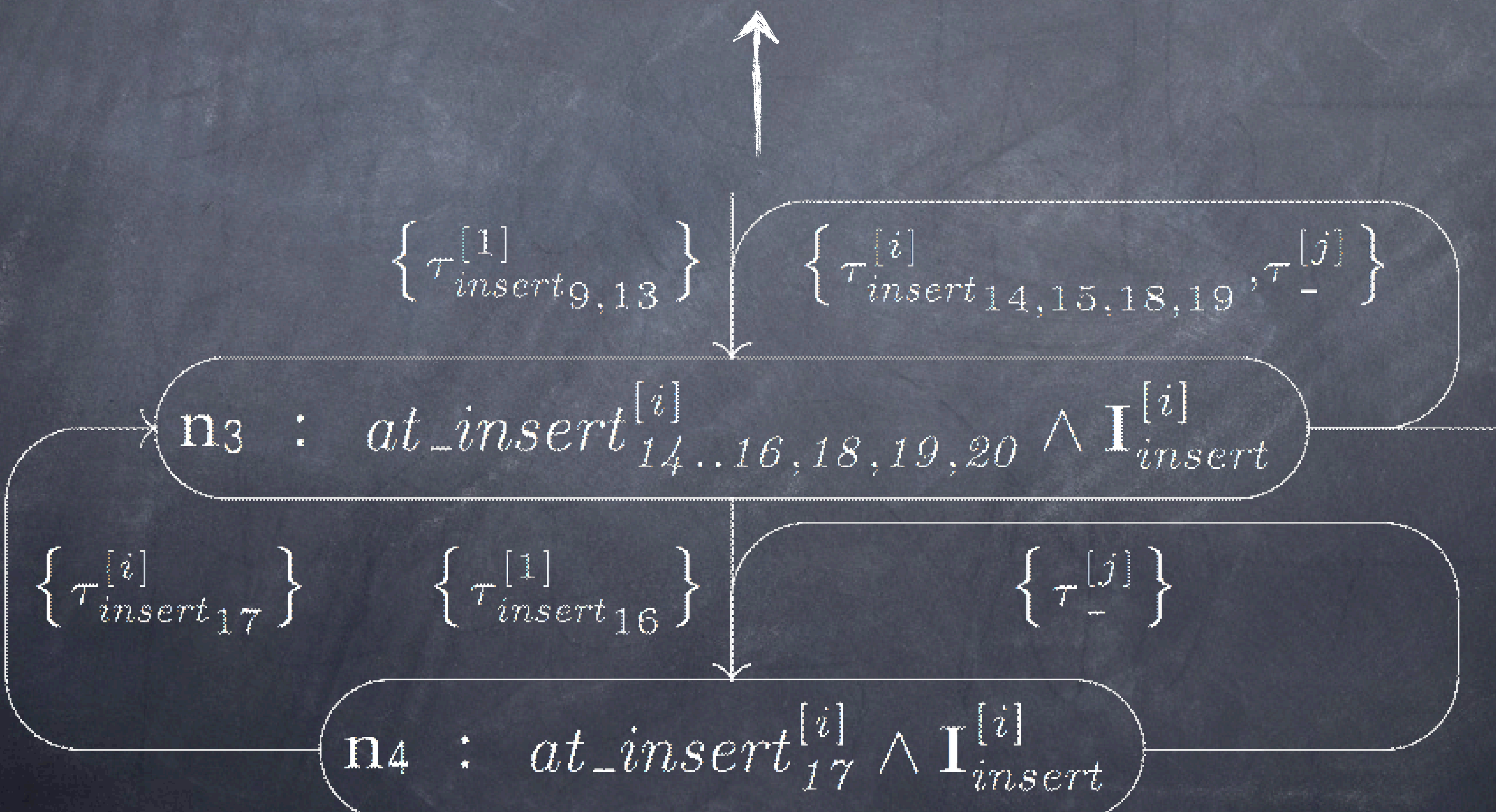
Fine-grained lock-coupling CSL

```
14: while curr.key < k do
15:     pred.locks[i].unlock()          //@ m_r := m_r − (pred, forward[i])
16:     pred := curr
17:     curr := pred.forward[i]
18:     curr.locks[i].lock()            //@ m_r := m_r ∪ (curr, forward[i])
19: end while
```



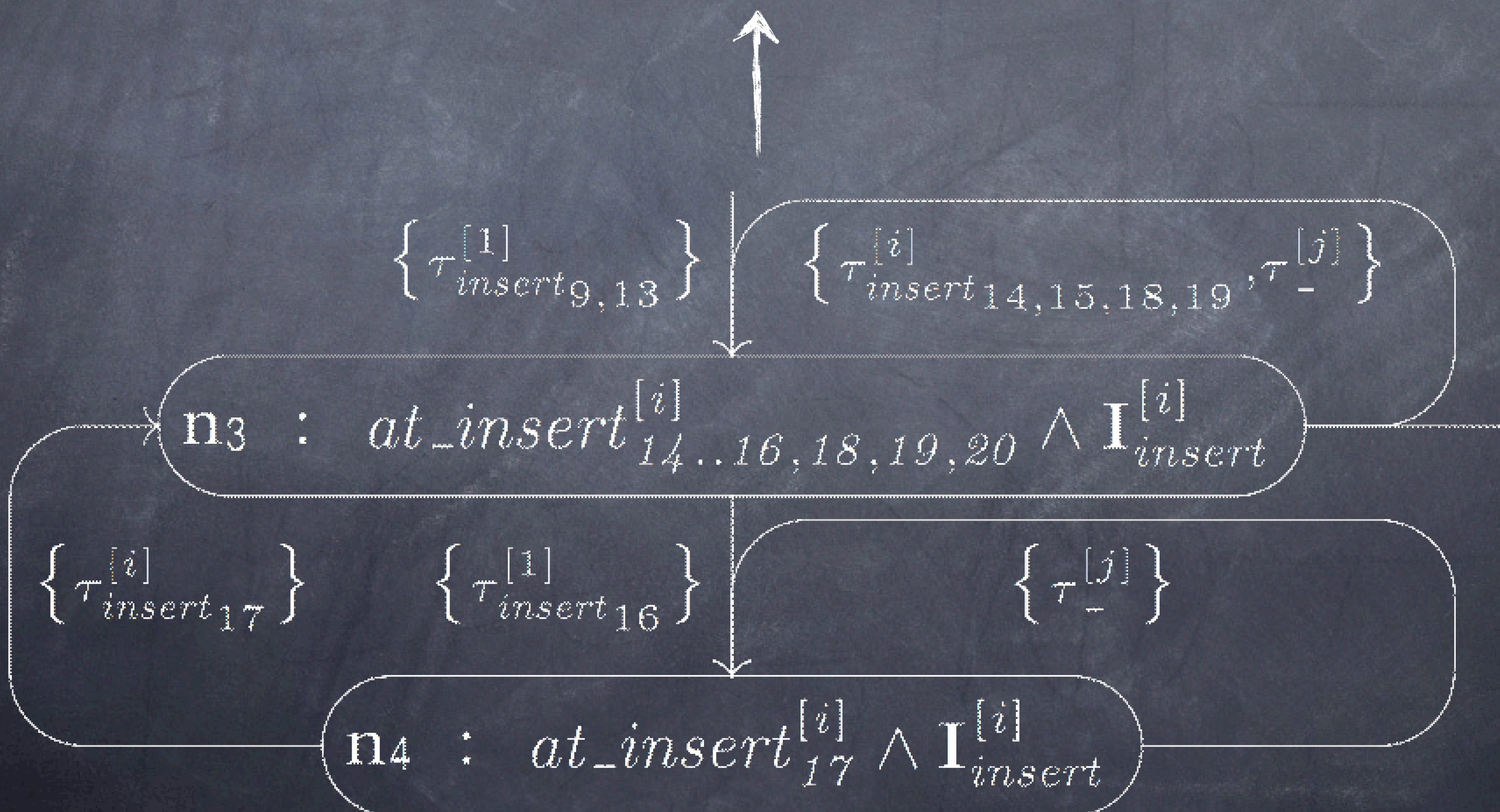
Fine-grained lock-coupling CSL

Verification conditions



Fine-grained lock-coupling CSL

Verification conditions ✓



Fine-grained lock-coupling CSL

Verification conditions ✓

$$\Psi$$

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Fine-grained lock-coupling CSL

Verification conditions ✓

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Future work

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- This is just the beginning

Questions ?