Decision Procedures for the Temporal Verification of Concurrent Lists

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Motivation

Why do we need decision procedures for concurrent data structures?

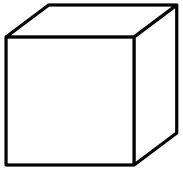
Verification of Concurrent Data-structures

Main Idea

Verification of Concurrent Data-structures

Main Idea

Concurrent DataStructure



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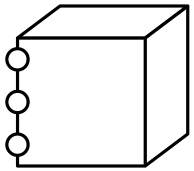

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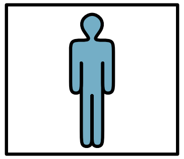

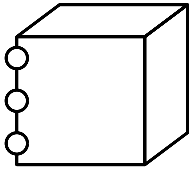

Most General Client

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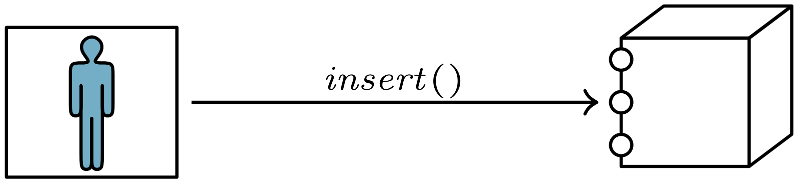

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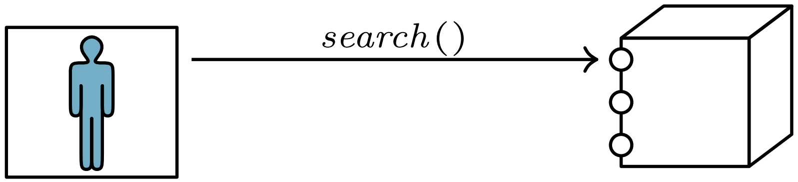

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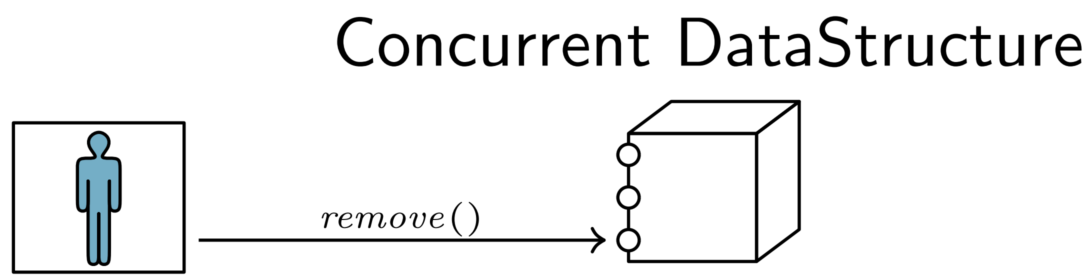

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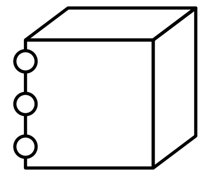

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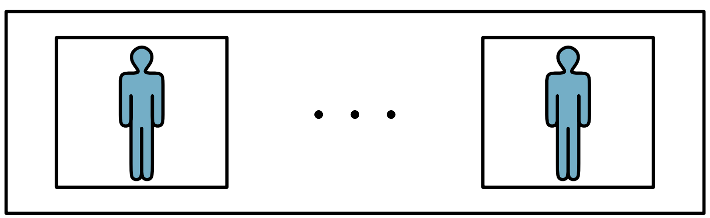

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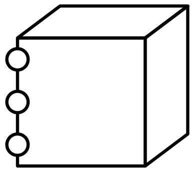

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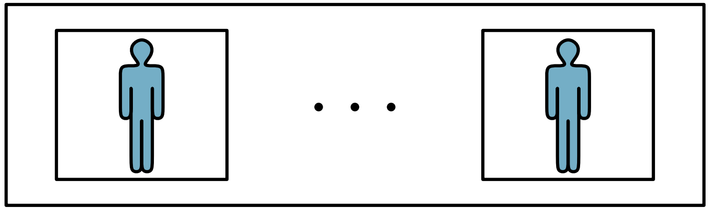

$$P[N] : P(1) || \cdots || P(N)$$

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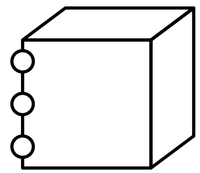

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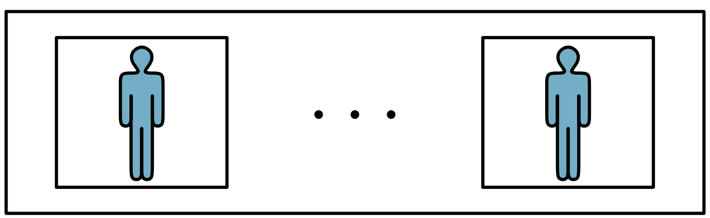

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$$+$$

ghost variables

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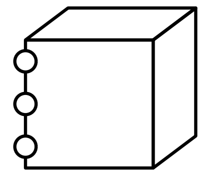

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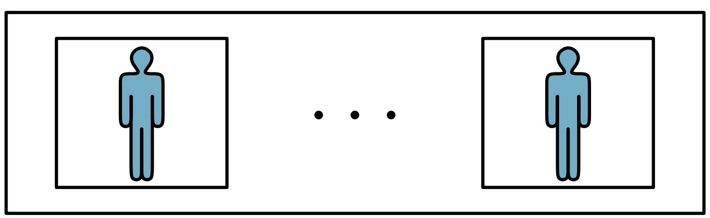

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Property

$$\varphi^{(k)}$$

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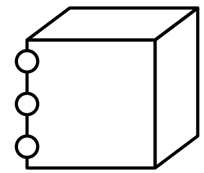

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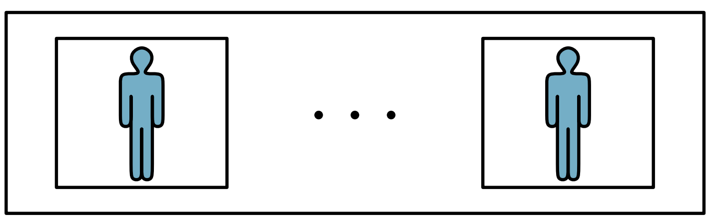

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LTL ($\Box$,$\Diamond$,$\mathcal{U}$,. . .)

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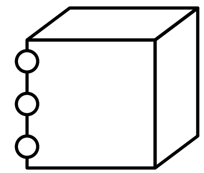

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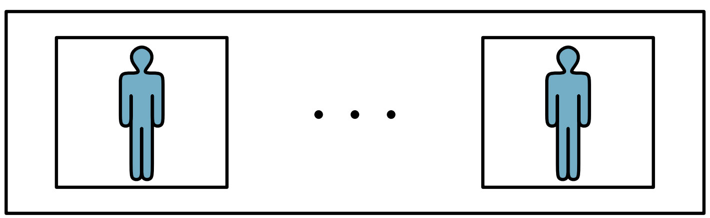

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Diagram

$\mathcal{D}$

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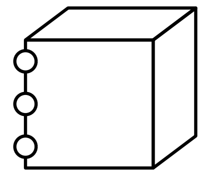

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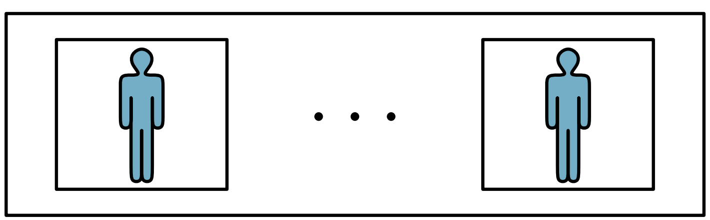

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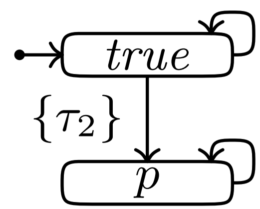
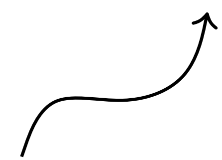


Property

$\varphi^{(k)}$



LTL ($\Box$,$\Diamond$,$\mathcal{U}$,$\ldots$)

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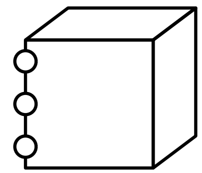

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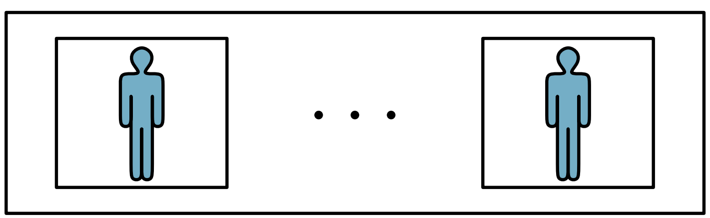

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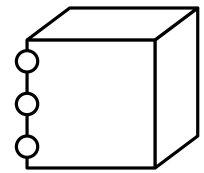

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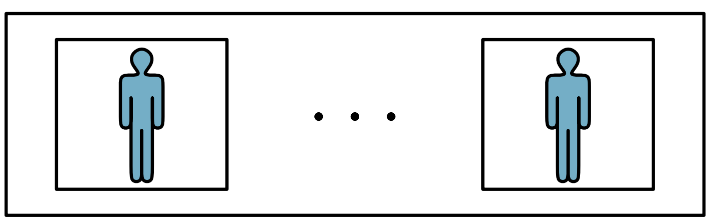

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Diagram

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$\models \quad \mathcal{D} \quad \models \quad \varphi^{(k)}$

Verification Conditions:

- Initiation
- Consecution
- Acceptance
- Fairness

Satisfaction (Model Checking)

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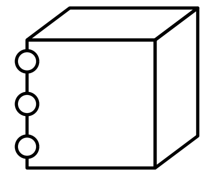

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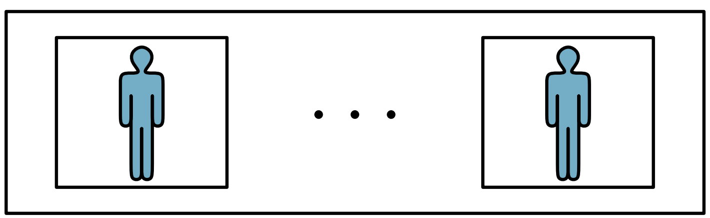

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Satisfaction
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Decision Procedures
(first order propositional logic)

Concurrent Lock-coupling Lists

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- A concurrent implementation of sets

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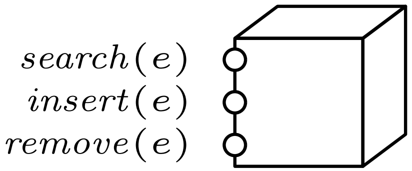

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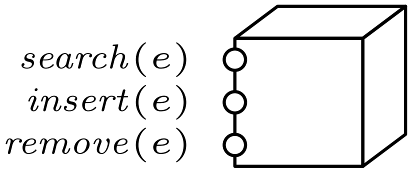


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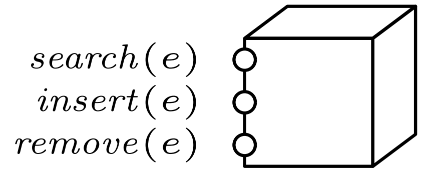


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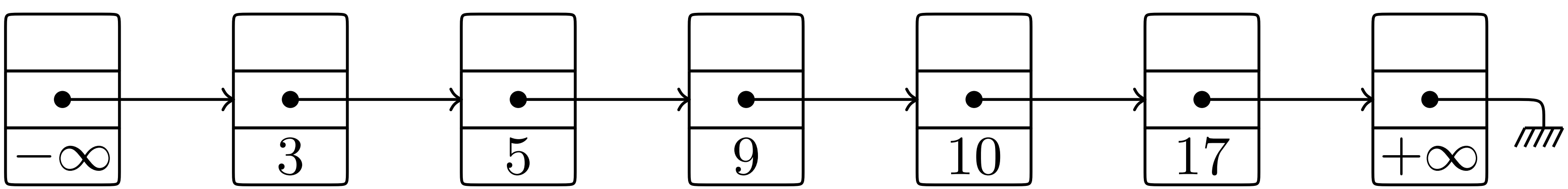

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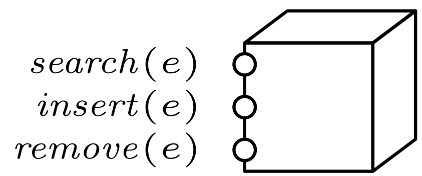


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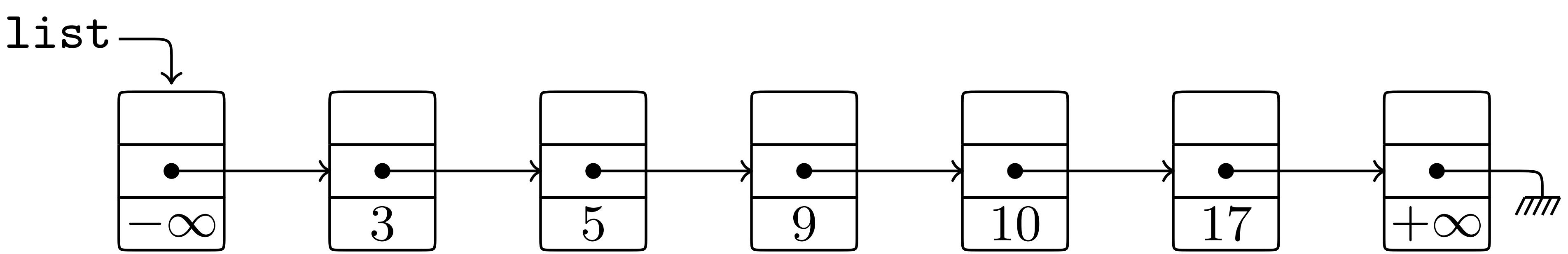

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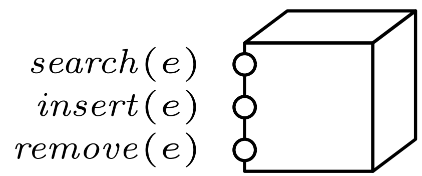


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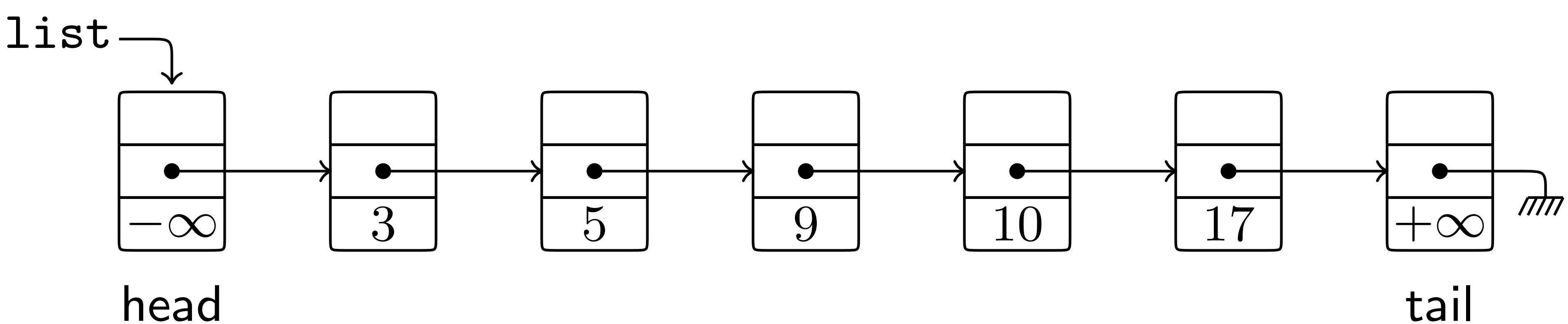

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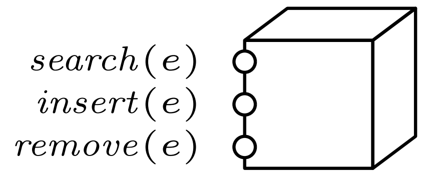

search(8)

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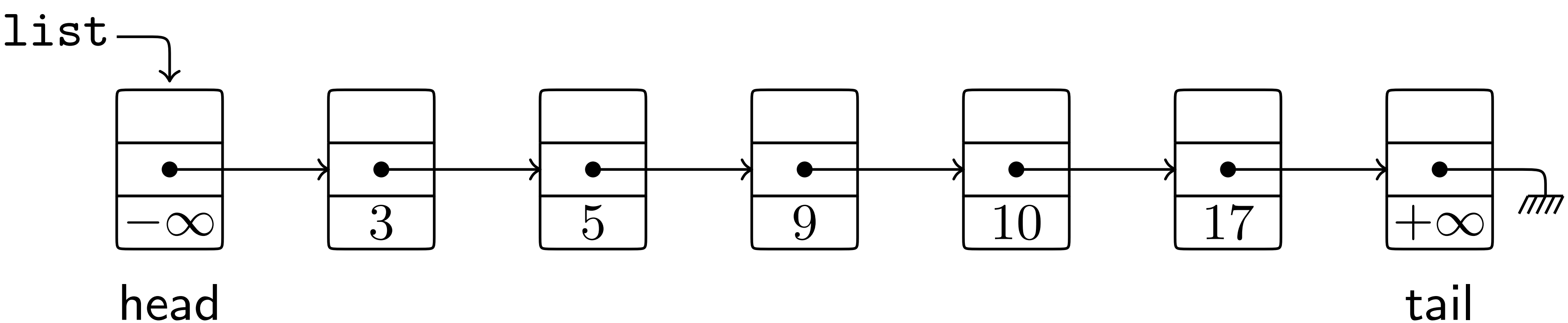

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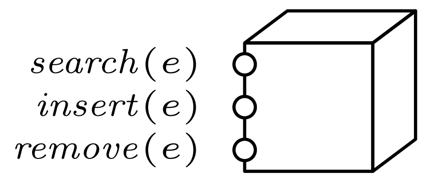

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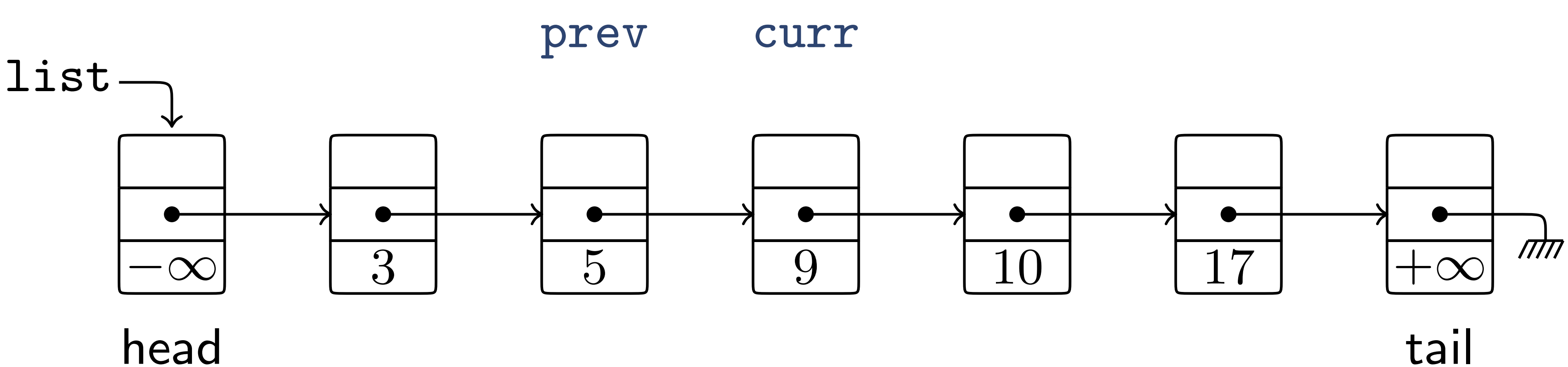

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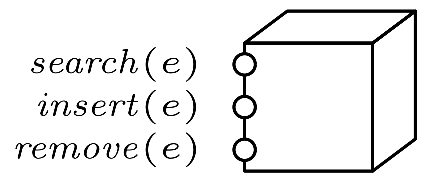

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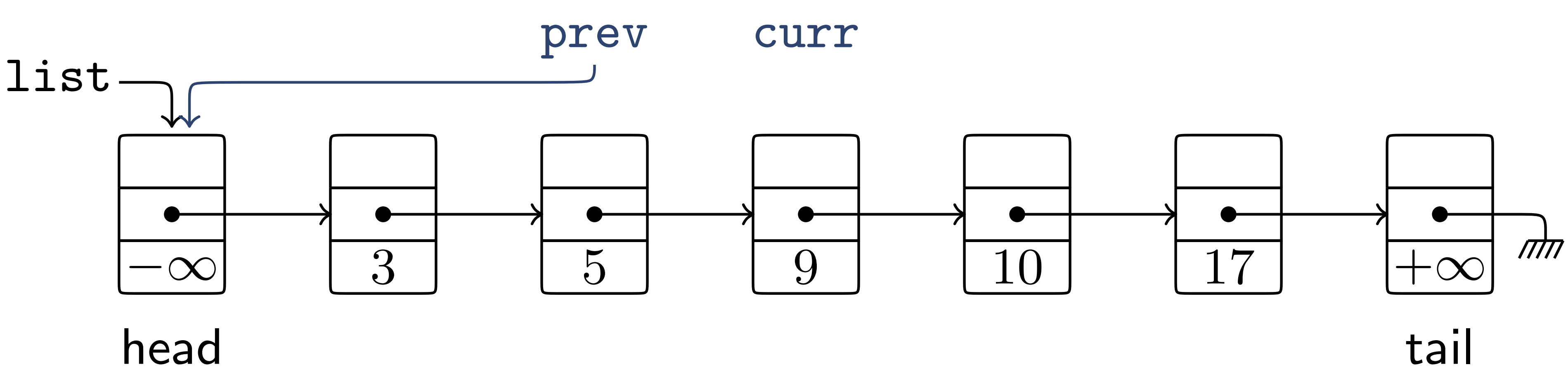

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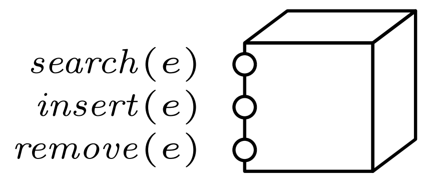

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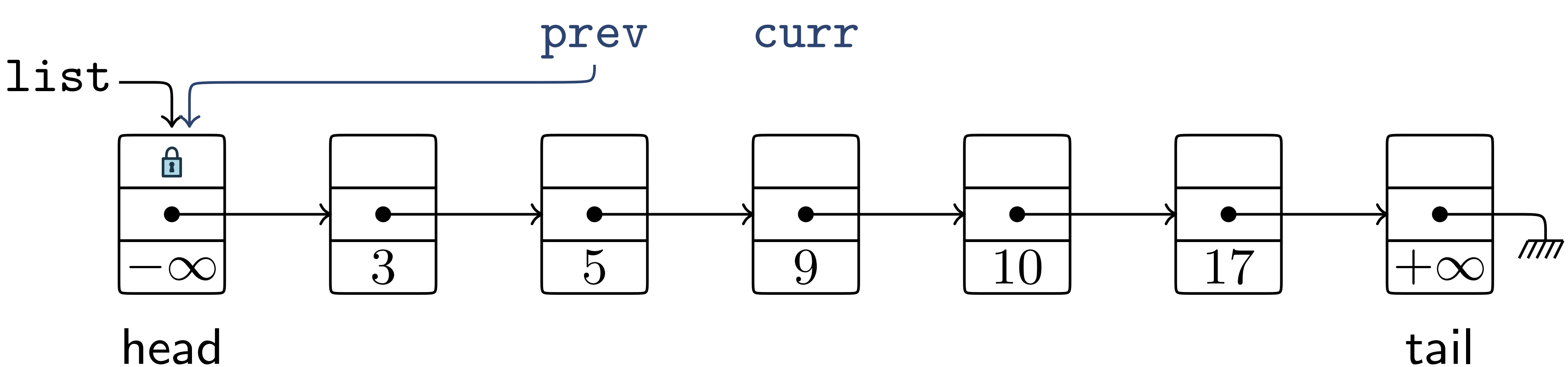

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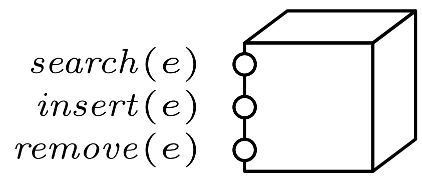

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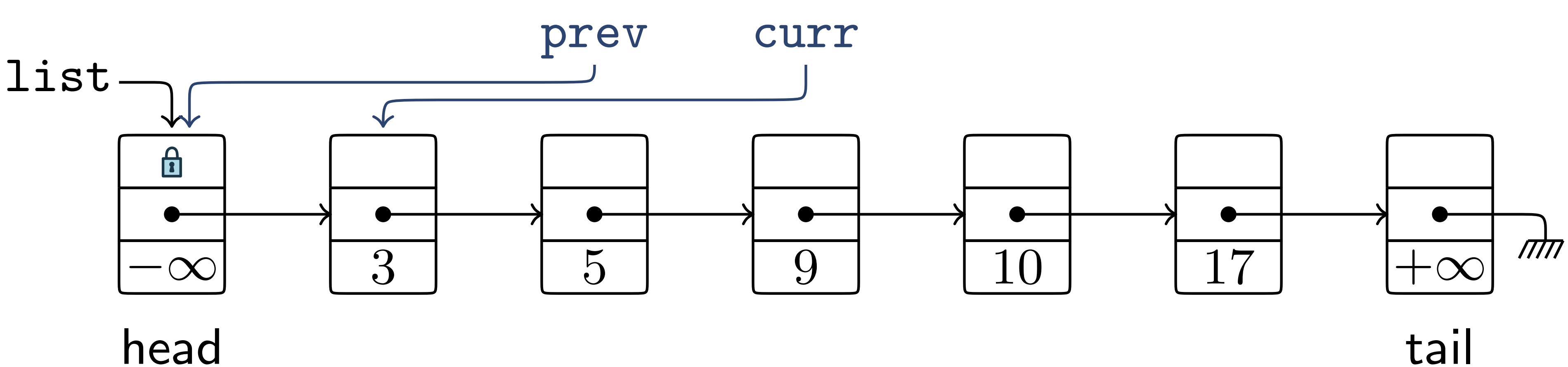

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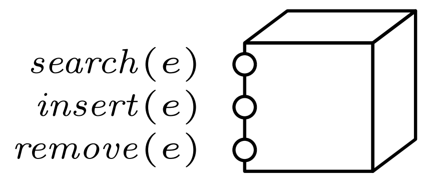

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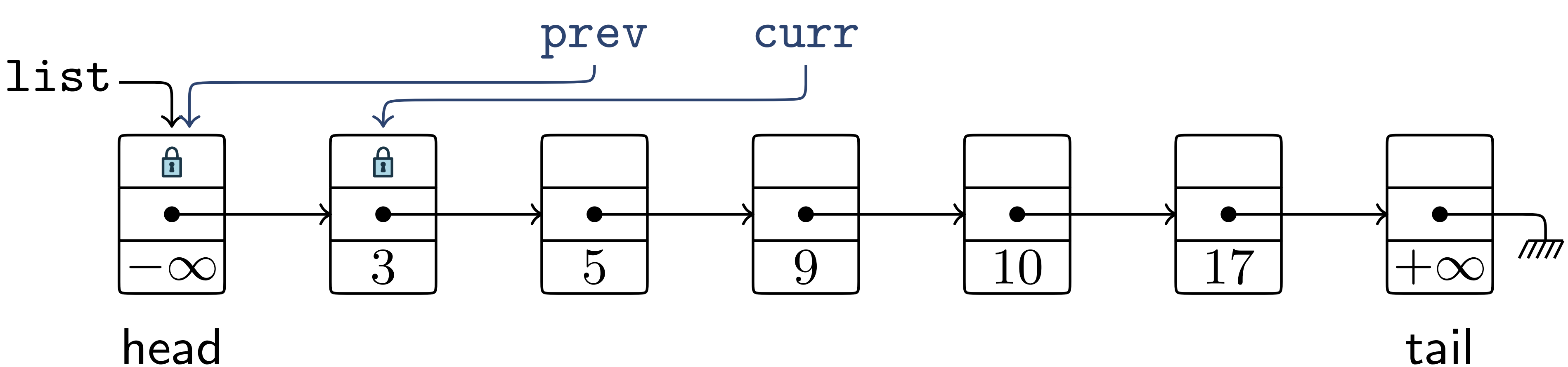

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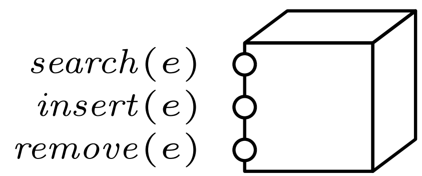

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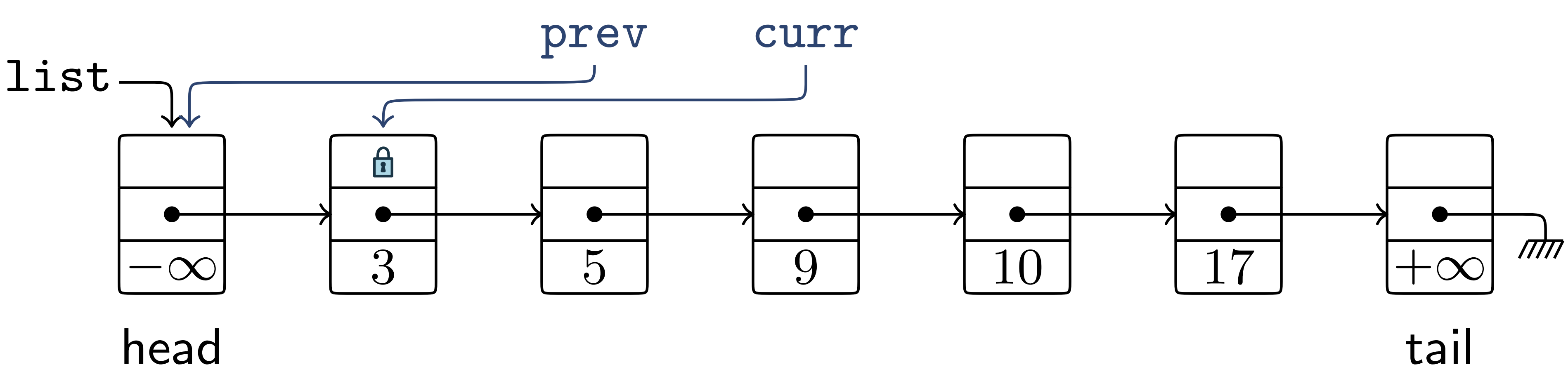

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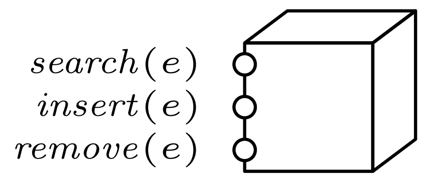

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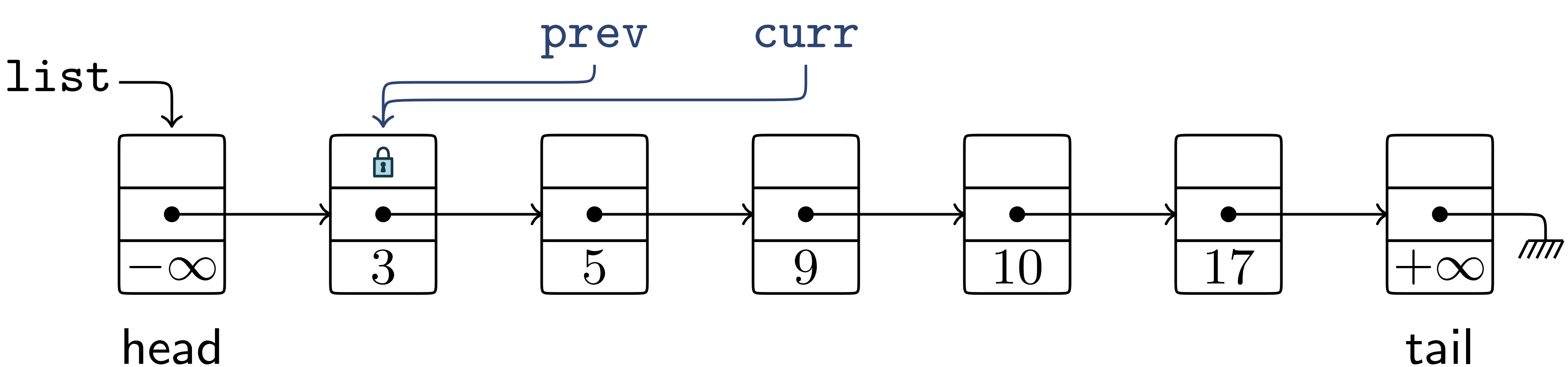

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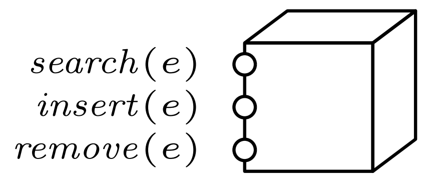

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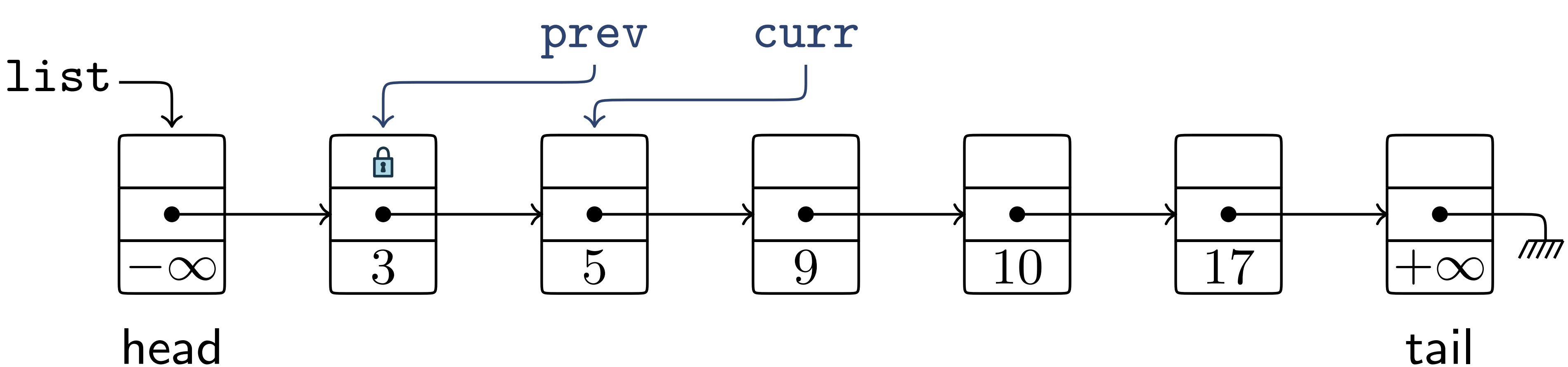

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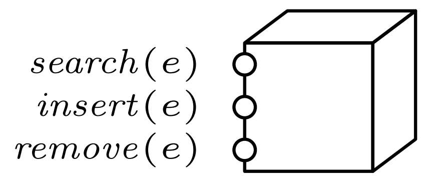

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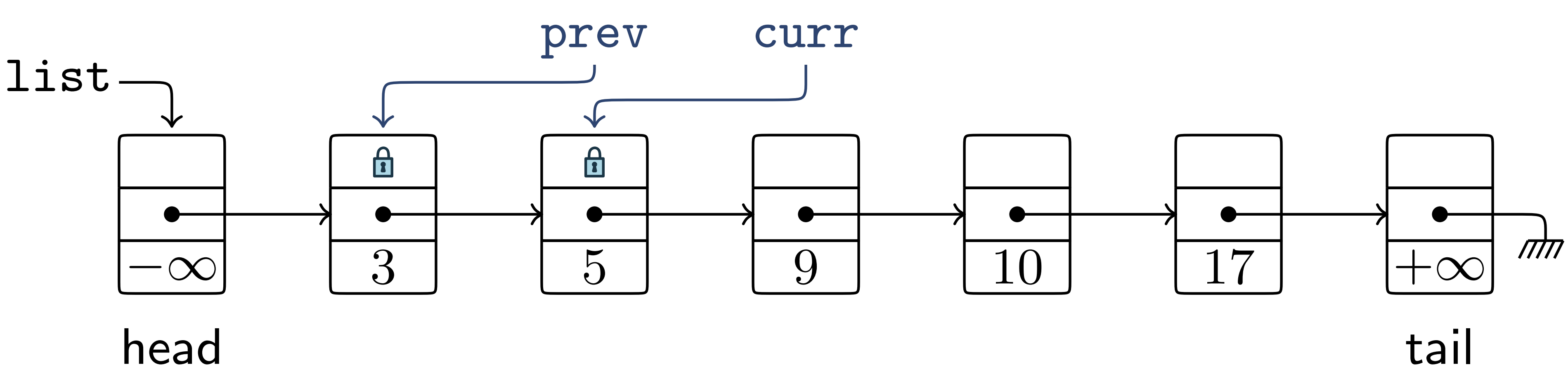

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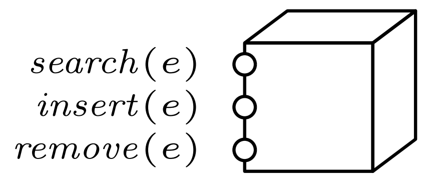

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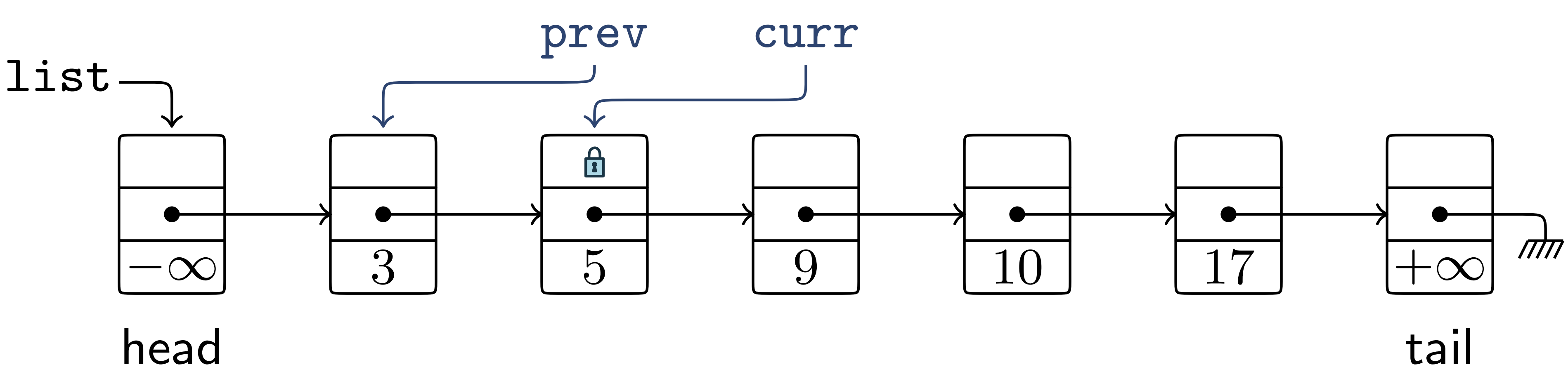
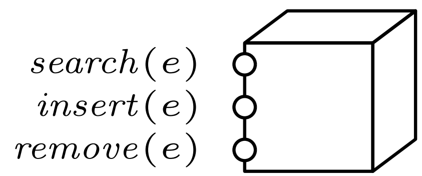
     Lock lock;
}
```

```
class List {
     Node list;
}
```



Concurrent Lock-coupling Lists

- A concurrent implementation of sets



search(8)

```
class Node {
      Value val;
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}
```

```
class List {
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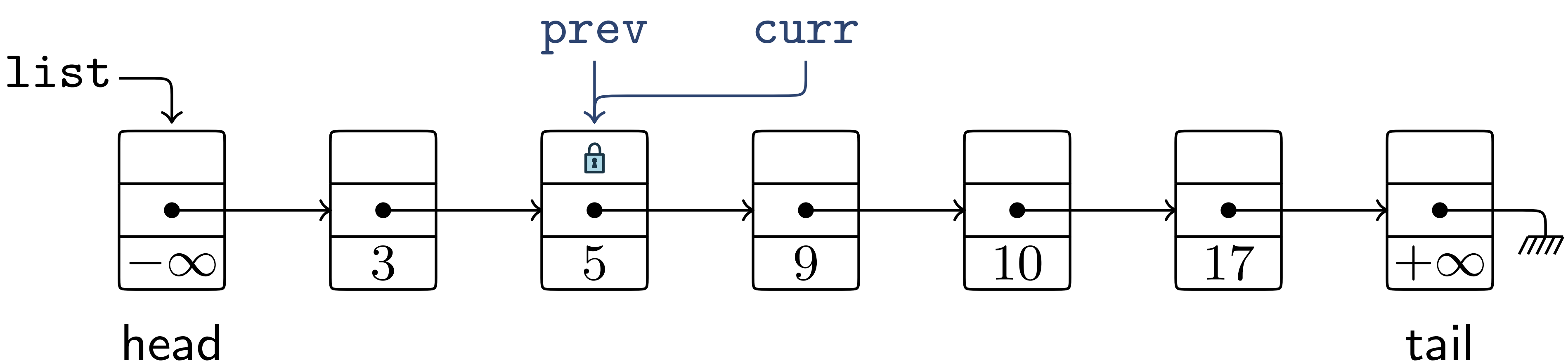
}
```



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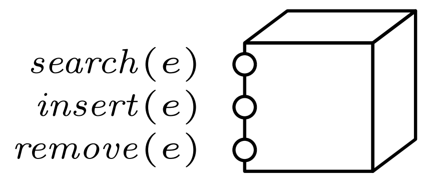

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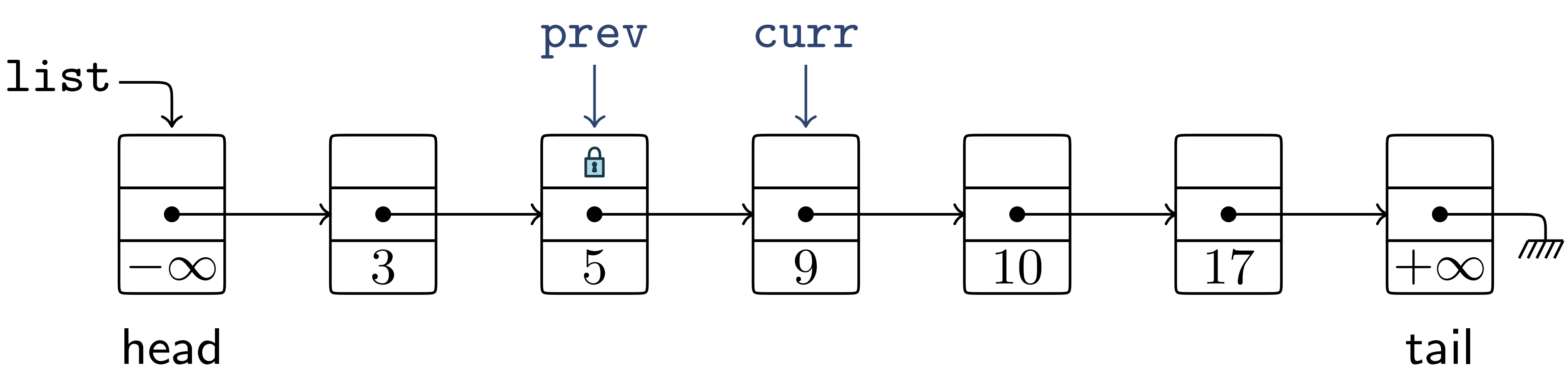

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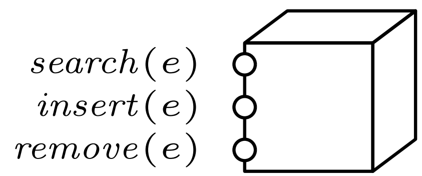

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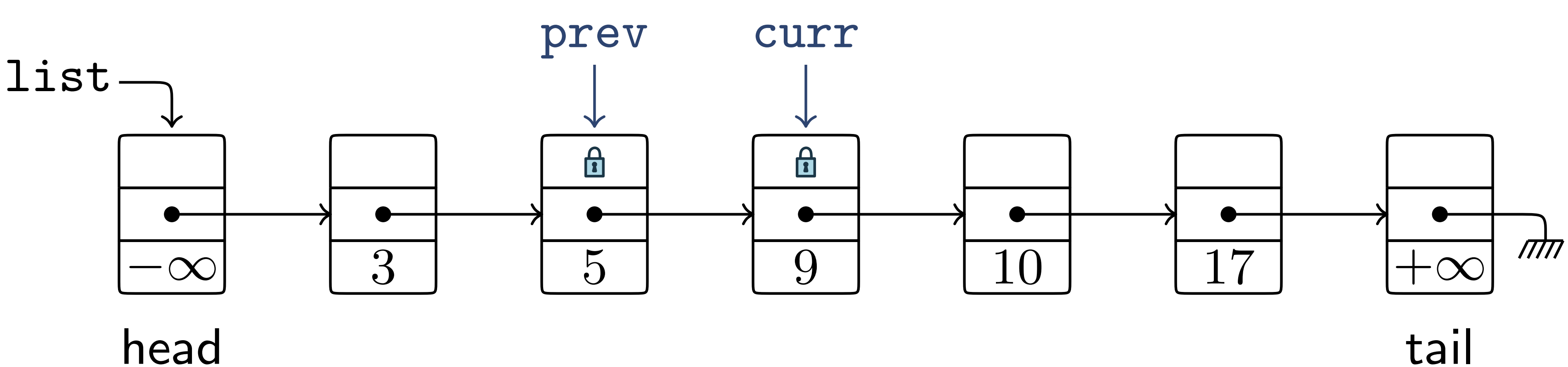

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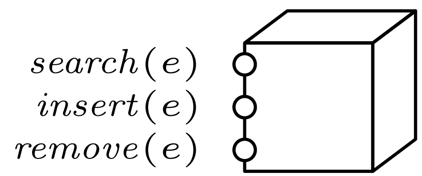

$insert(8)$

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class List {
      Node list;
}
```

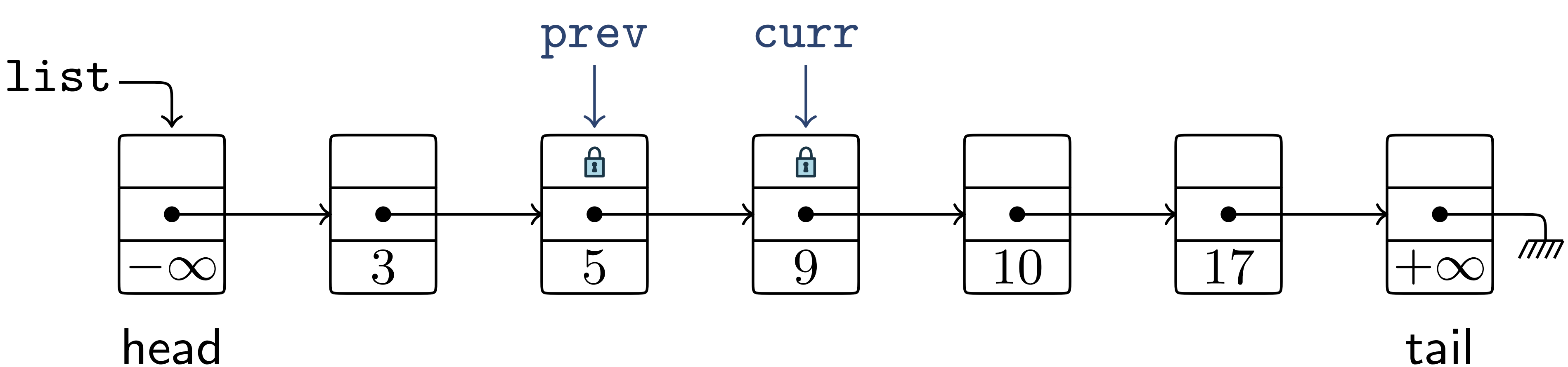

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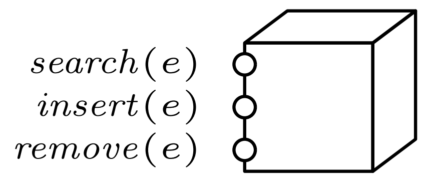

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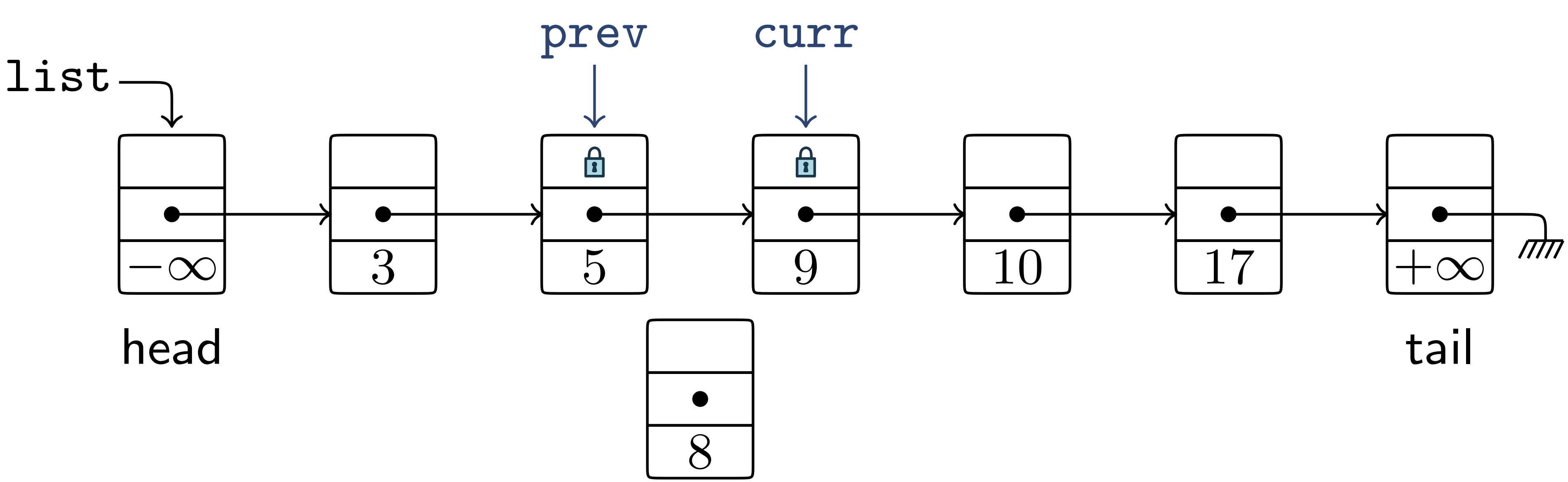

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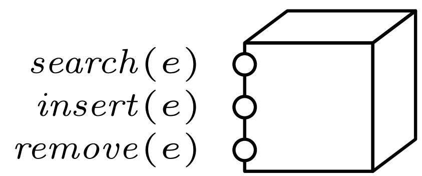

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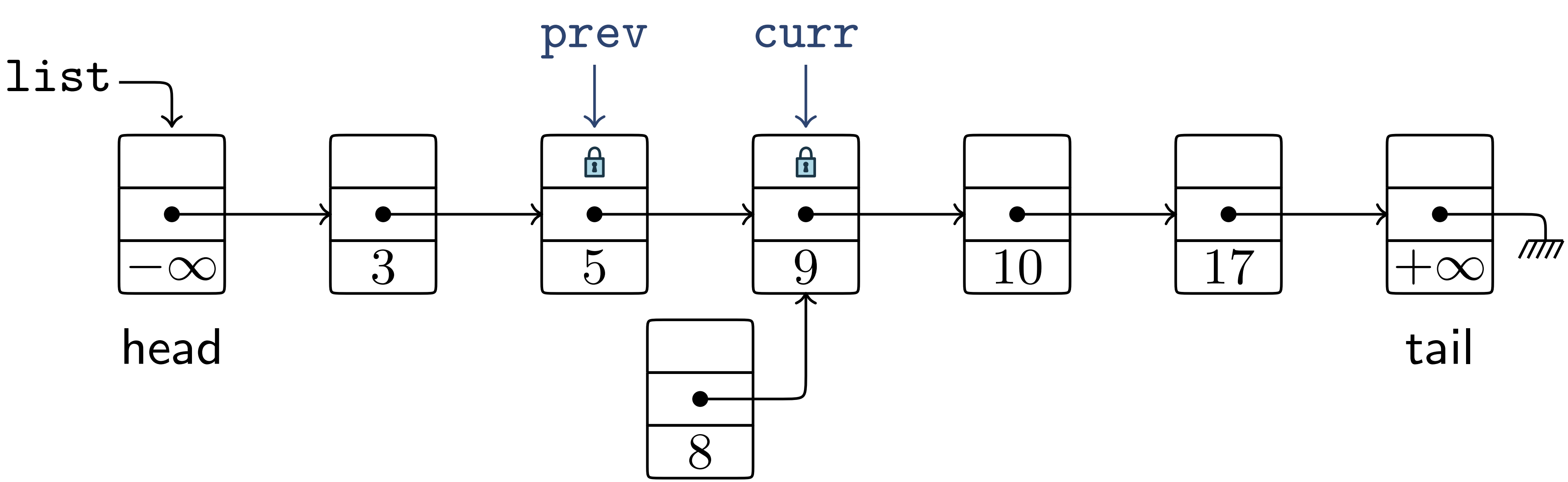

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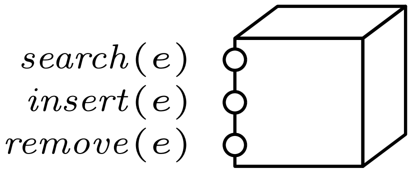

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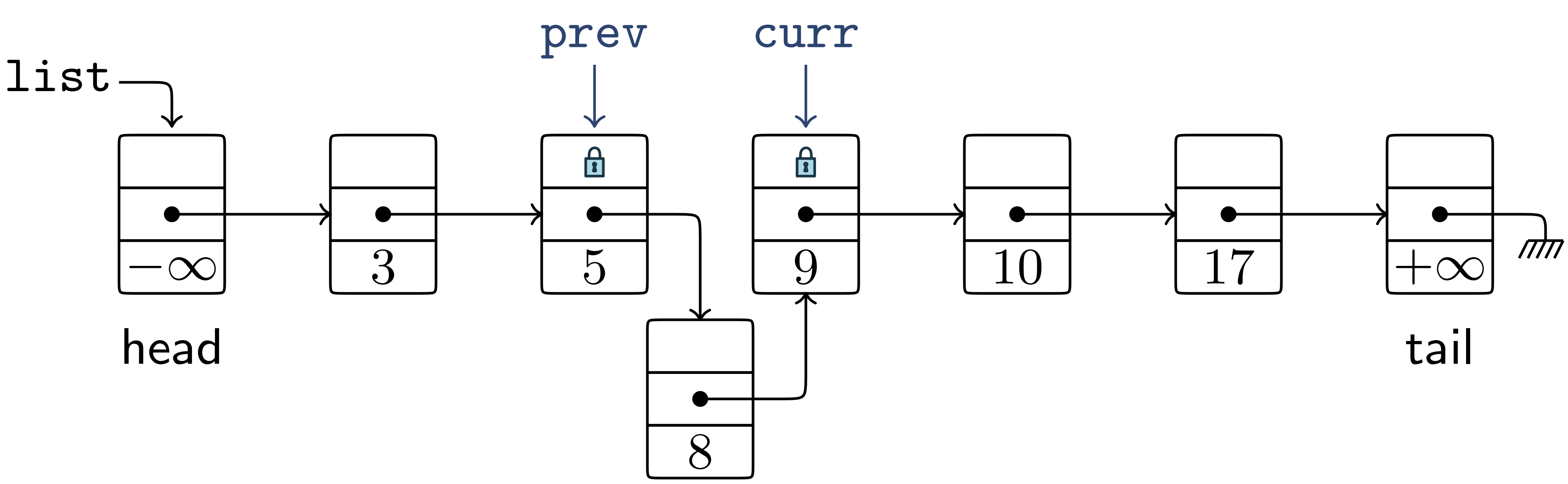

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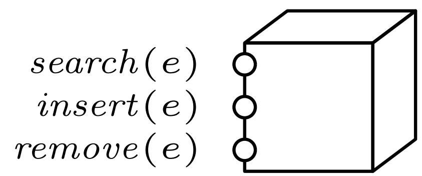


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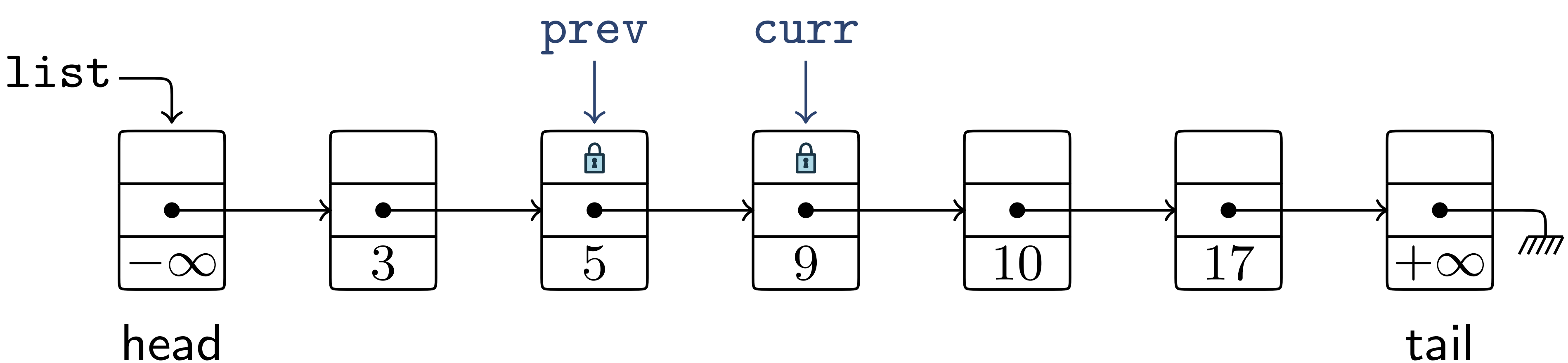

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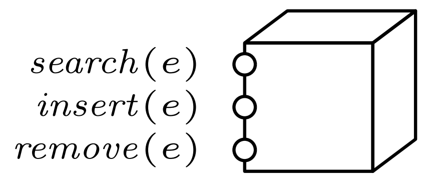

$remove(9)$

```
class Node {
      Value val;
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      Lock lock;
}
```
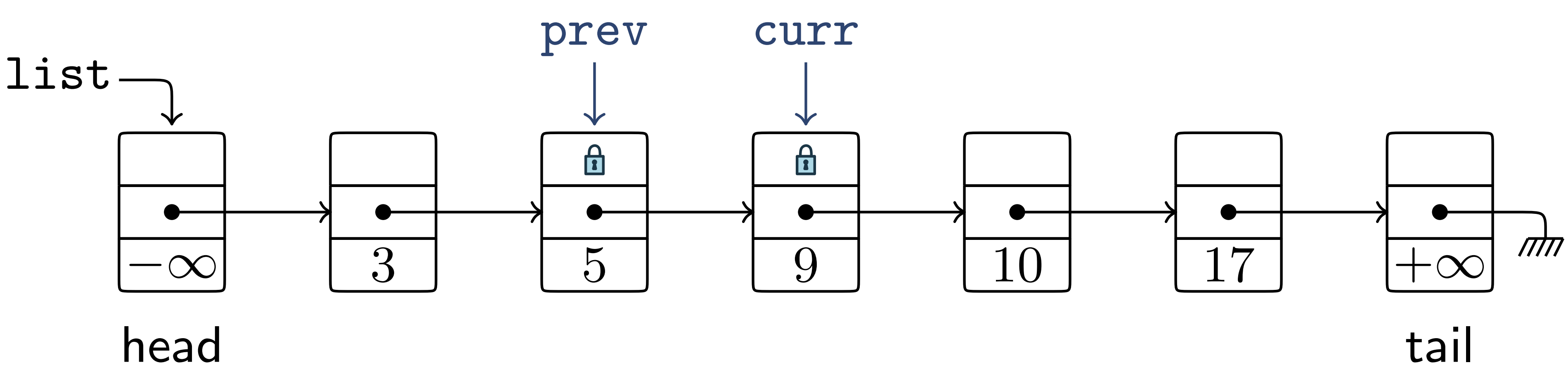

```
class List {
      Node list;
}
```



Concurrent Lock-coupling Lists

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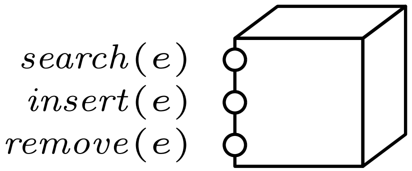

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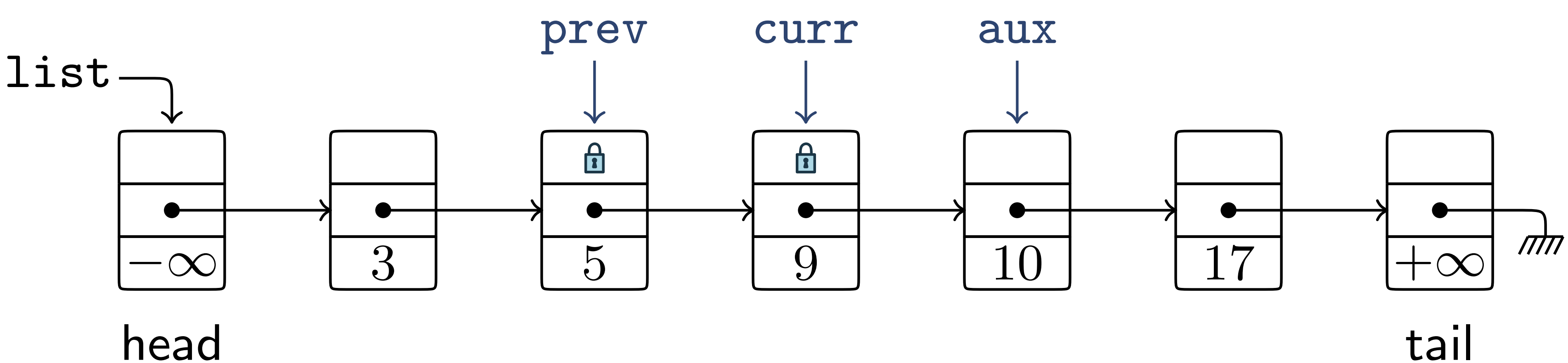

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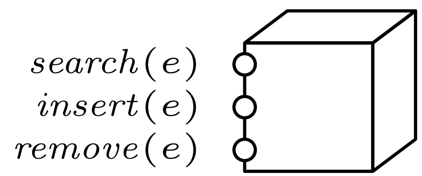

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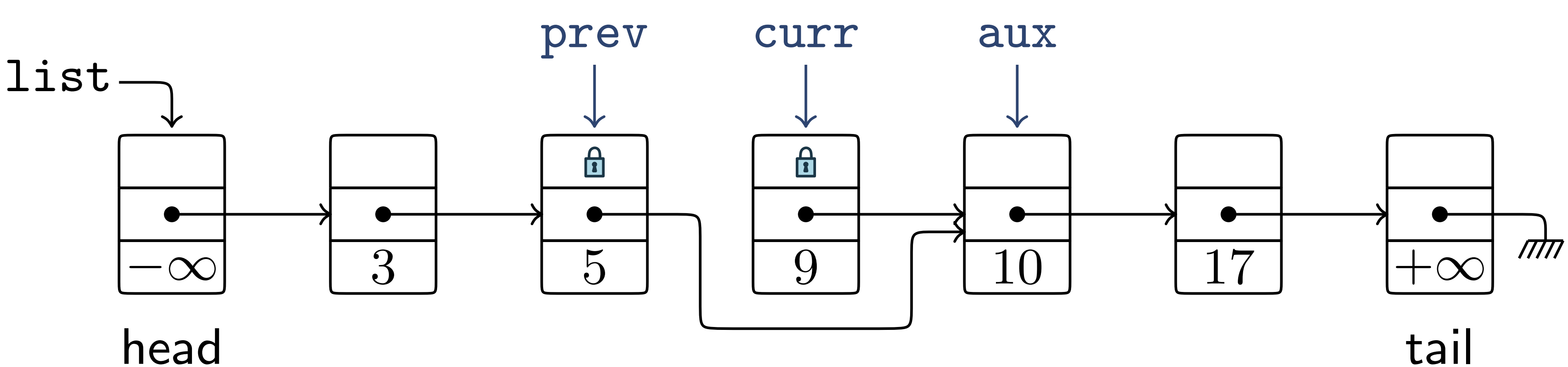

Verification Diagram for "Last Terminates"

Verification Diagram for "Last Terminates"

$$\mathcal{S}[N] \models \psi^{(k)}$$

Verification Diagram for "Last Terminates"

Concurrent execution of N "most general clients"

$$\mathcal{S}[N] \models \psi^{(k)}$$

Verification Diagram for "Last Terminates"

Concurrent execution of N "most general clients"

$$\mathcal{S}[N] \models \psi^{(k)}$$

Thread k holding the rightmost lock, terminates

Verification Diagram for "Last Terminates"

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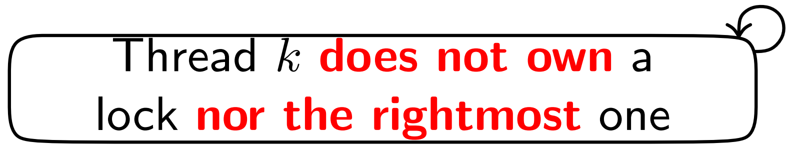
Verification Diagram for "Last Terminates"



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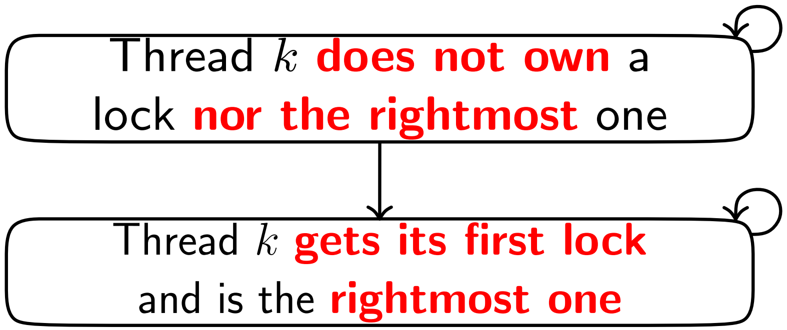

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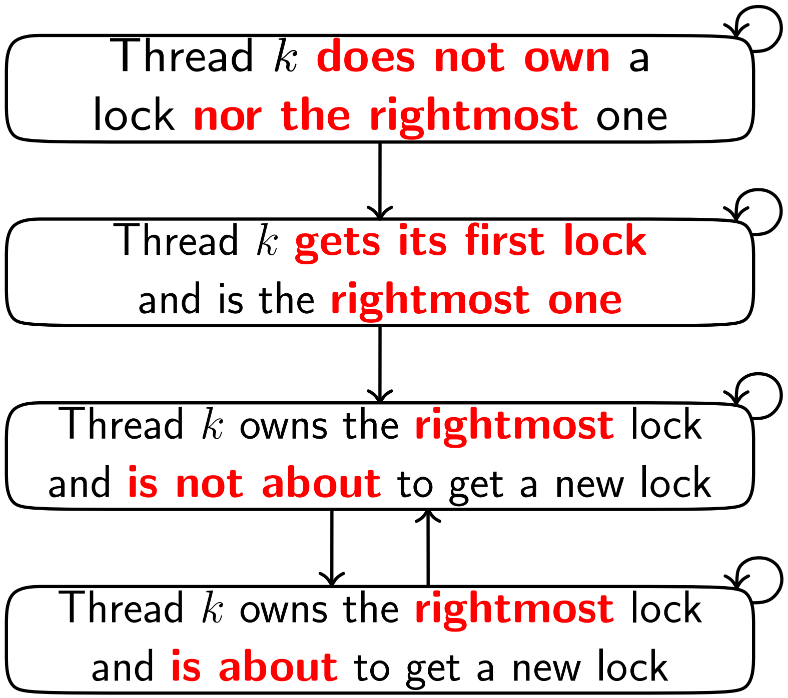

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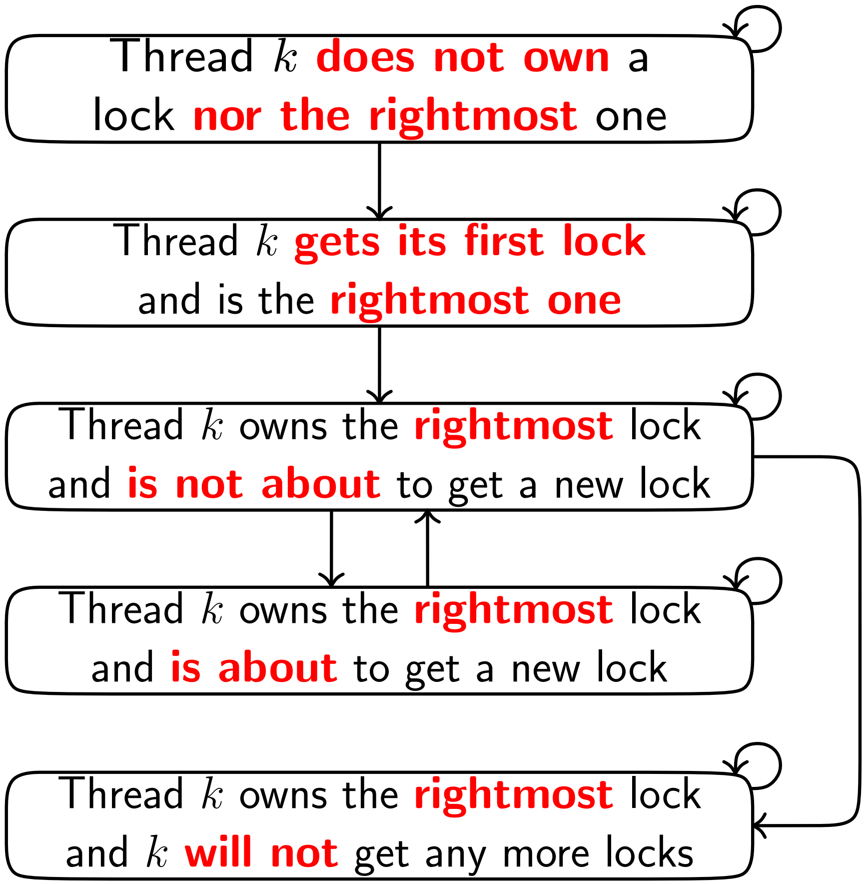

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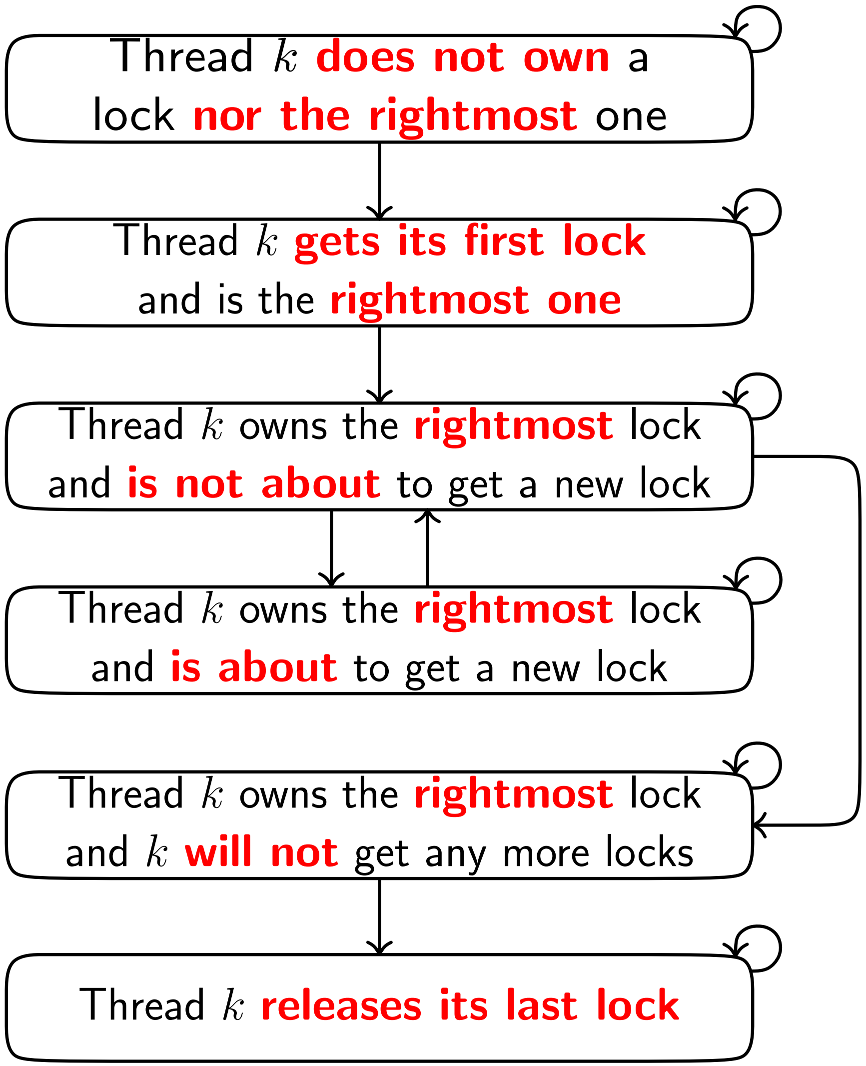

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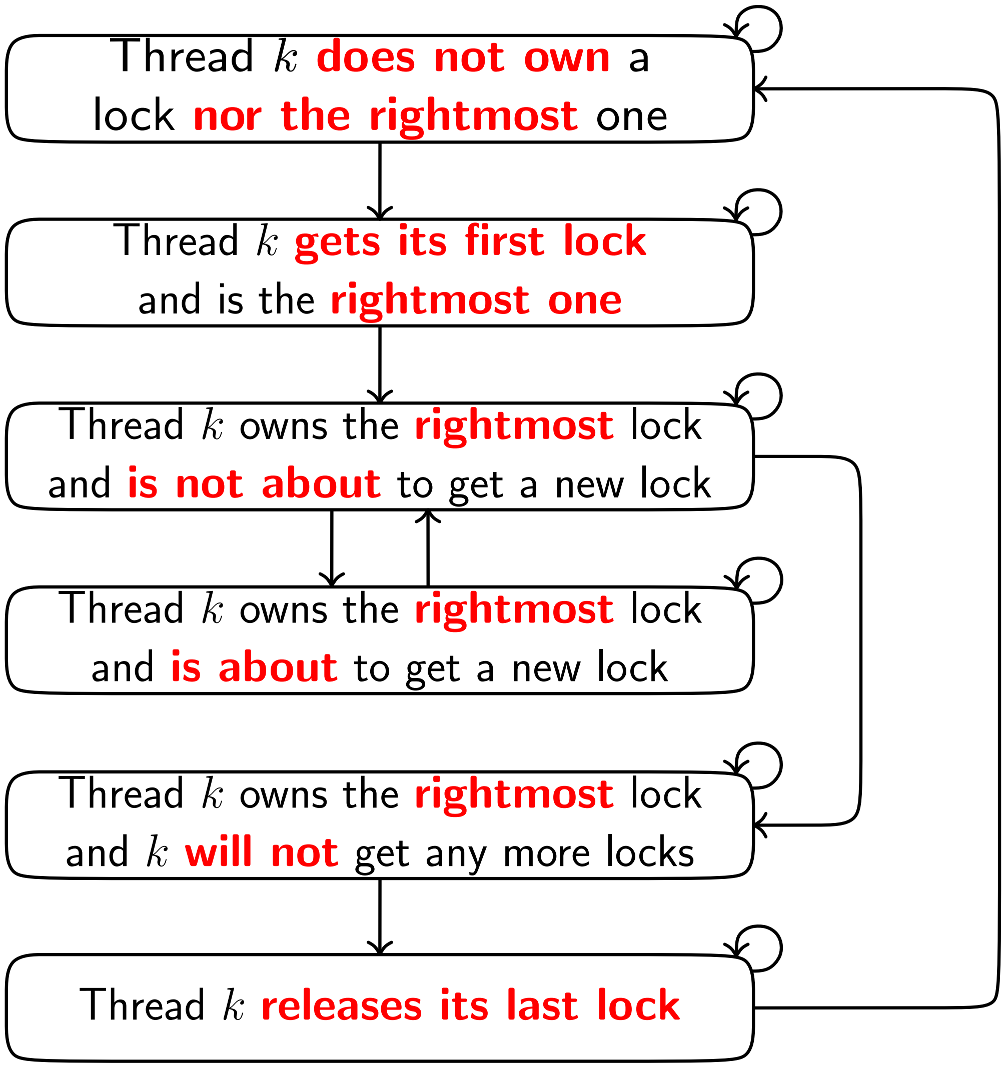

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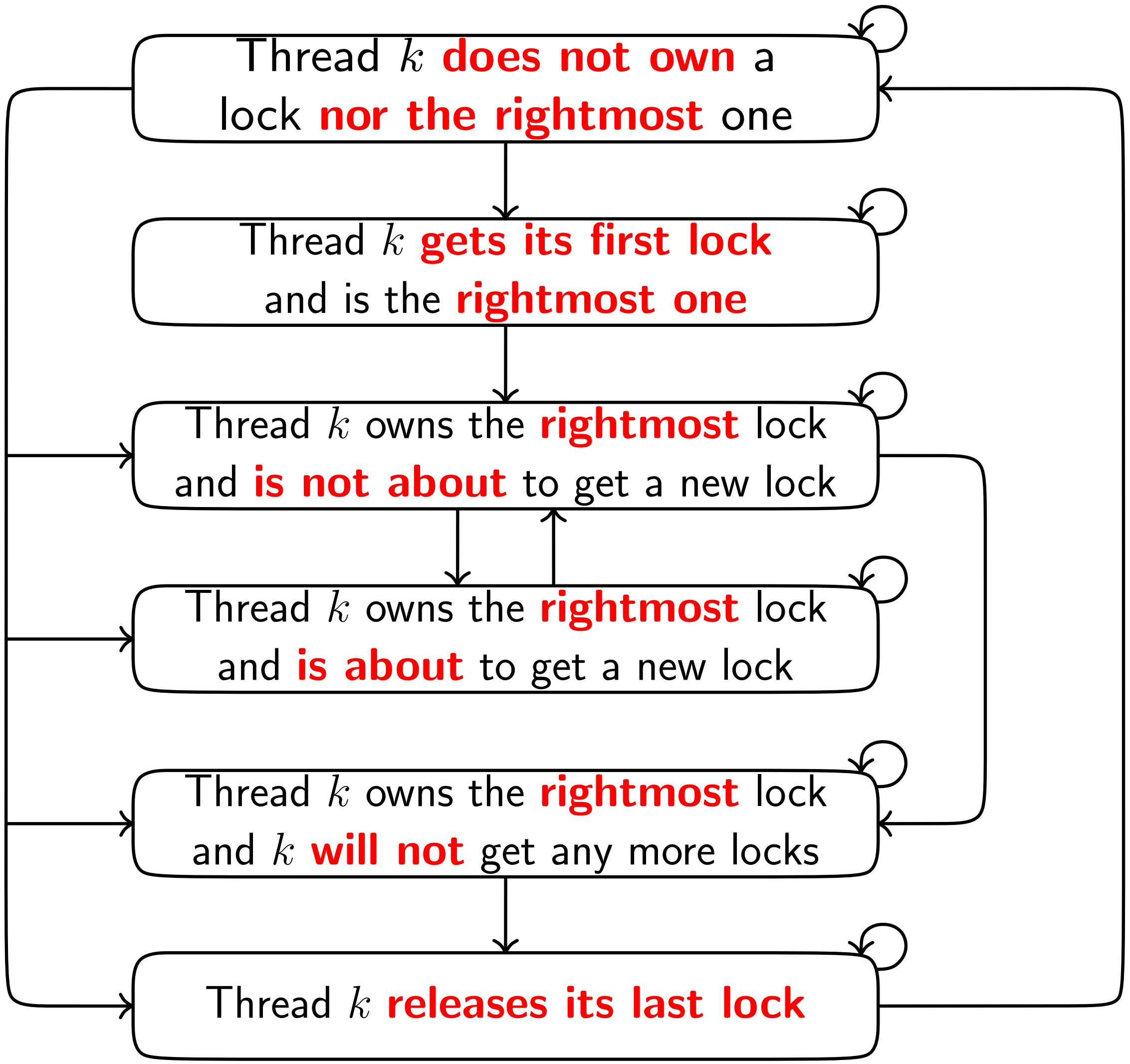

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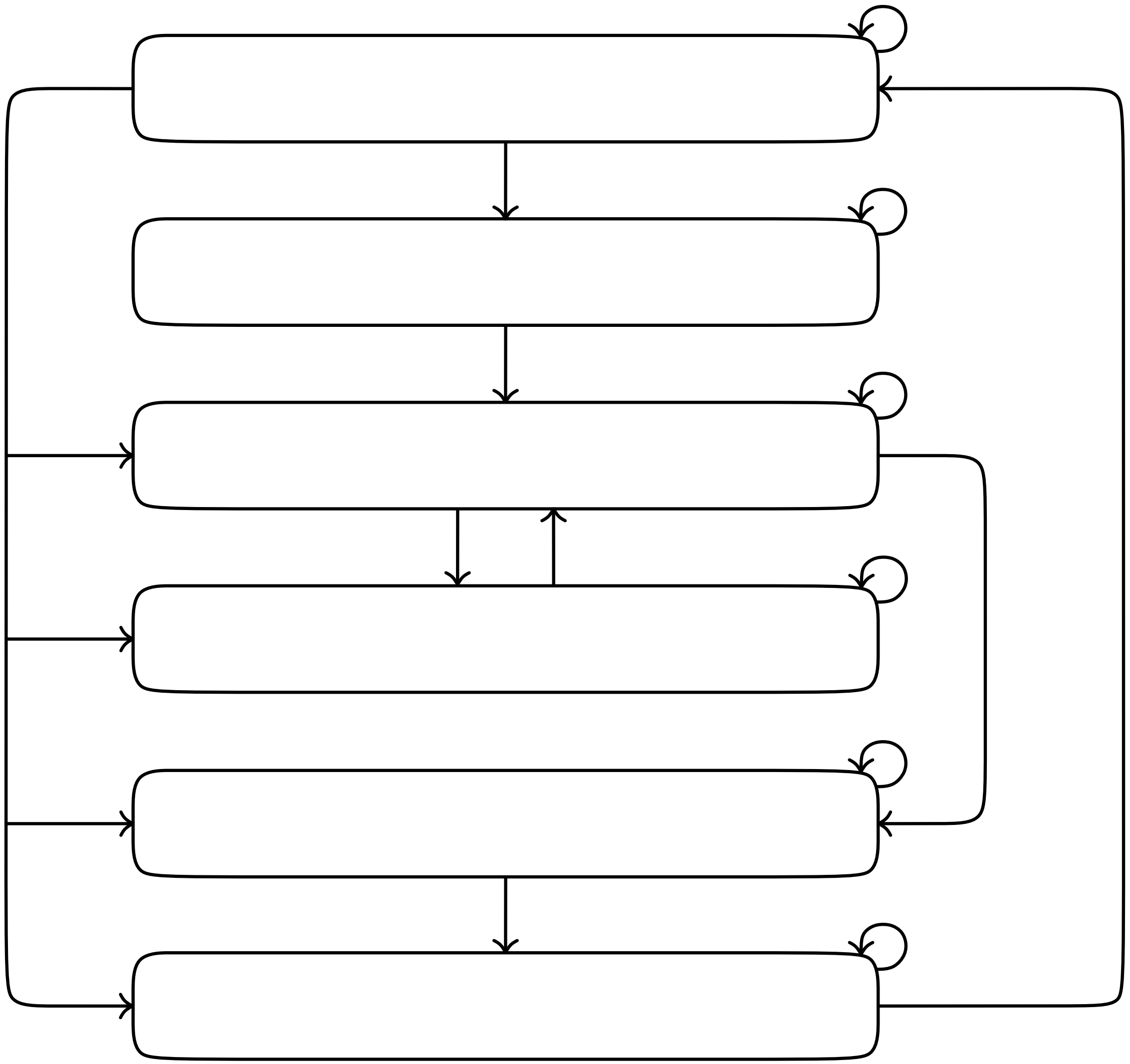

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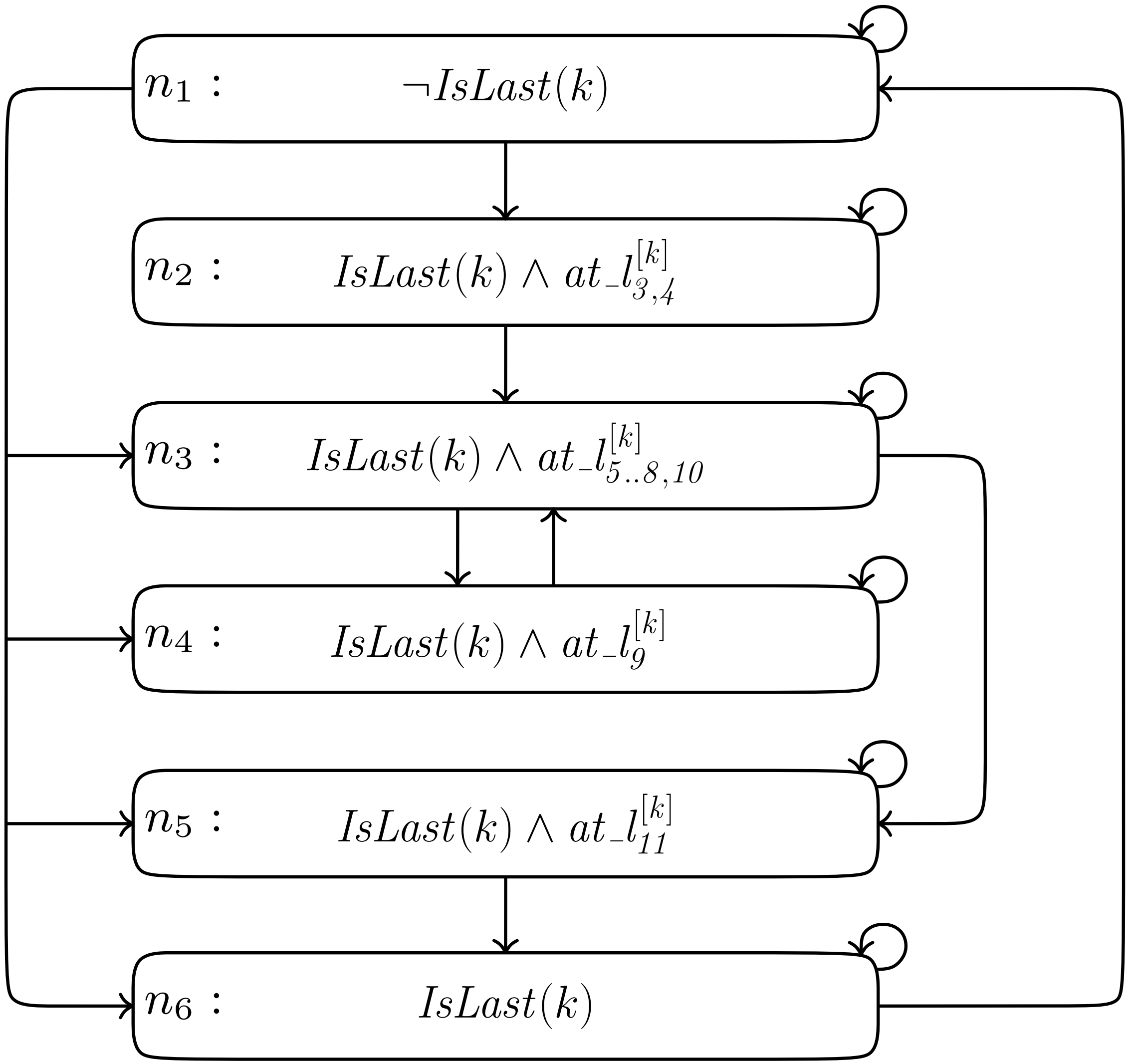

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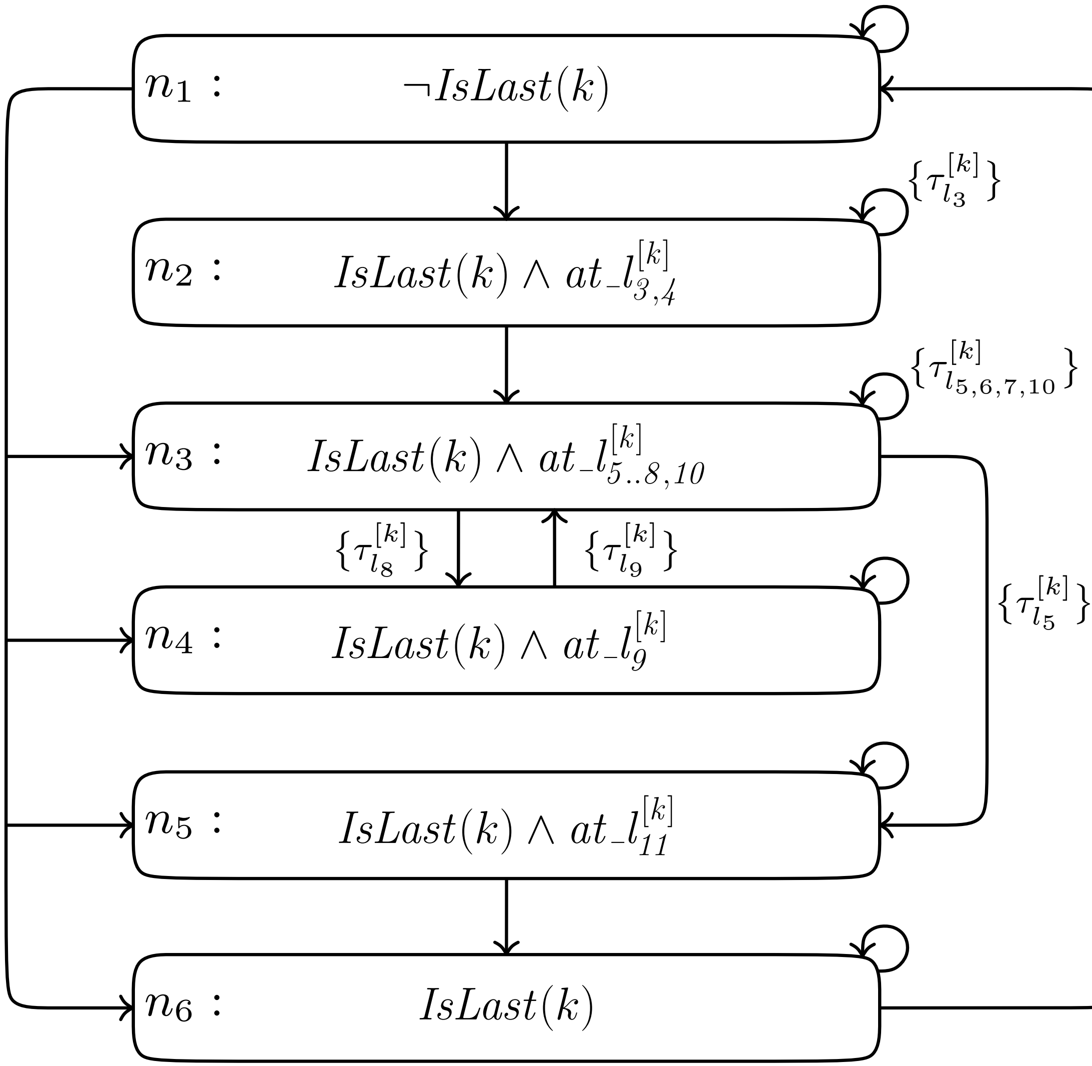

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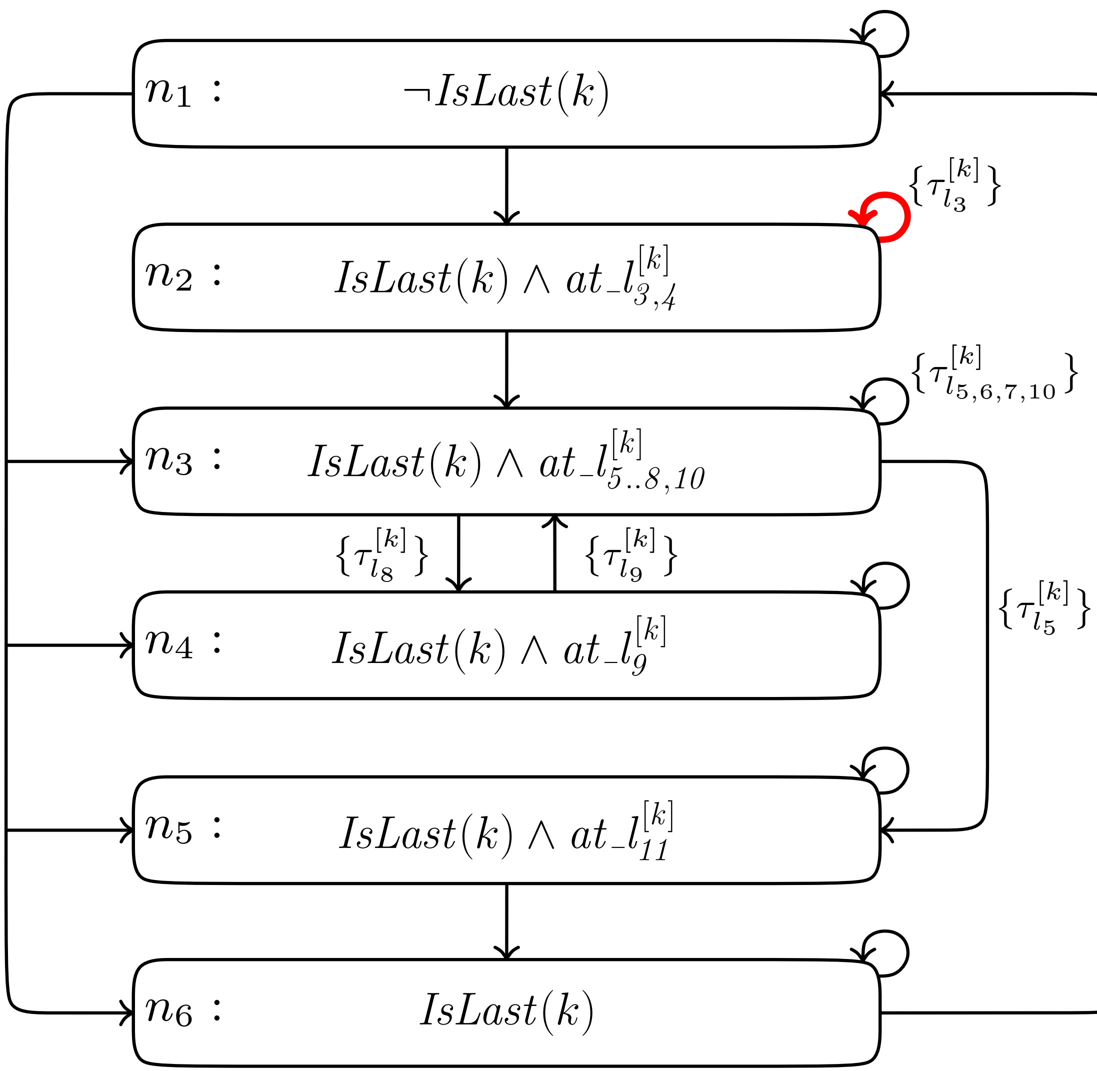

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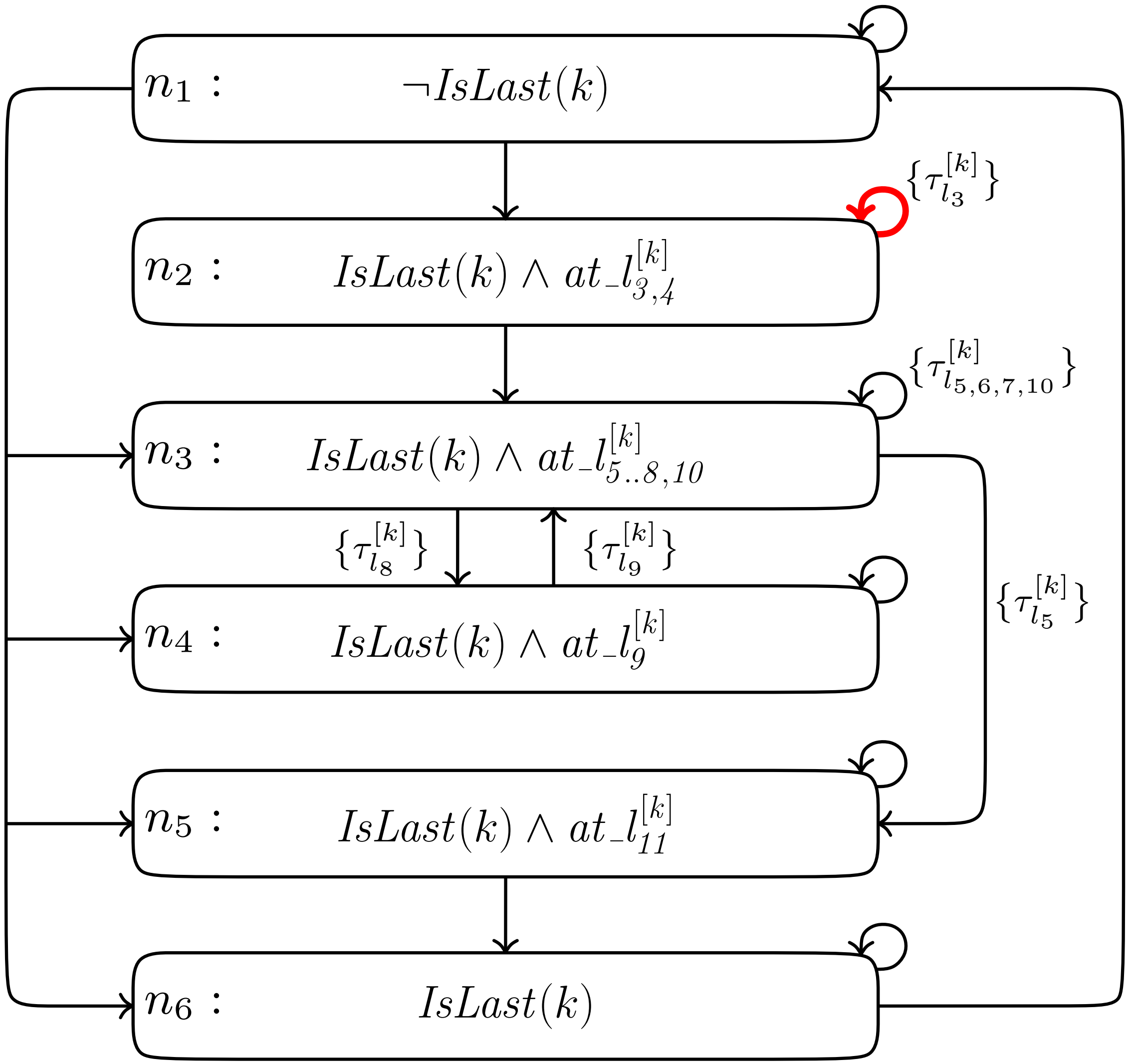
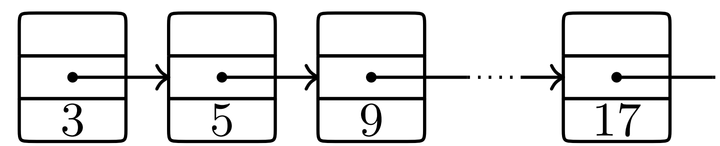


$$\mathcal{S}[N] \vDash \psi^{(k)}$$



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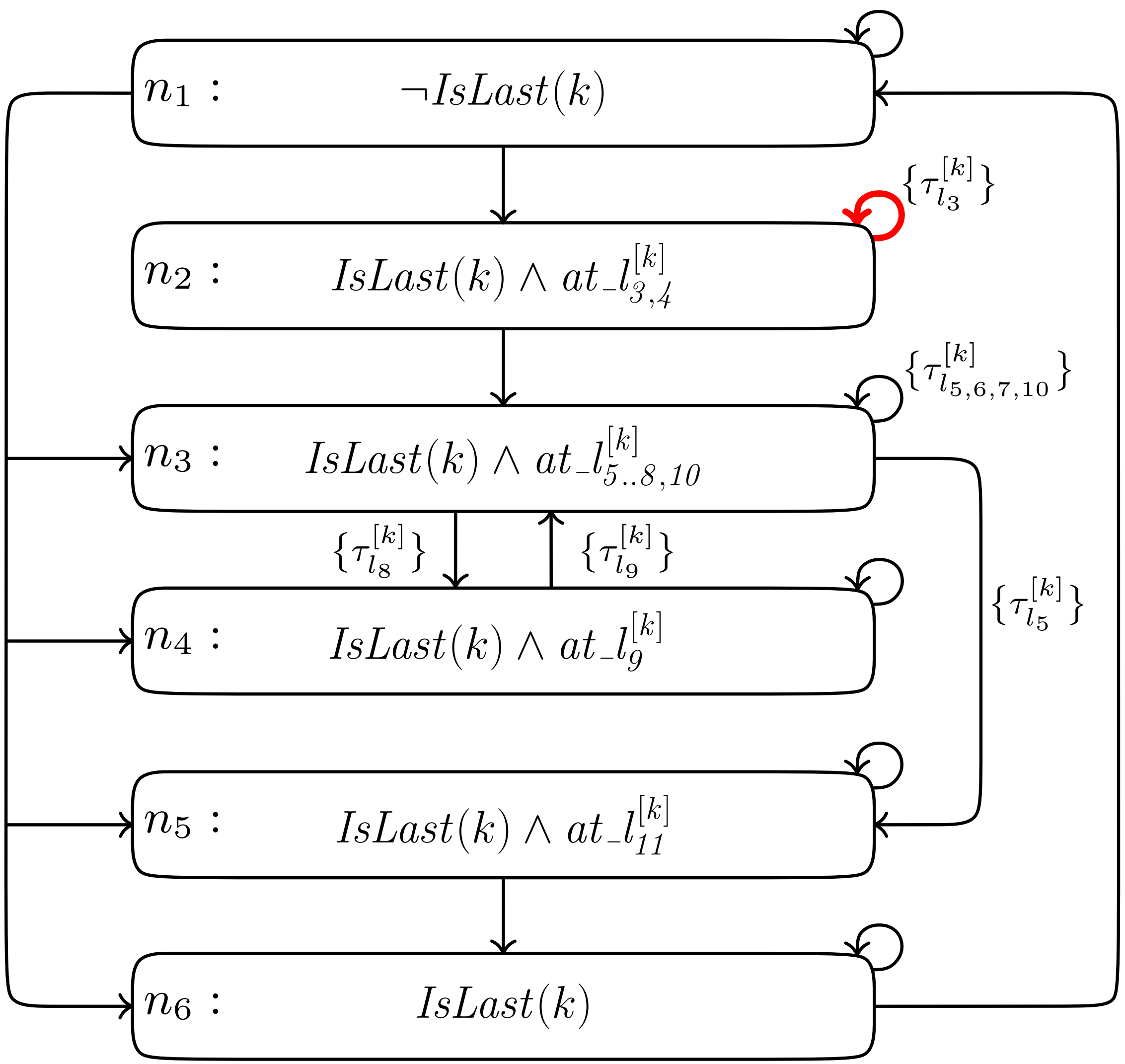

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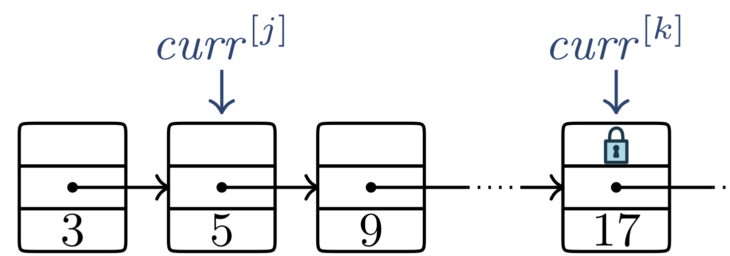

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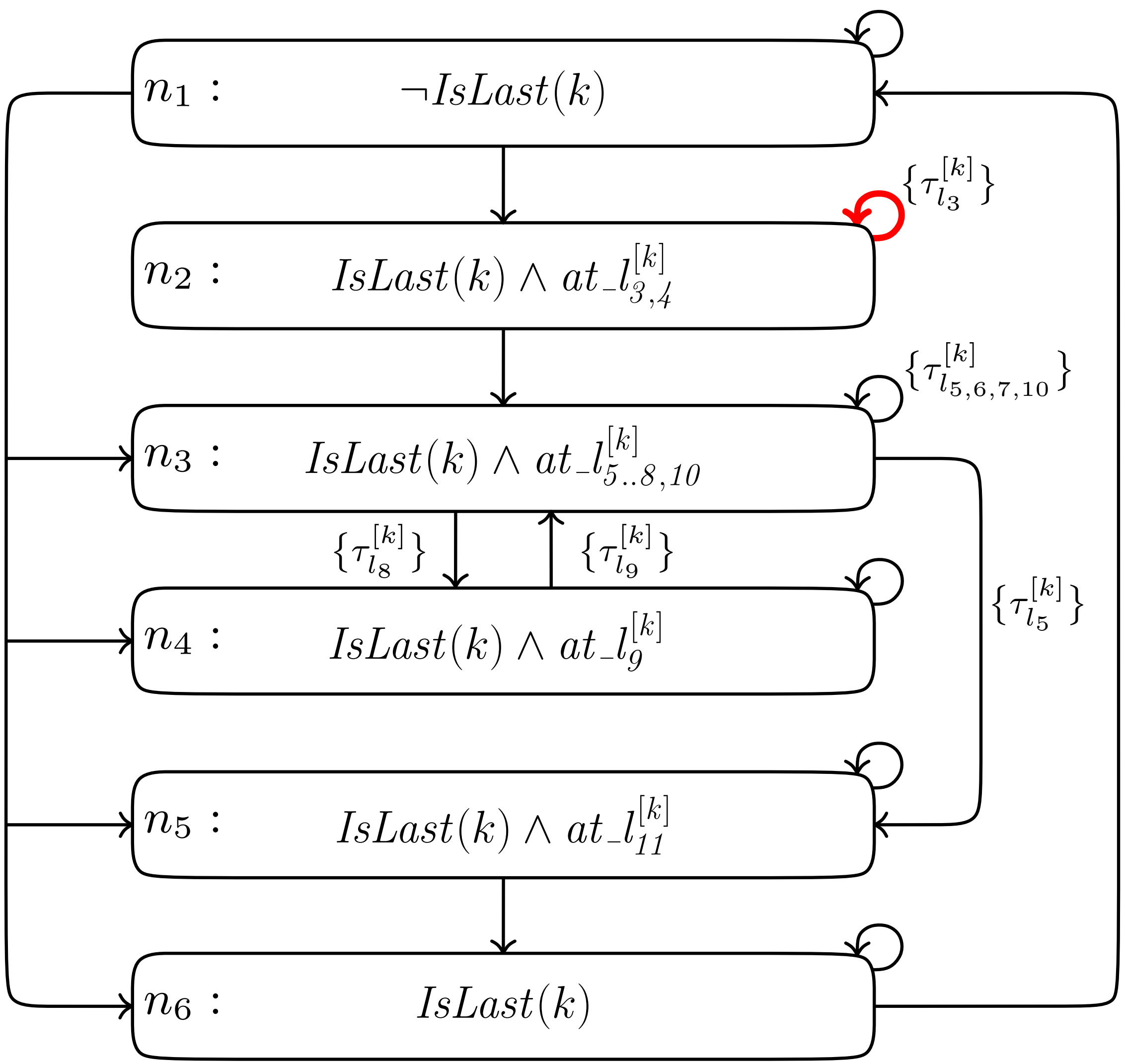

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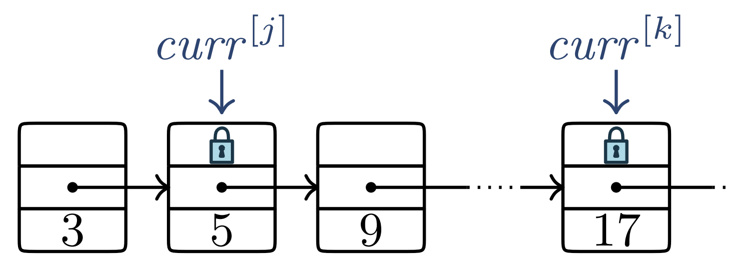

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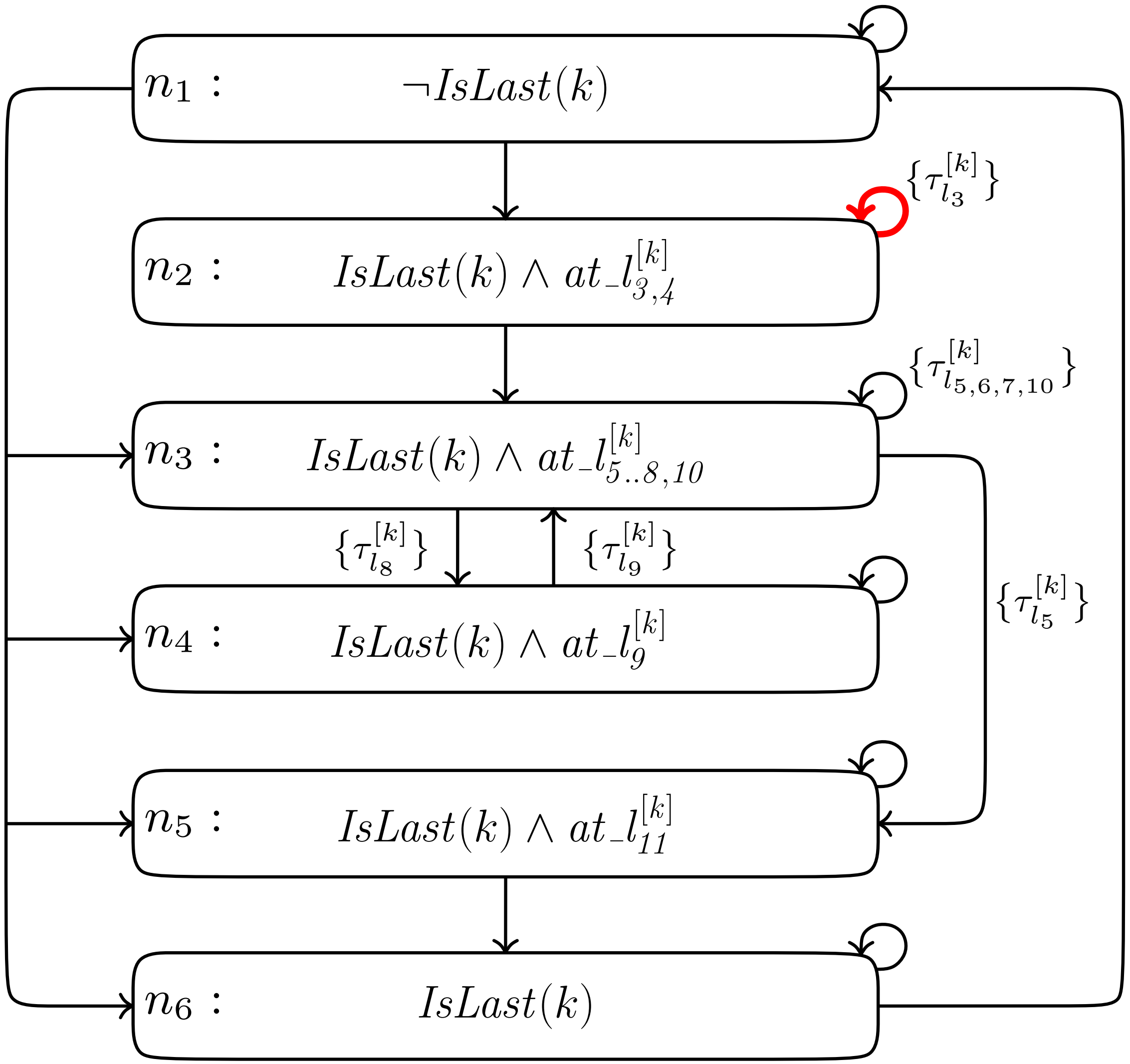

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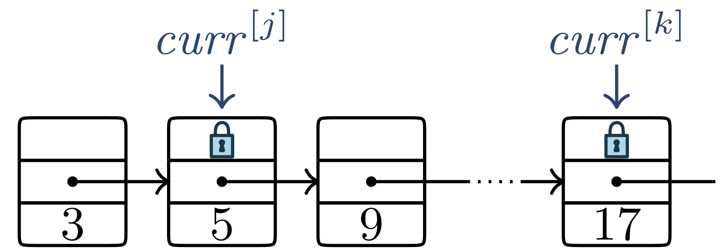
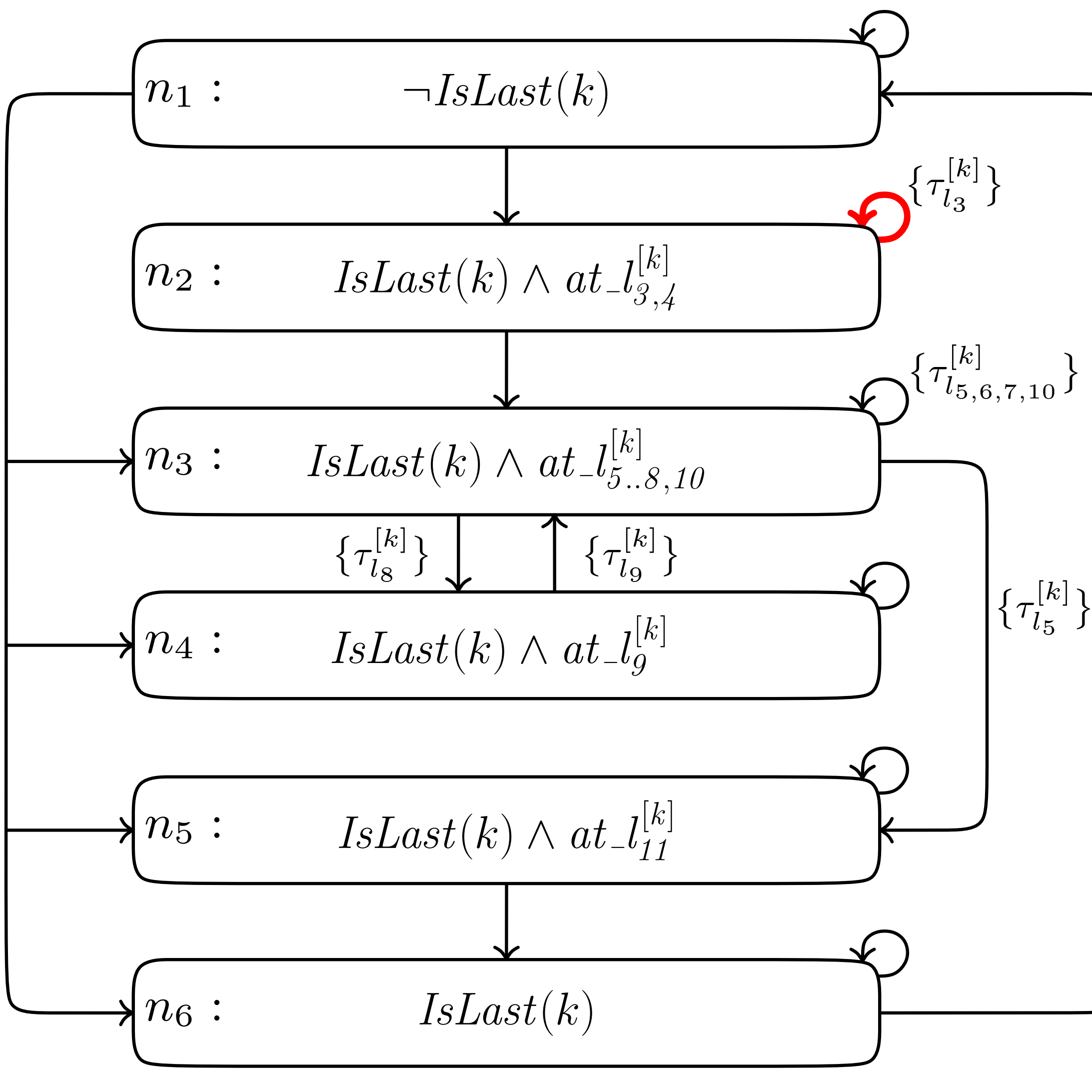


$$\begin{pmatrix} IsLast(k) \wedge \ j \neq k \wedge \\ at\_l^{[k]}_{3,4} \wedge at\_l^{[j]}_{9} \wedge \\ curr^{[j]}.lockid = \oslash \end{pmatrix} \wedge \ curr^{[j]}.lock(j) \rightarrow \begin{pmatrix} IsLast(k') \wedge at'\_l^{[k']}_{3,4} \wedge \\ at'\_l^{[j']}_{10} \wedge pres(V - curr^{[j]}) \wedge \\ curr'^{[j']}.lockid = j' \end{pmatrix}$$

Verification Diagram for "Last Terminates"



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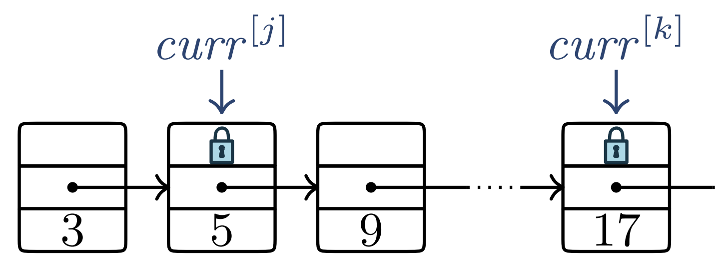

Reason about:

locks

$$\begin{pmatrix} IsLast(k) \wedge \ j \neq k \ \wedge \\ at\_l_{3,4}^{[k]} \wedge at\_l_{9}^{[j]} \wedge \\ curr^{[j]}.lockid = \oslash \end{pmatrix} \wedge curr^{[j]}.lock(j) \rightarrow \begin{pmatrix} IsLast(k') \wedge at'\_l_{3,4}^{[k']} \ \wedge \\ at'\_l_{10}^{[j']} \wedge pres(V - curr^{[j]}) \ \wedge \\ curr'^{[j']}.lockid = j' \end{pmatrix}$$

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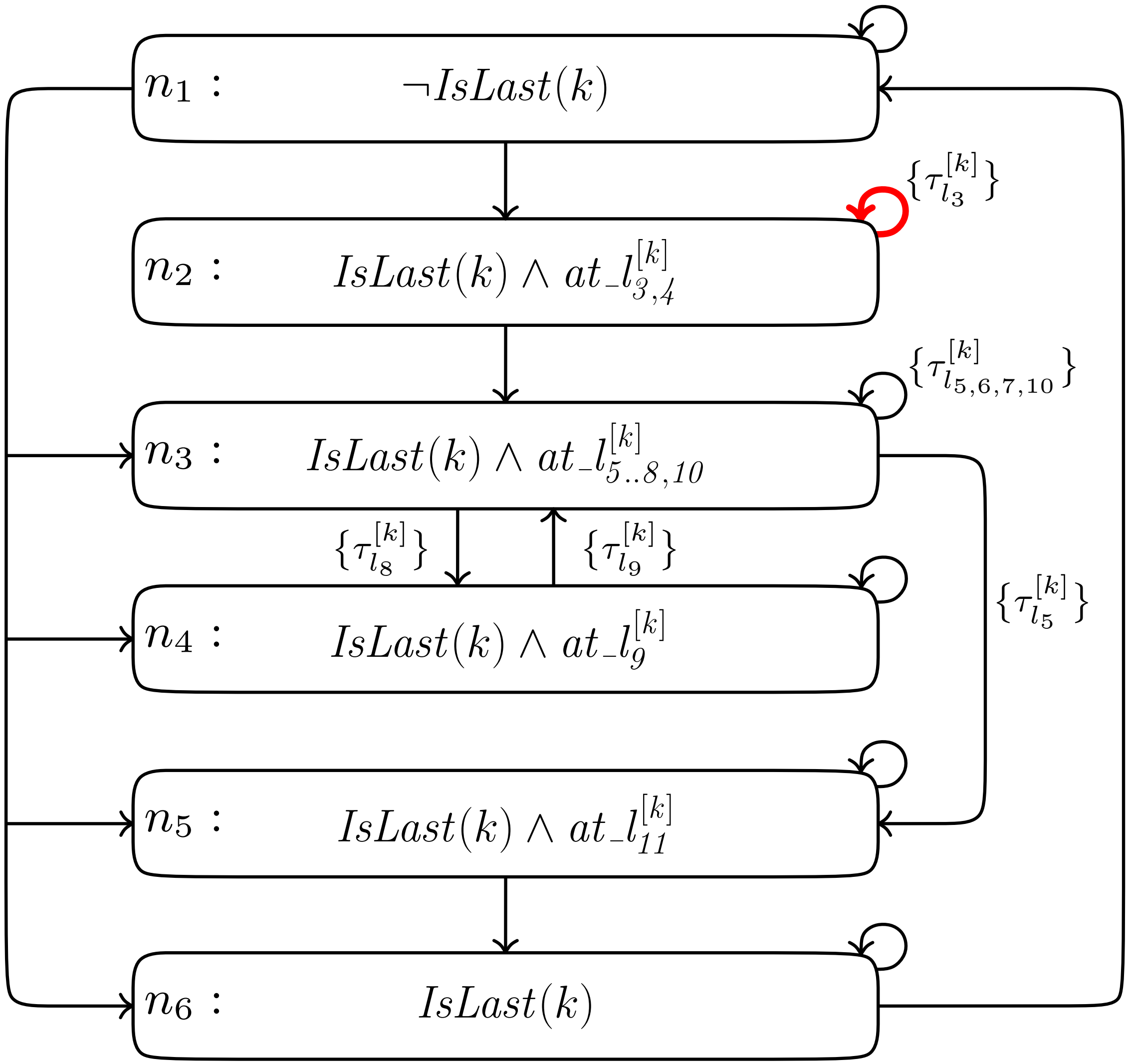

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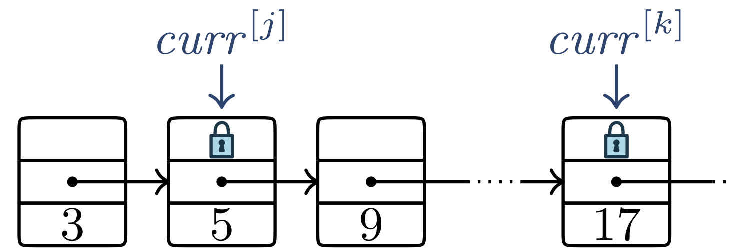

Reason about:

thread identifiers

$$\begin{pmatrix} IsLast(k) \wedge j \neq k \wedge \\ at\_l_{3,4}^{[k]} \wedge at\_l_9^{[j]} \wedge \\ curr^{[j]}.lockid = \emptyset \end{pmatrix} \wedge \; curr^{[j]}.lock(j) \rightarrow \begin{pmatrix} IsLast(k') \wedge at'\_l_{3,4}^{[k']} \wedge \\ at'\_l_{10}^{[j']} \wedge pres(V - curr^{[j]}) \wedge \\ curr'^{[j']}.lockid = j' \end{pmatrix}$$

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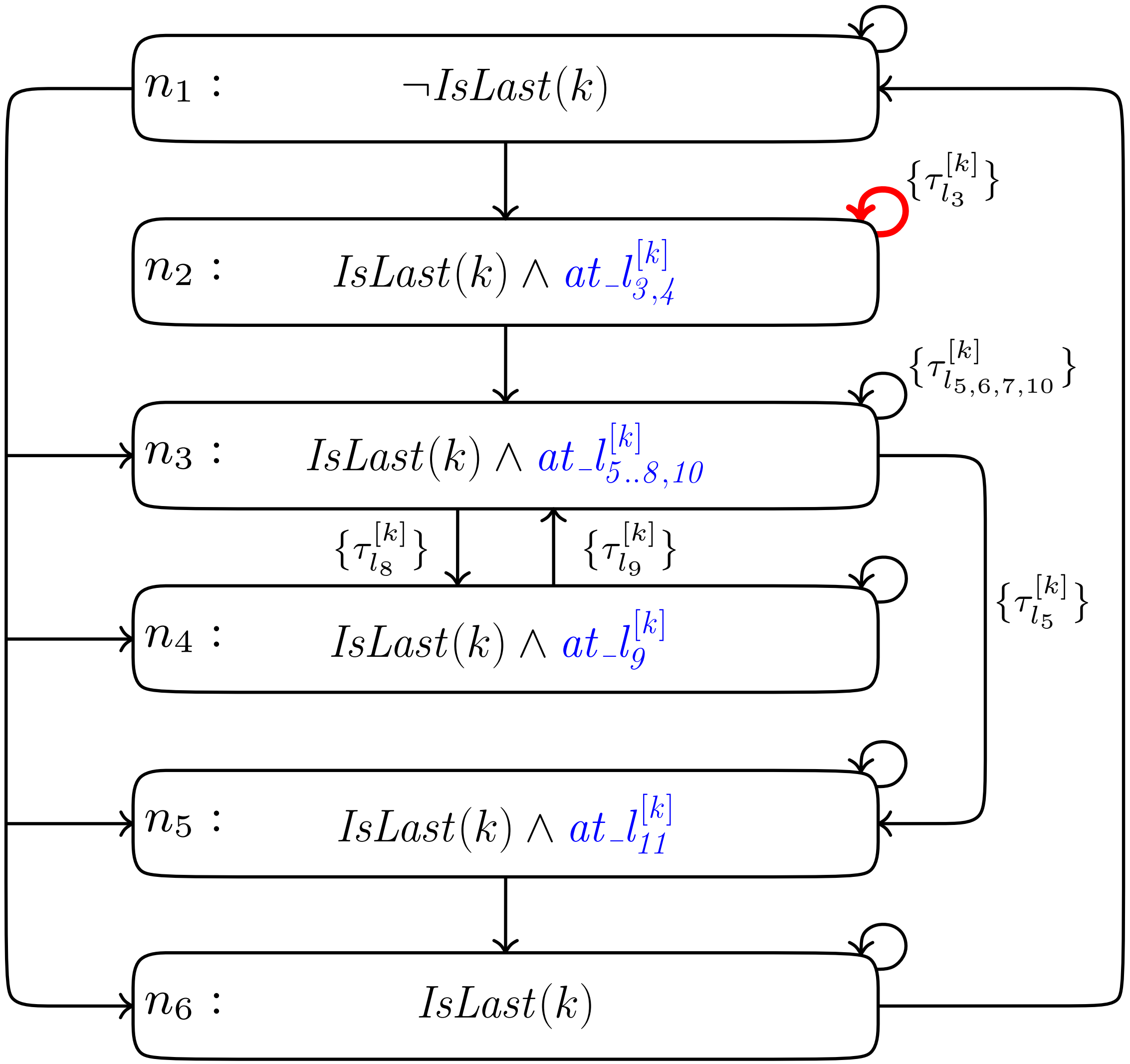
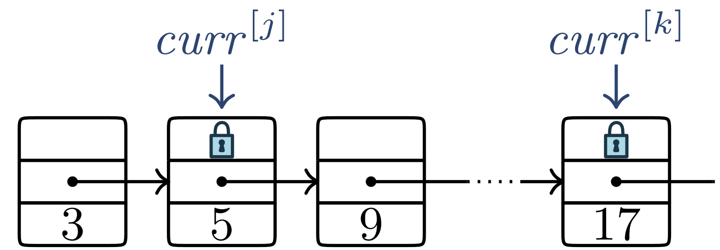


$$\mathcal{S}[N] \vDash \psi^{(k)}$$



Reason about:

program position

$$\begin{pmatrix} IsLast(k) \wedge \ j \neq k \wedge \\ at\_l^{[k]}_{3,4} \wedge at\_l^{[j]}_{9} \wedge \\ curr^{[j]}.lockid = \emptyset \end{pmatrix} \wedge \ curr^{[j]}.lock(j) \rightarrow \begin{pmatrix} IsLast(k') \wedge at'\_l^{[k']}_{3,4} \wedge \\ at'\_l^{[j']}_{10} \wedge pres(V - curr^{[j]}) \wedge \\ curr'^{[j']}.lockid = j' \end{pmatrix}$$

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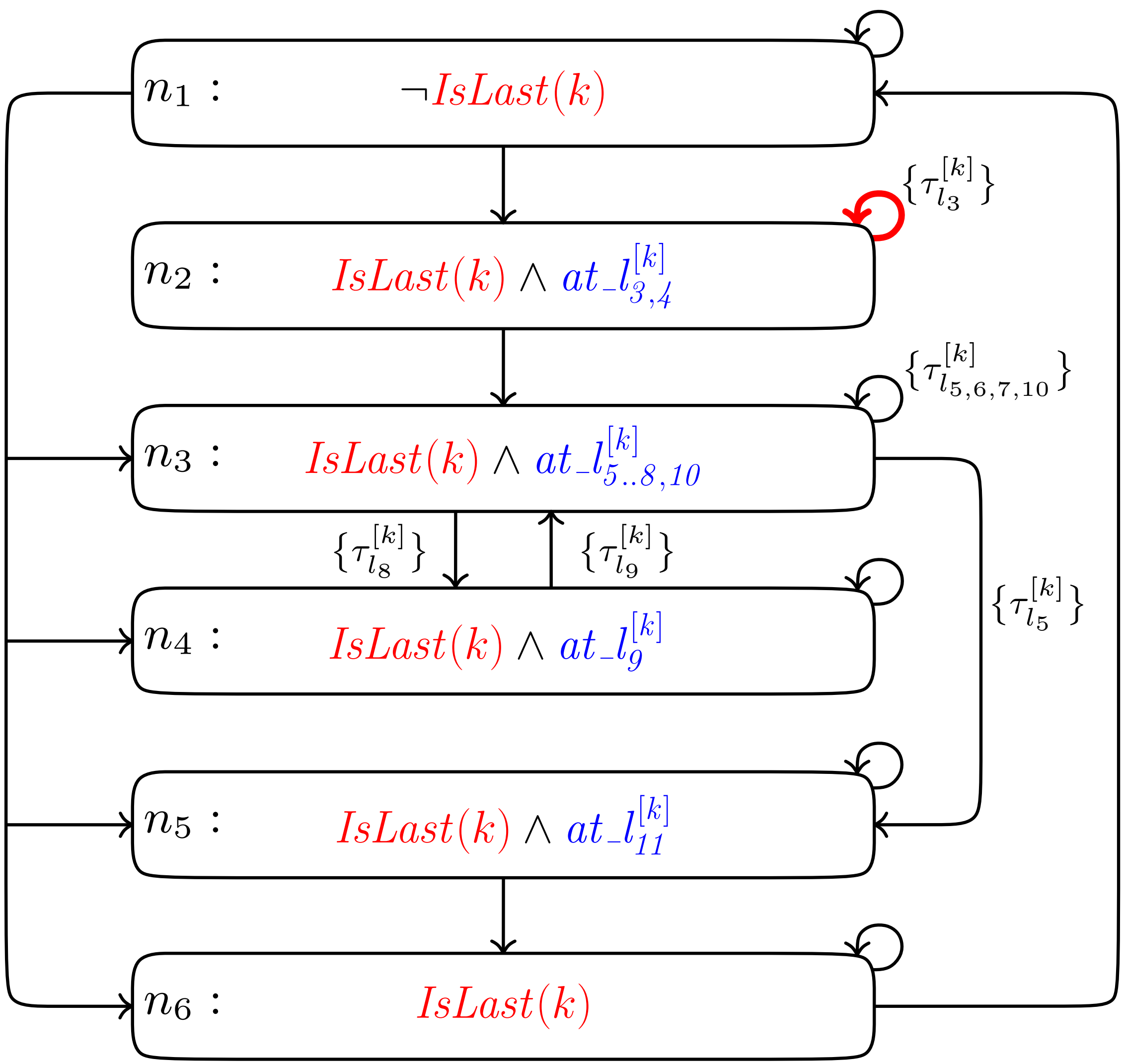

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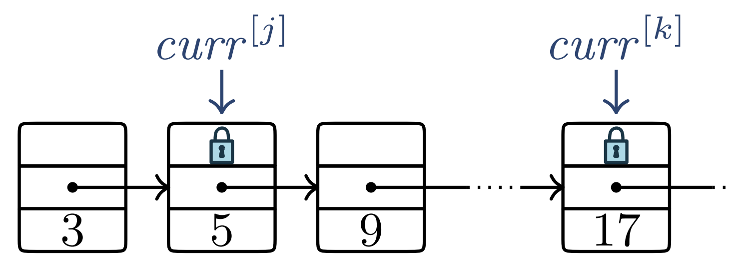

Reason about:

reachability, cells, memory

$$\begin{pmatrix} IsLast(k) \wedge \ j \neq k \wedge \\ at\_l_{3,4}^{[k]} \wedge at\_l_{9}^{[j]} \wedge \\ curr^{[j]}.lockid = \emptyset \end{pmatrix} \wedge \ curr^{[j]}.lock(j) \rightarrow \begin{pmatrix} IsLast(k') \wedge at'\_l_{3,4}^{[k']} \wedge \\ at'\_l_{10}^{[j']} \wedge pres(V - curr^{[j]}) \wedge \\ curr'^{[j']}.lockid = j' \end{pmatrix}$$

Our Contribution

- **TLL3**, a theory for concurrent linked lists
- We show TLL3 **decidable**
- We propose a combination based **decision procedure**

TLL3: A Theory for Single-Linked Concurrent Lists

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- TLL3 is a union of other theories

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Σ_{addr}

TLL3: A Theory for Single-Linked Concurrent Lists

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$\Sigma_{\mathsf{addr}} \cup \Sigma_{\mathsf{elem}}$

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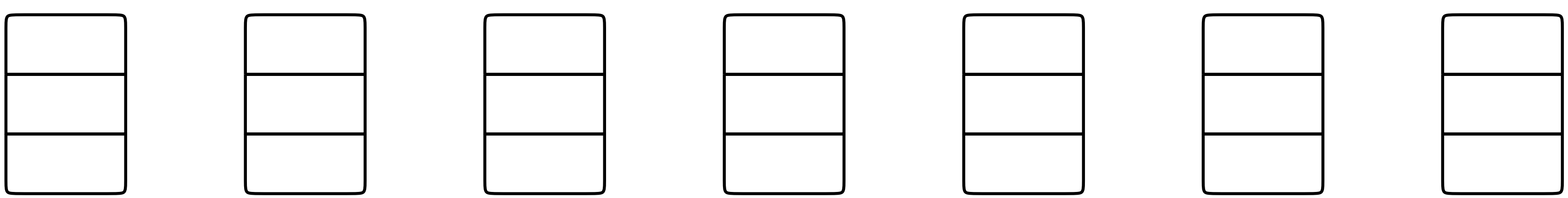

TLL3: A Theory for Single-Linked Concurrent Lists

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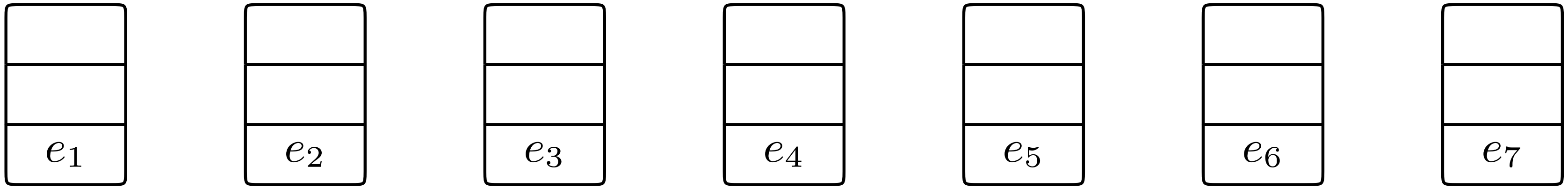

TLL3: A Theory for Single-Linked Concurrent Lists

- TLL3 is a union of other theories

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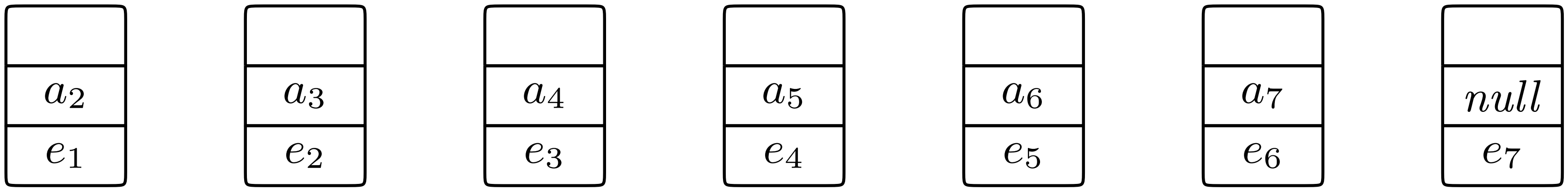

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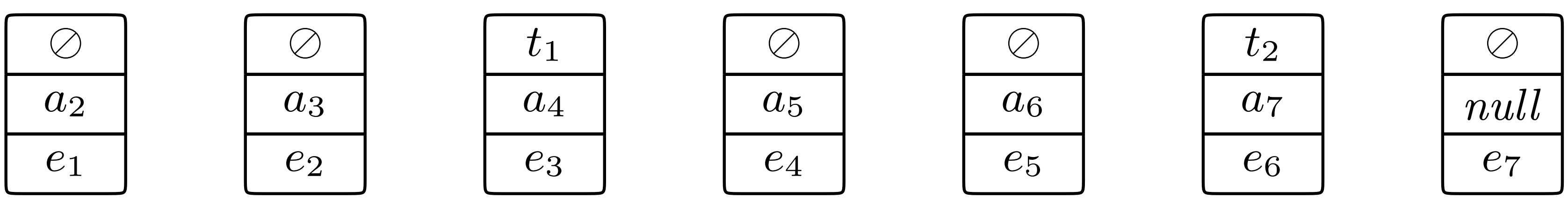

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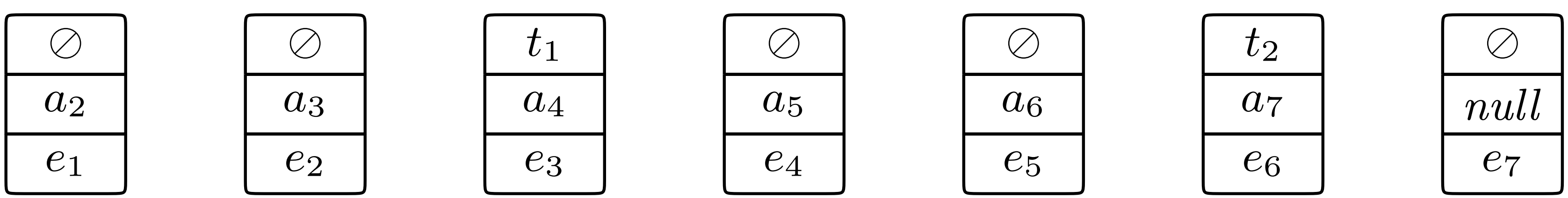

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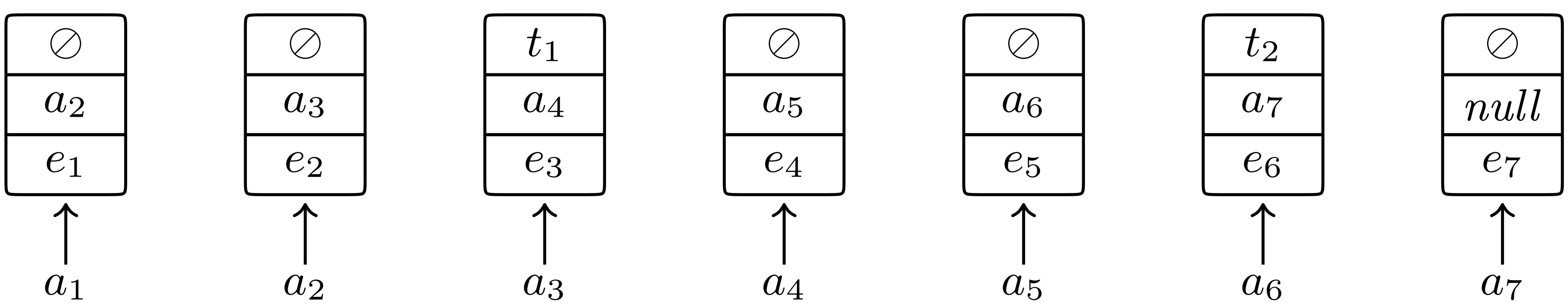

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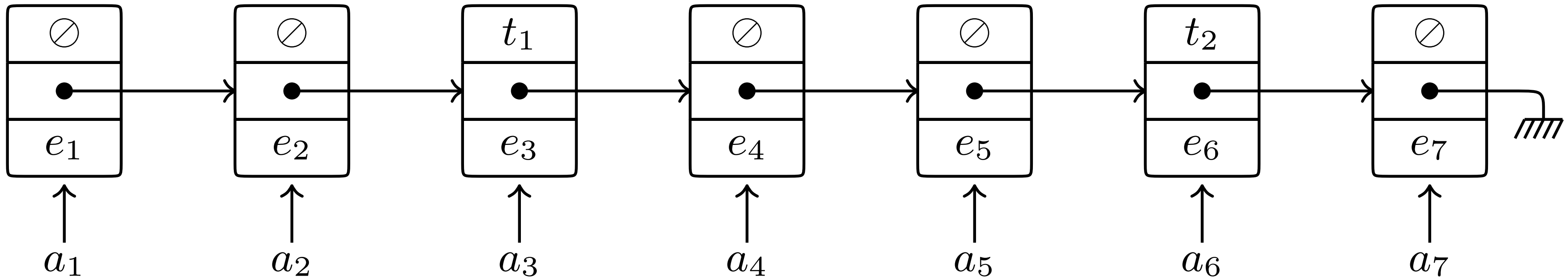

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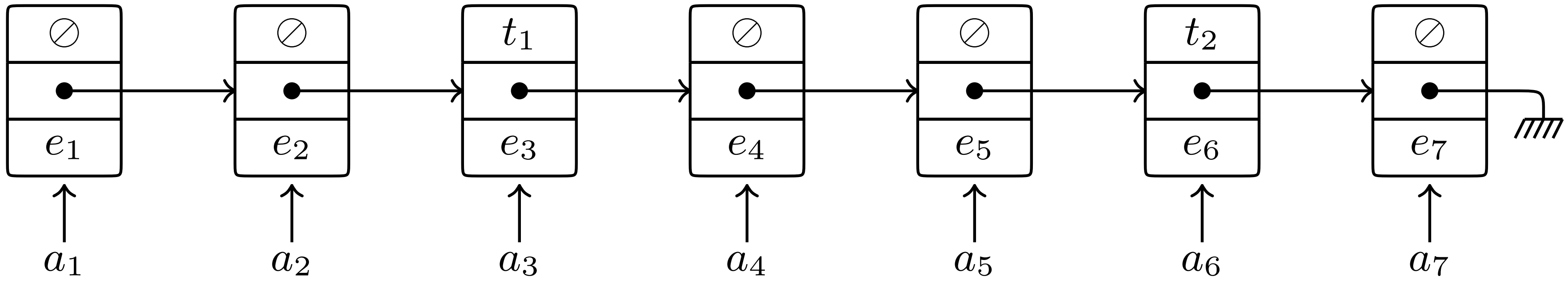

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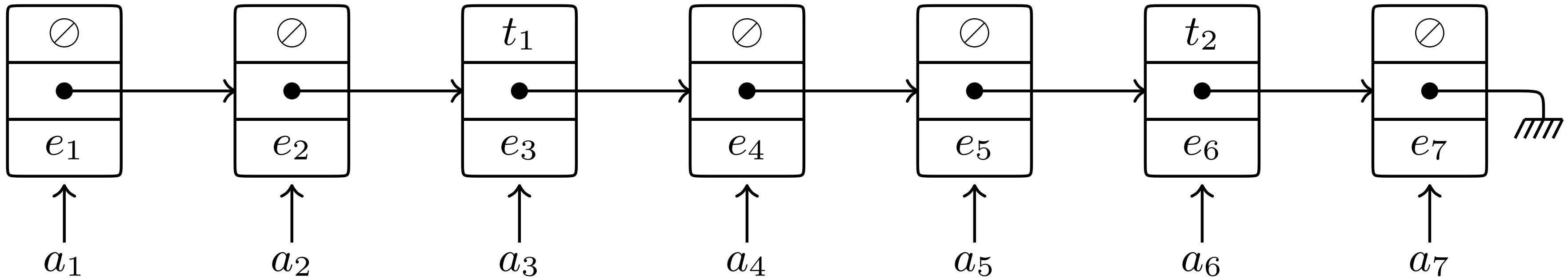

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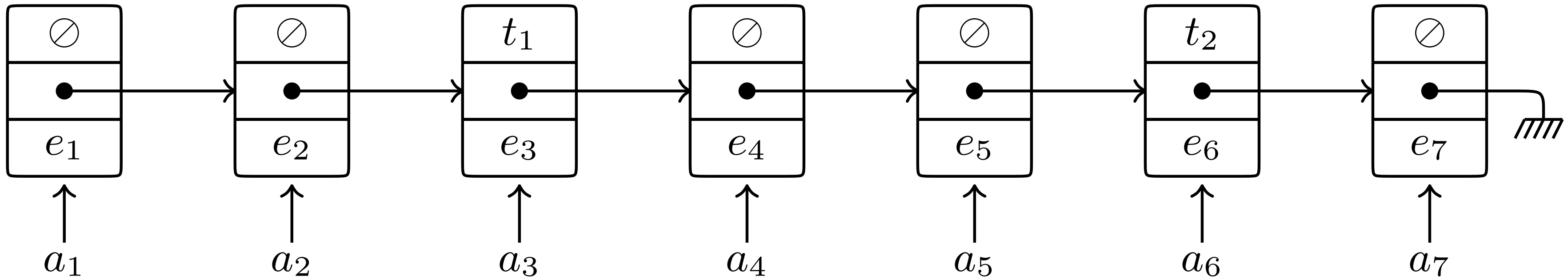

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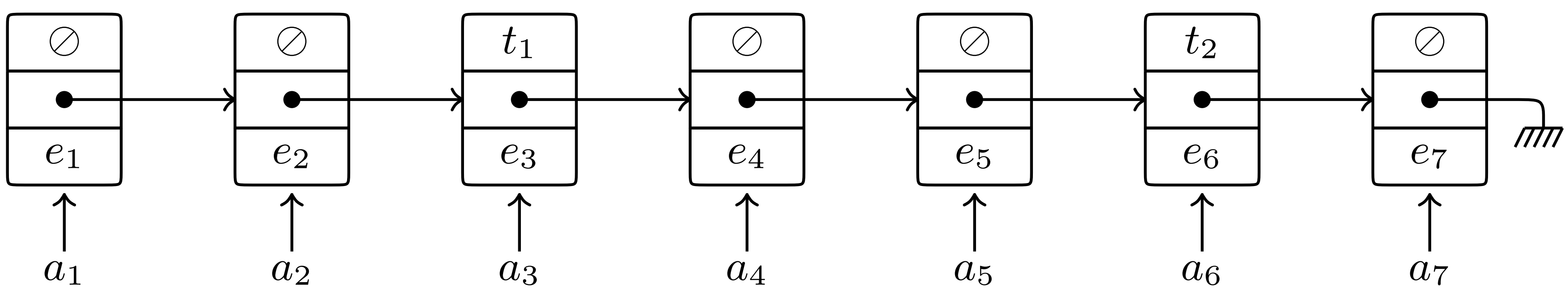
path $=$ a non-repeating sequence of addresses



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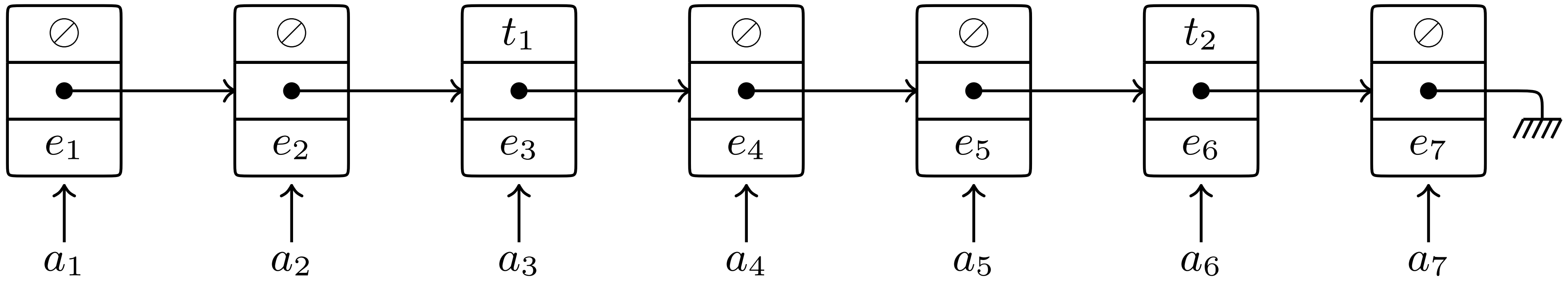
path = a non-repeating sequence of addresses

$$[a_1, a_2, a_3]$$



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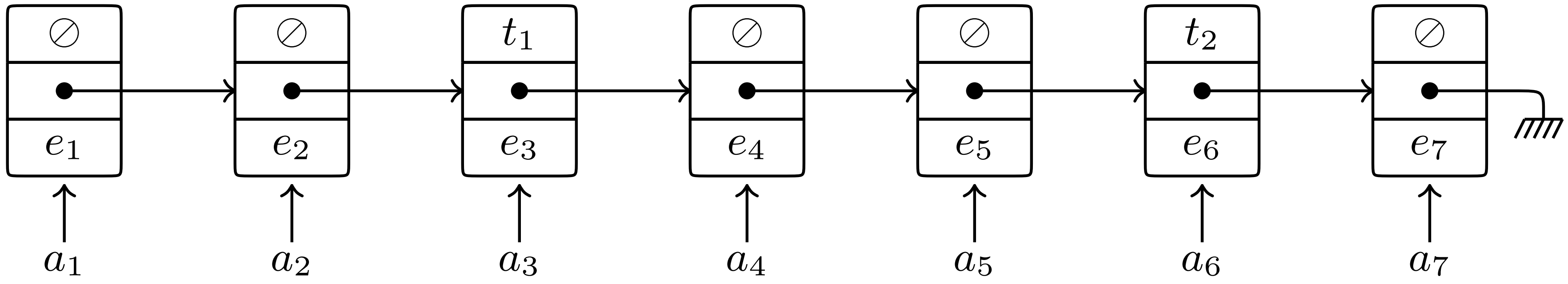

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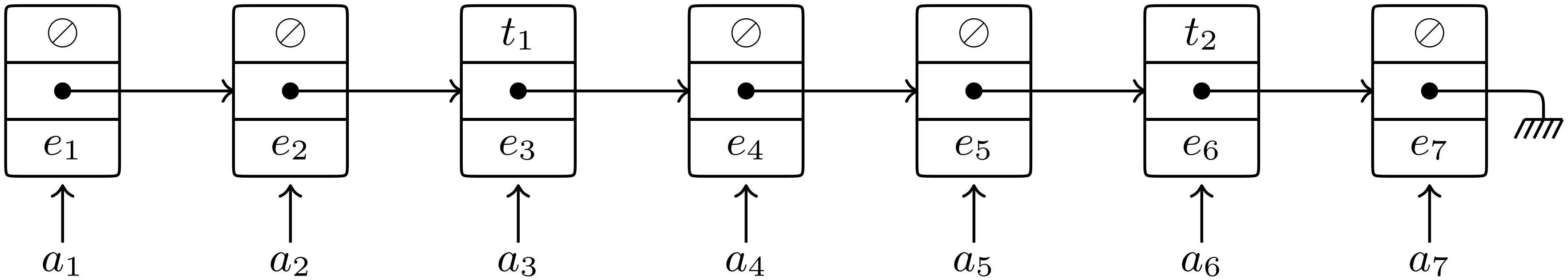
$$append([a_1, a_2], [a_3, a_4, a_5], [a_1, a_2, a_3, a_4, a_5])$$



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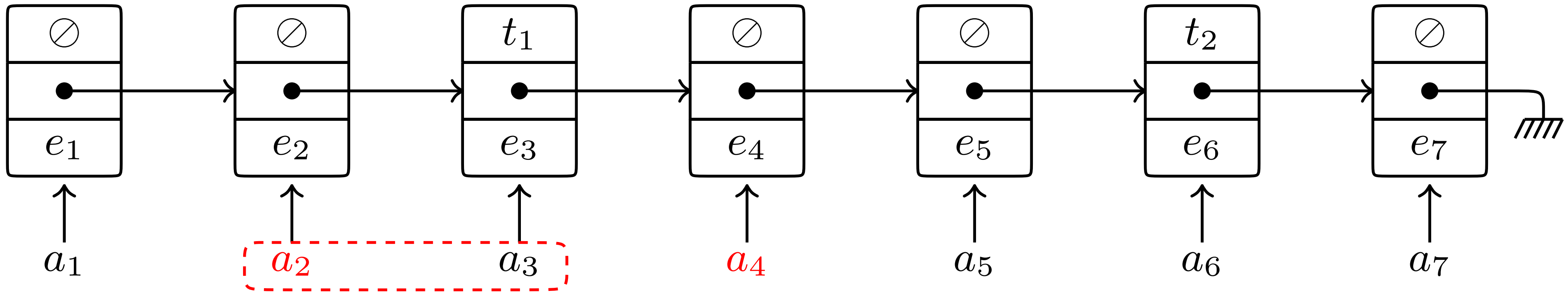
$$reach(m, a_2, a_4, [a_2, a_3])$$



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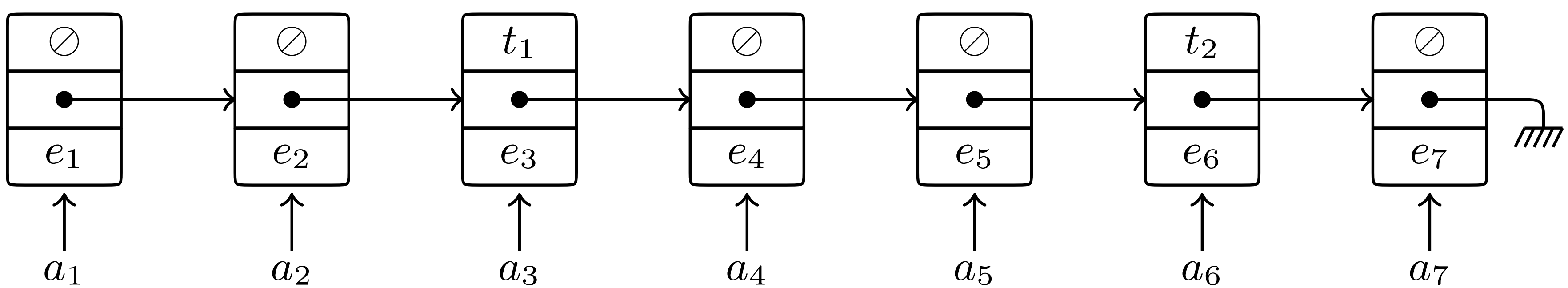

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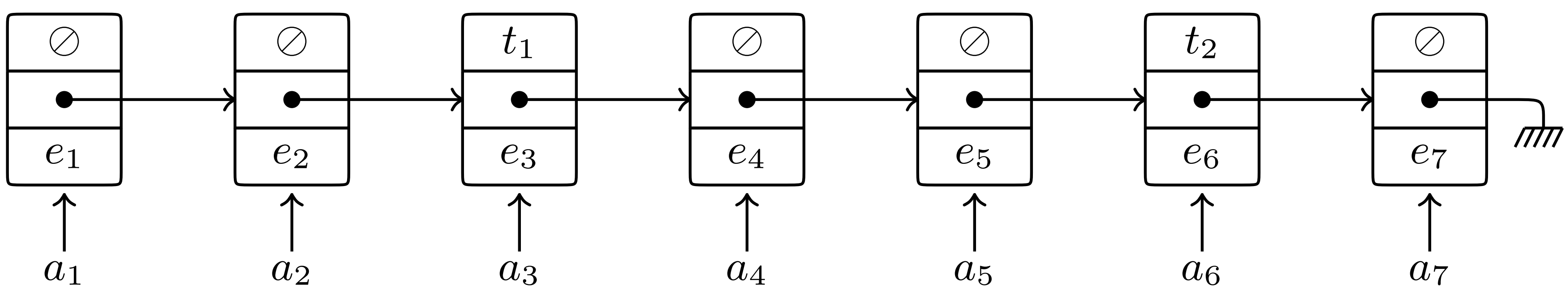

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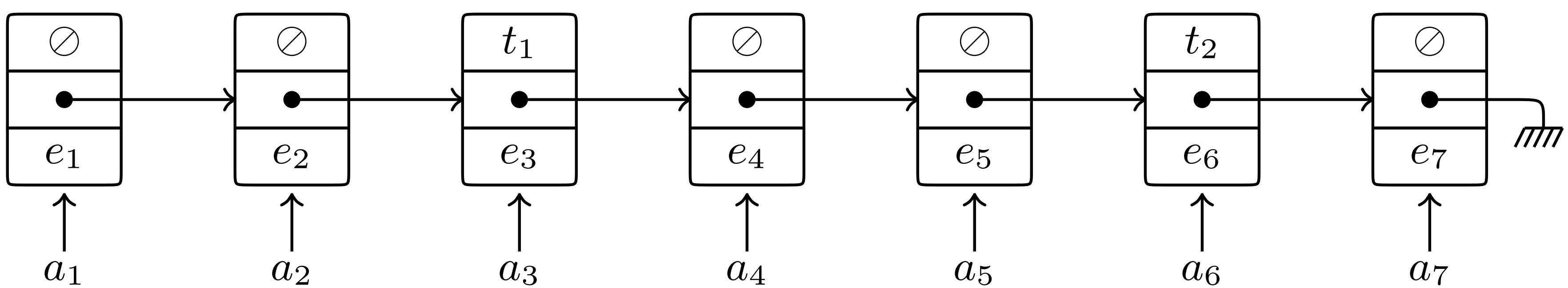
$$path2set([a_2, a_3]) = \{a_2, a_3\}$$



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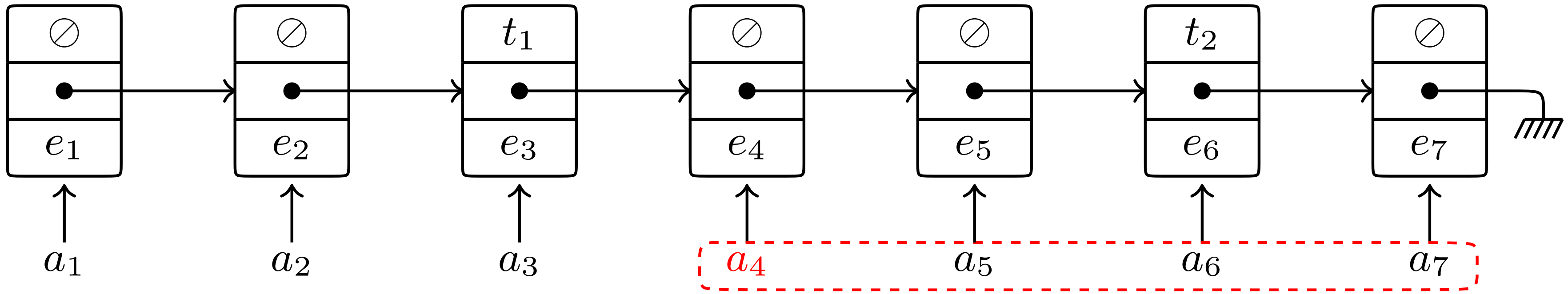
$$addr2set(m, a_4) = \{a_4, a_5, a_6, a_7\}$$



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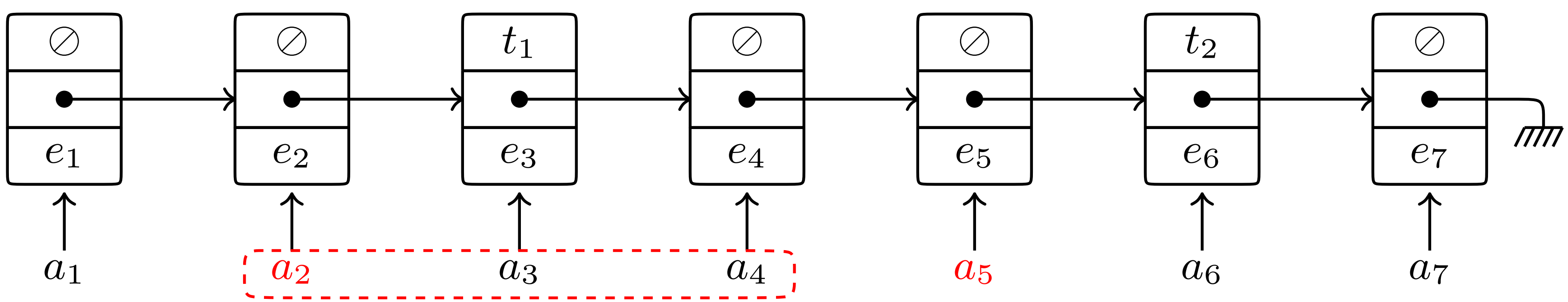
$$getp(m, a_2, a_5) = [a_2, a_3, a_4]$$



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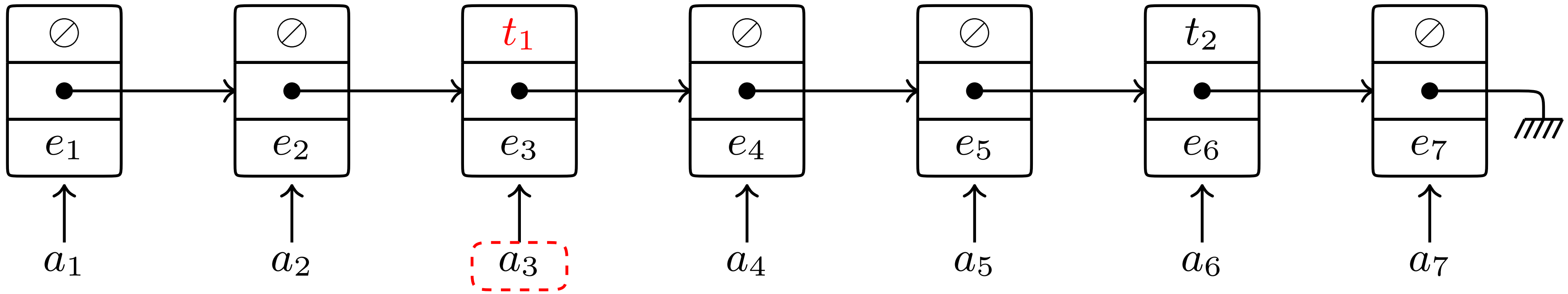
$$\mathit{firstlocked}(m, [a_1, a_2, a_3, a_4, a_5, a_6, a_7]) = a_3$$



Decidability by Small Model Property (SMP)

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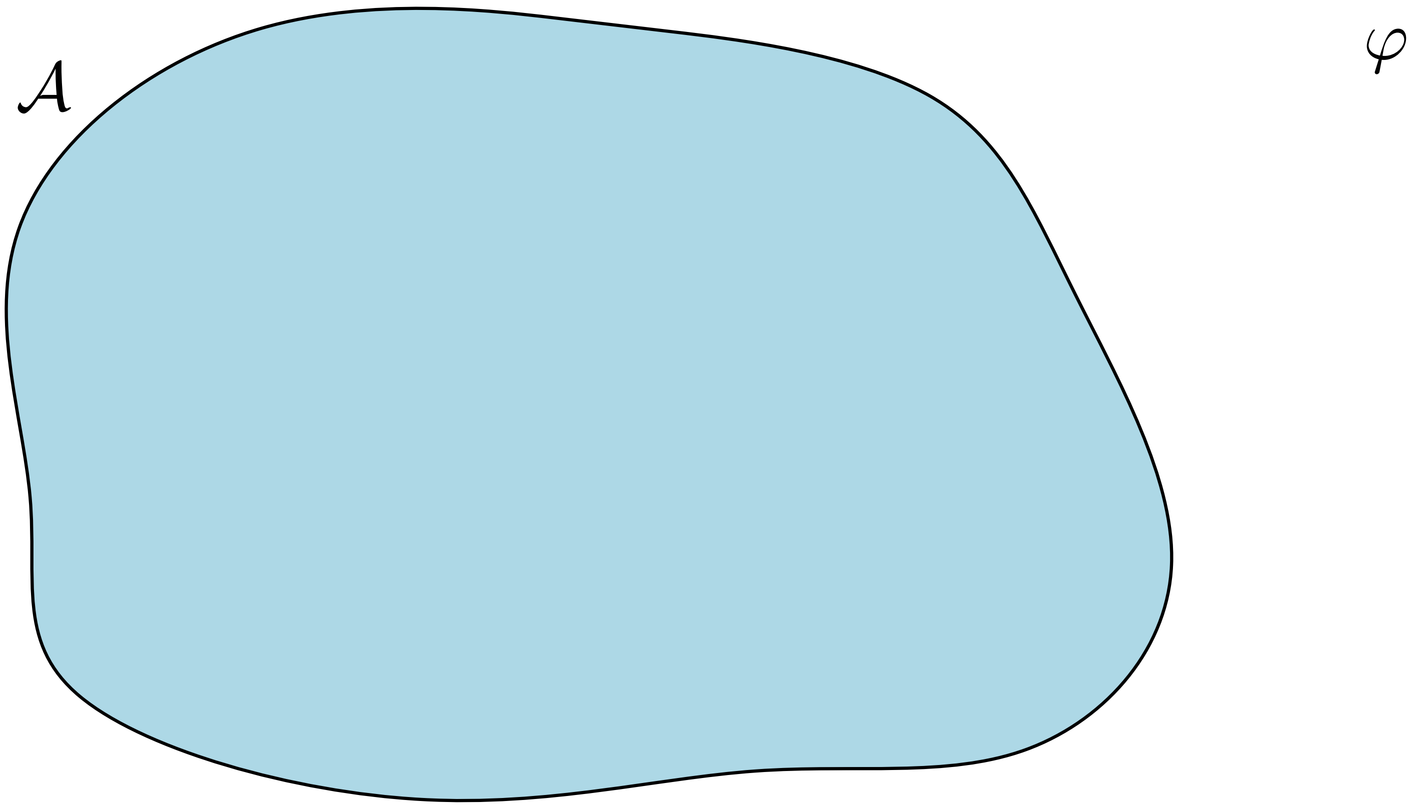

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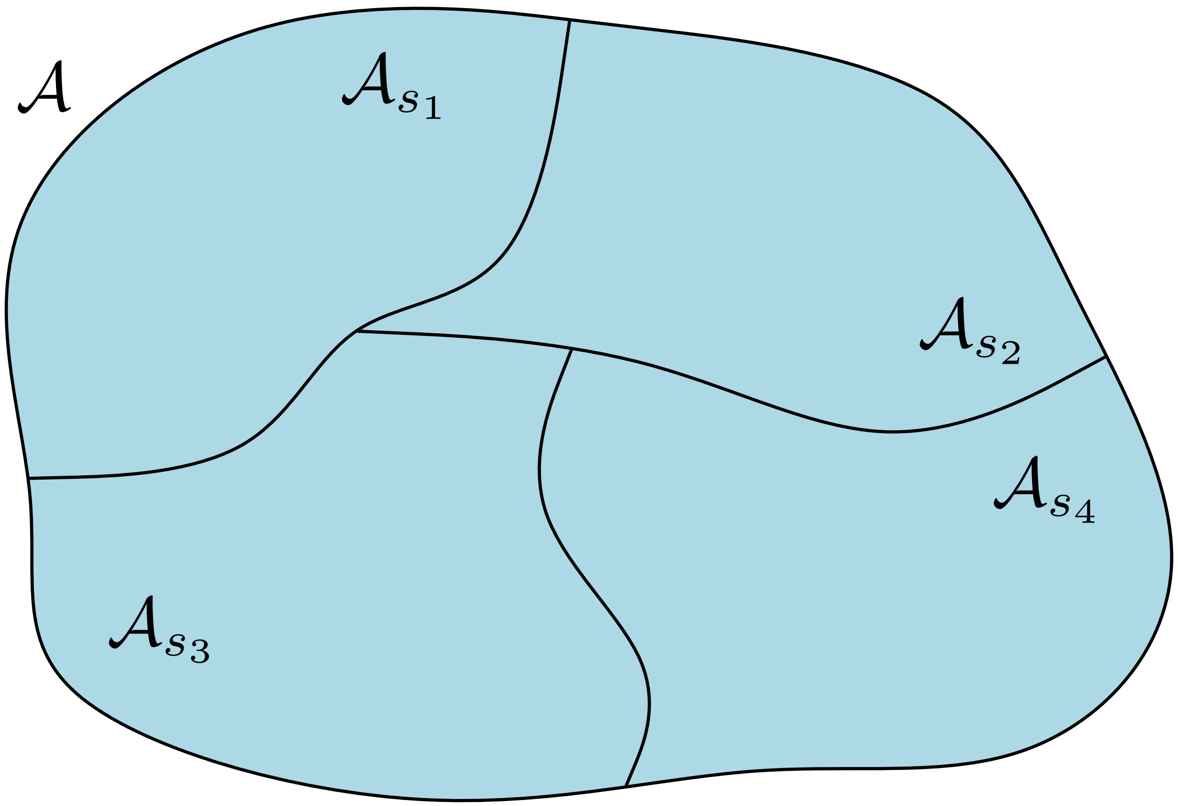

φ

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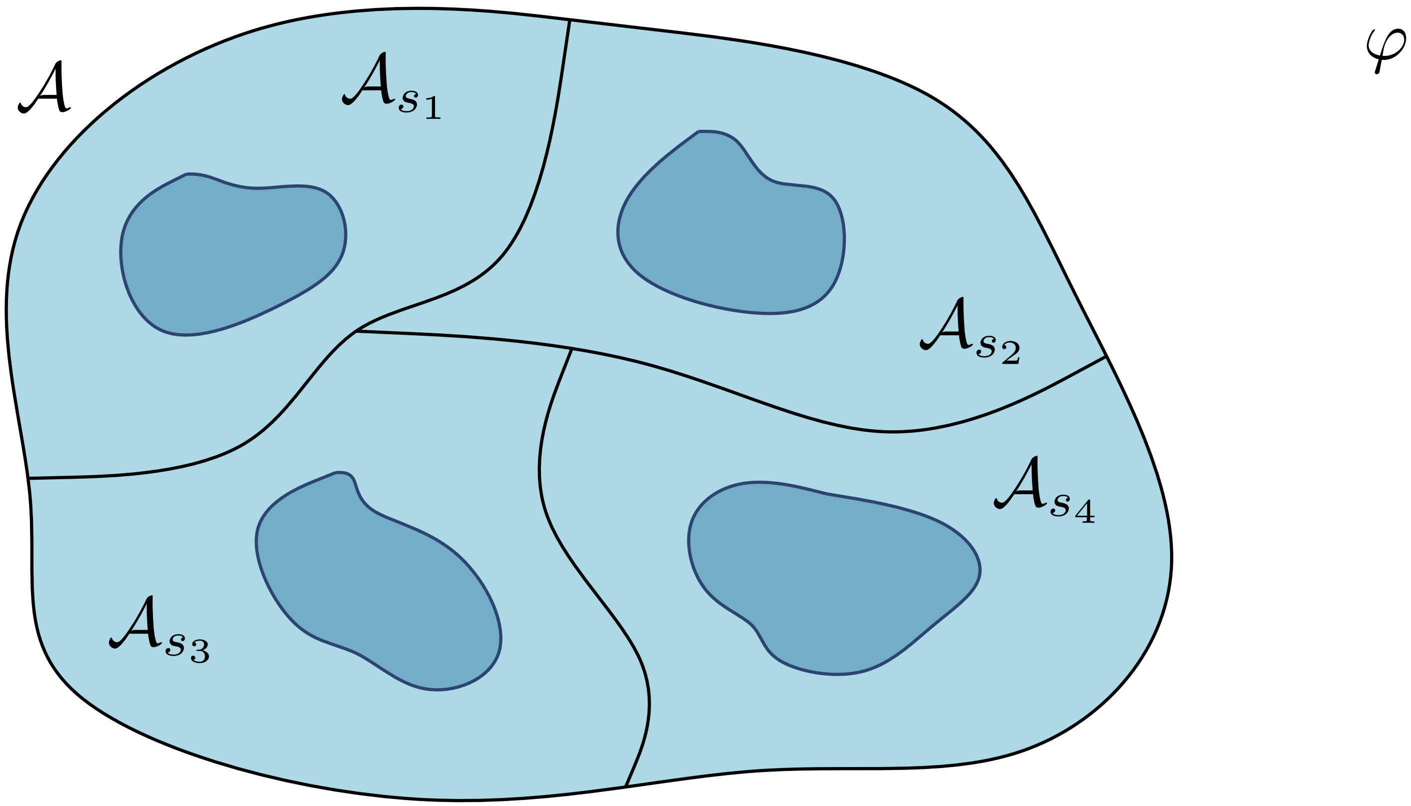

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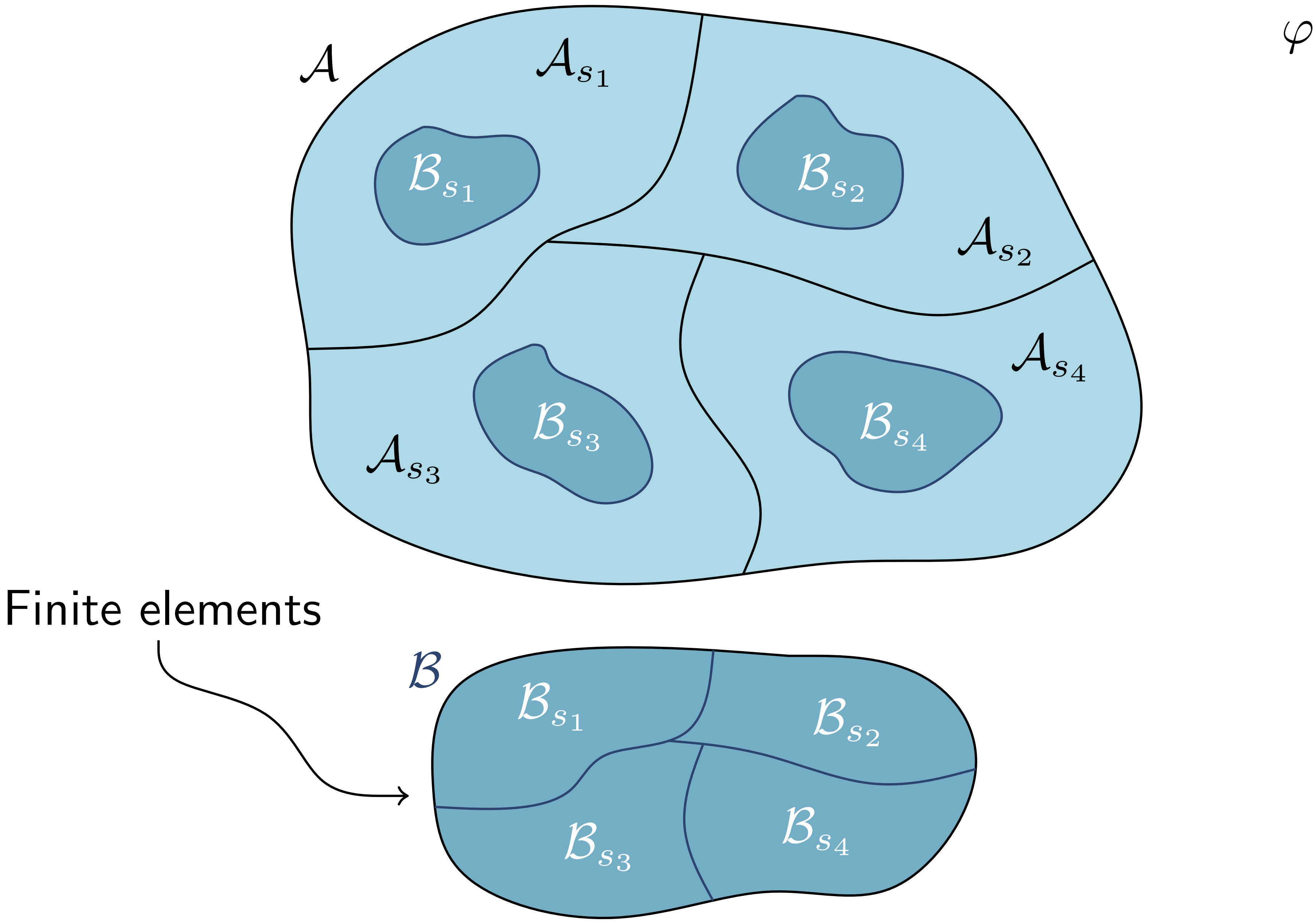
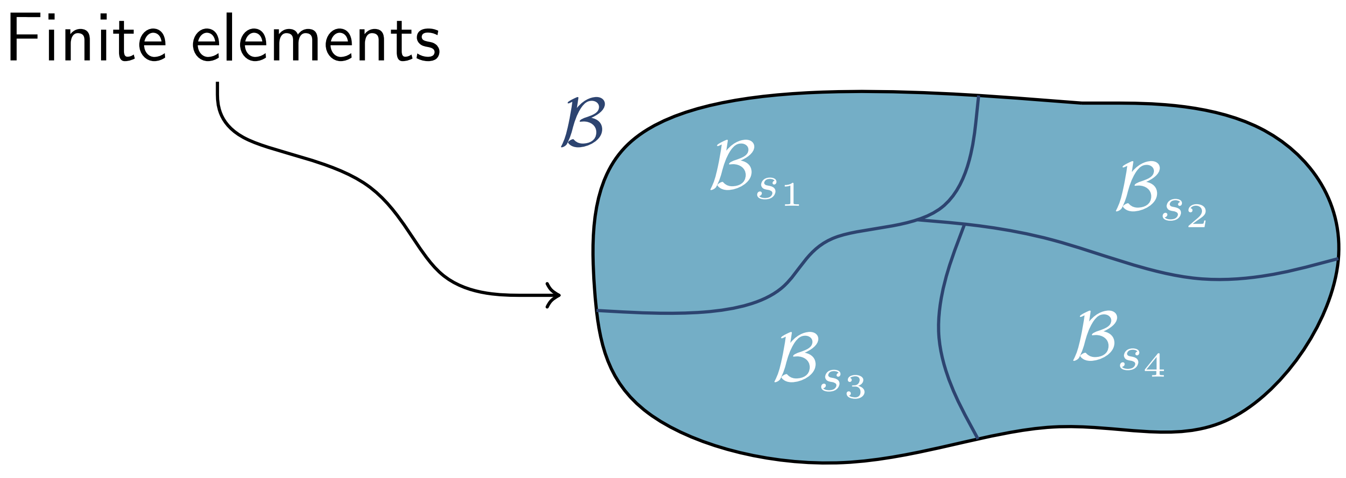


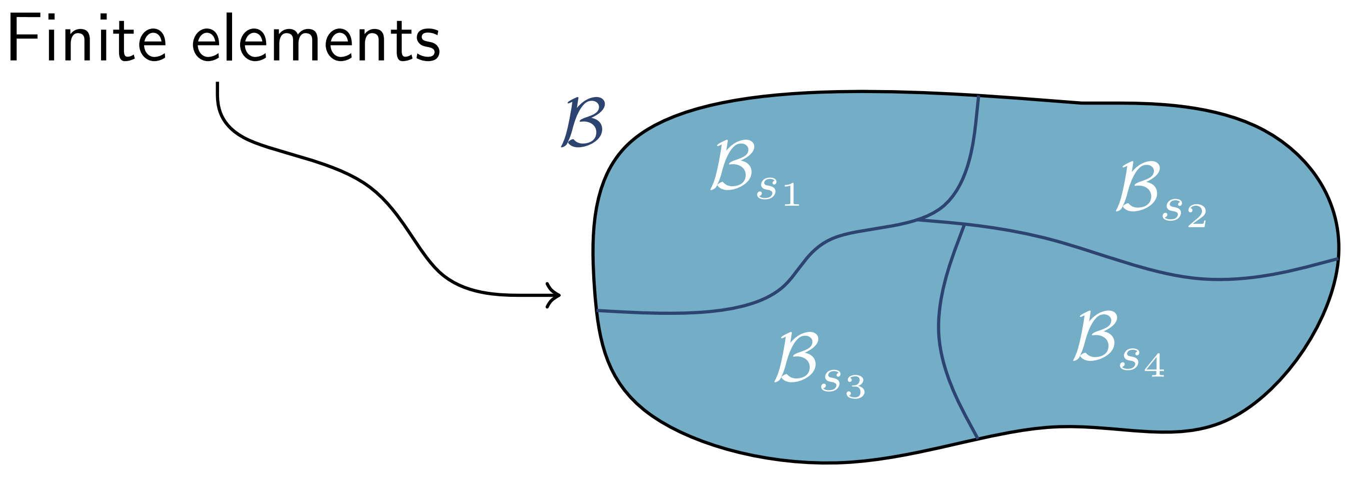
Decidability by Small Model Property (SMP)

- Let Γ be a conjunction of TLL3-literals



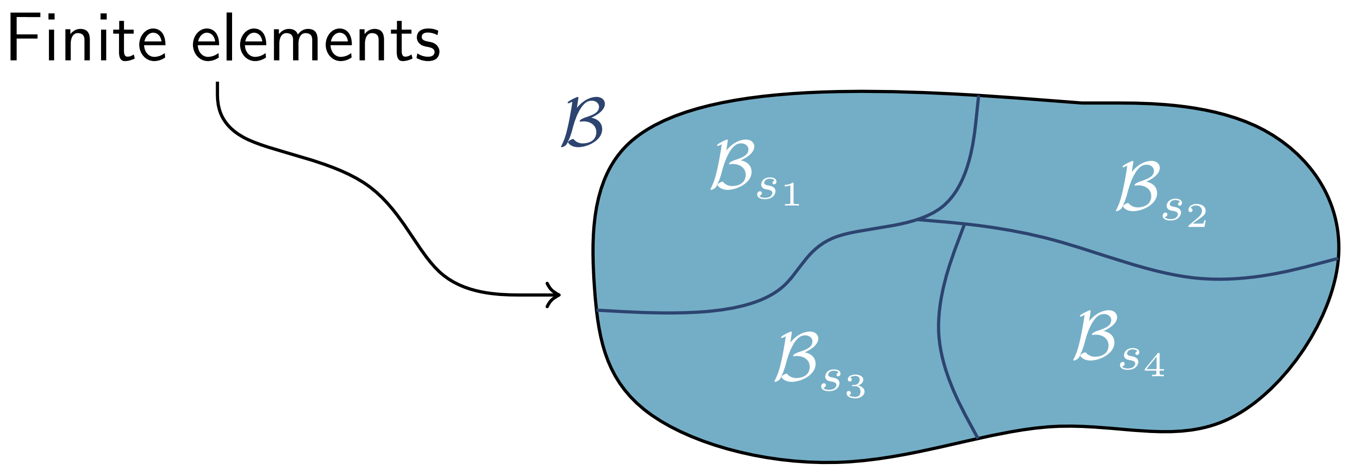
Decidability by Small Model Property (SMP)

- Let Γ be a conjunction of TLL3-literals
- If Γ is satisfied in an arbitrary TLL3 interpretation $\mathcal{A}$



Decidability by Small Model Property (SMP)

- Let Γ be a conjunction of TLL3-literals
- If Γ is satisfied in an arbitrary TLL3 interpretation $\mathcal{A}$
- We construct a new finite interpretation $\mathcal{B}$ **bounded by** Γ



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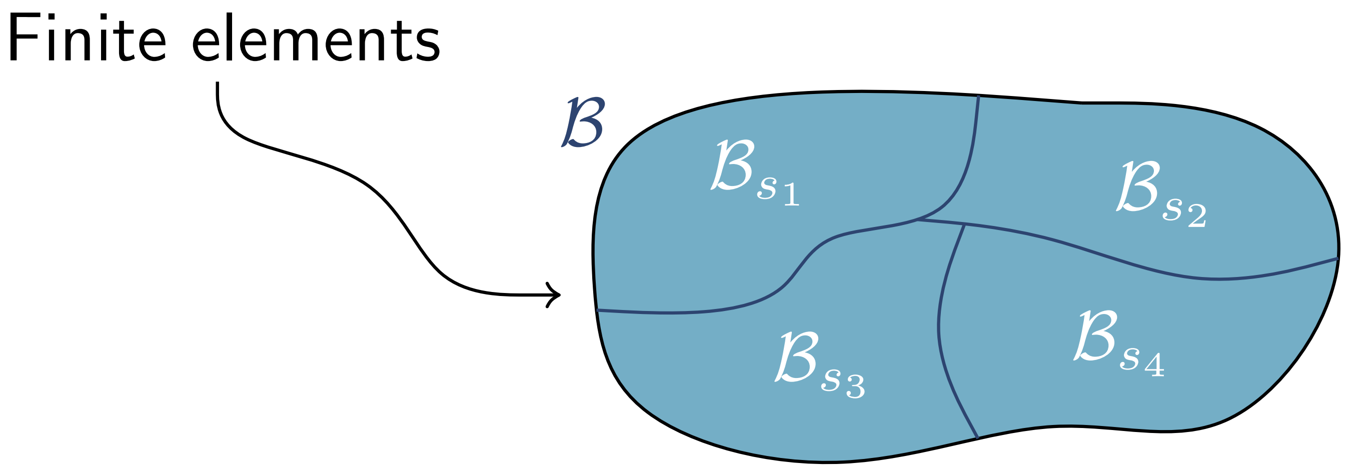
TLL3 is **decidable** by enumerating all possible elements



A Decision Procedure for TLL3 (main idea)

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- We use a **many-sorted variant of Nelson-Oppen**

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To apply it, we require:

- A **decision procedure** for each theory
- Theories **share only sorts**
- When two theories are combined:
  - Both are **stable infinite**
  - One is **polite** to the other with respect to shared sorts

Stable Infinite & Politeness

Stable Infinite & Politeness

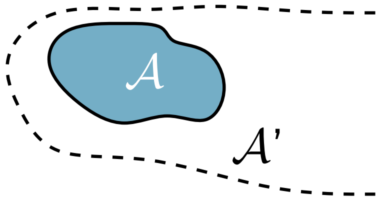
- **Stable infinite**

- **Polite** with respect to sorts $s_1, \ldots s_n$

Stable Infinite & Politeness

- **Stable infinite**

  for all QF-formula φ, exists an infinite interpretation $\mathcal{A}$'



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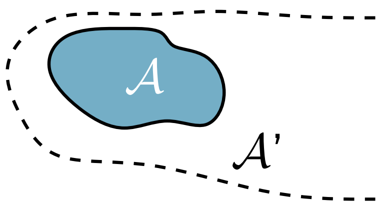

- **Polite** with respect to sorts $s_1, \ldots s_n$
  - **Smooth**
  - **Finite witnessable**

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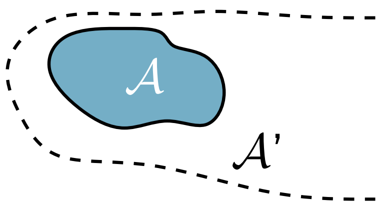

- **Polite** with respect to sorts $s_1, \ldots s_n$
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    if $\mathcal{A}$ is a model of φ, with domains $\mathcal{A}_{s_1} \quad \cdots \quad \mathcal{A}_{s_n}$

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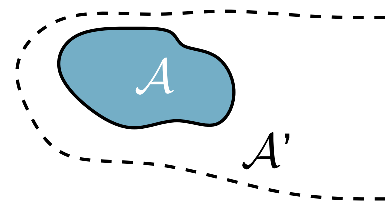

- **Polite** with respect to sorts $s_1, \ldots s_n$
  - **Smooth**

    if $\mathcal{A}$ is a model of φ, with domains $|\mathcal{A}_{s_1}| \cdots |\mathcal{A}_{s_n}|$

    then, for every $k_1 \cdots k_n$

    $$\begin{array}{ccc} |\mathcal{A}_{s_1}| & \cdots & |\mathcal{A}_{s_n}| \\ \leq & & \leq \\ k_1 & \cdots & k_n \end{array}$$

  - **Finite witnessable**

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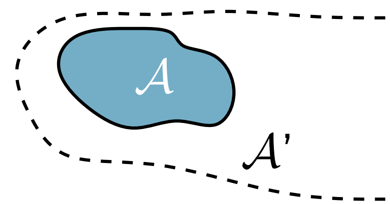

- **Polite** with respect to sorts $s_1, \ldots s_n$
  - **Smooth**

    if $\mathcal{A}$ is a model of φ, with domains

    then, for every

    there is a model $\mathcal{B}$ of φ, such that

$$\begin{array}{ccc} |\mathcal{A}_{s_1}| & \cdots & |\mathcal{A}_{s_n}| \\ \geq & & \geq \\ k_1 & \cdots & k_n \\ = & & = \\ |\mathcal{B}_{s_1}| & \cdots & |\mathcal{B}_{s_n}| \end{array}$$

  - **Finite witnessable**

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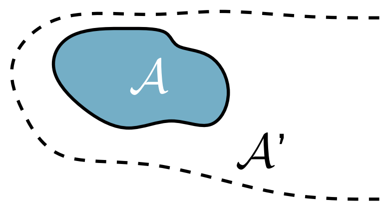
  for all QF-formula φ, exists an infinite interpretation $\mathcal{A}'$



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    if $\mathcal{A}$ is a model of φ, with domains $|\mathcal{A}_{s_1}| \cdots |\mathcal{A}_{s_n}|$

    then, for every $k_1 \cdots k_n$

    there is a model $\mathcal{B}$ of φ, such that $|\mathcal{B}_{s_1}| \cdots |\mathcal{B}_{s_n}|$

    $$\begin{array}{ccc} |\mathcal{A}_{s_1}| & \cdots & |\mathcal{A}_{s_n}| \\ \geq & & \geq \\ k_1 & \cdots & k_n \\ = & & = \\ |\mathcal{B}_{s_1}| & \cdots & |\mathcal{B}_{s_n}| \end{array}$$
  - **Finite witnessable**

    f

Stable Infinite & Politeness

- **Stable infinite**

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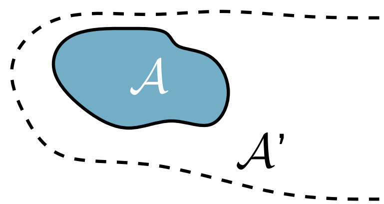

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    $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; \not\geq \qquad\quad \not\geq$

    then, for every $k_1 \cdots k_n$

    $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\;\; || \qquad\quad ||$

    there is a model $\mathcal{B}$ of φ, such that $|\mathcal{B}_{s_1}| \cdots |\mathcal{B}_{s_n}|$
  - **Finite witnessable**

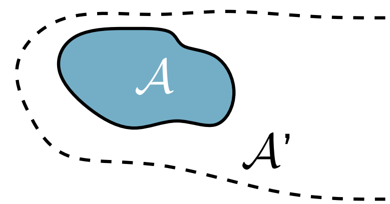
    f

    φ

Stable Infinite & Politeness

- **Stable infinite**

  for all QF-formula φ, exists an infinite interpretation $\mathcal{A}'$



- **Polite** with respect to sorts $s_1, \dots s_n$
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    if $\mathcal{A}$ is a model of φ, with domains $|\mathcal{A}_{s_1}| \cdots |\mathcal{A}_{s_n}|$

    $$\begin{array}{ccc} |\mathcal{A}_{s_1}| & \cdots & |\mathcal{A}_{s_n}| \\ \not\wedge & & \not\wedge \\ k_1 & \cdots & k_n \\ || & & || \\ |\mathcal{B}_{s_1}| & \cdots & |\mathcal{B}_{s_n}| \end{array}$$

    then, for every $k_1 \cdots k_n$

    there is a model $\mathcal{B}$ of φ, such that $|\mathcal{B}_{s_1}| \cdots |\mathcal{B}_{s_n}|$
  - **Finite witnessable**

    $$\varphi \xrightarrow{\quad f \quad} (\exists \bar{v})\psi \quad \text{s.t. } [\bar{v} = V_\varphi \setminus V_\psi]$$

if ψ is **satisfiable**, then exists $\mathcal{B}$ with **one variable per value**

Checklist

- We use a **many-sorted variant of Nelson-Oppen**

To apply it, we require:

- A **decision procedure** for each theory
- Theories **share only sorts**
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Checklist

- We use a **many-sorted variant of Nelson-Oppen**

To apply it, we require:

- A **decision procedure** for each theory ✓ (except $T_{\text{reachability}}$ and Σ_{bridge})
- Theories **share only sorts**
- When two theories are combined:
  - Both are **stable infinite**
  - One is **polite** to the other with respect to shared sorts

Checklist

- We use a **many-sorted variant of Nelson-Oppen**

To apply it, we require:

- A **decision procedure** for each theory ✓ (except $T_{\text{reachability}}$ and Σ_{bridge})
- Theories **share only sorts** ✓
- When two theories are combined:
  - Both are **stable infinite**
  - One is **polite** to the other with respect to shared sorts

A Combination-based Decision Procedure for TLL3

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$$\mathsf{TLL3} = \cdots \oplus T_{\mathsf{reachability}} \oplus \cdots \oplus \textit{bridge functions and predicates}$$

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$$\mathsf{TLL3} = \cdots \oplus T_{\mathsf{reachability}} \oplus \cdots \oplus \textit{bridge functions and predicates}$$

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A Combination-based Decision Procedure for TLL3

$\mathsf{TLL3} = \cdots \oplus T_{\mathsf{reachability}} \oplus \cdots \oplus$ *bridge functions and predicates*

$\widehat{\mathsf{TLL3}}$

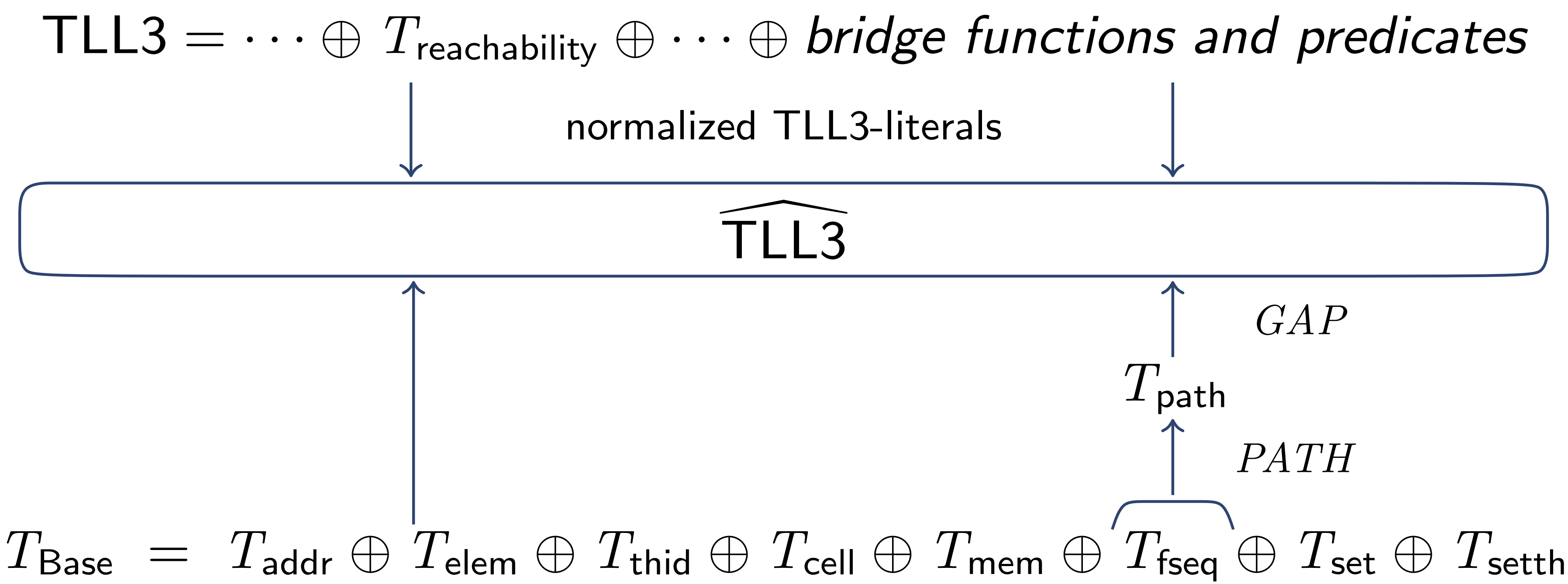
GAP

T_{path}

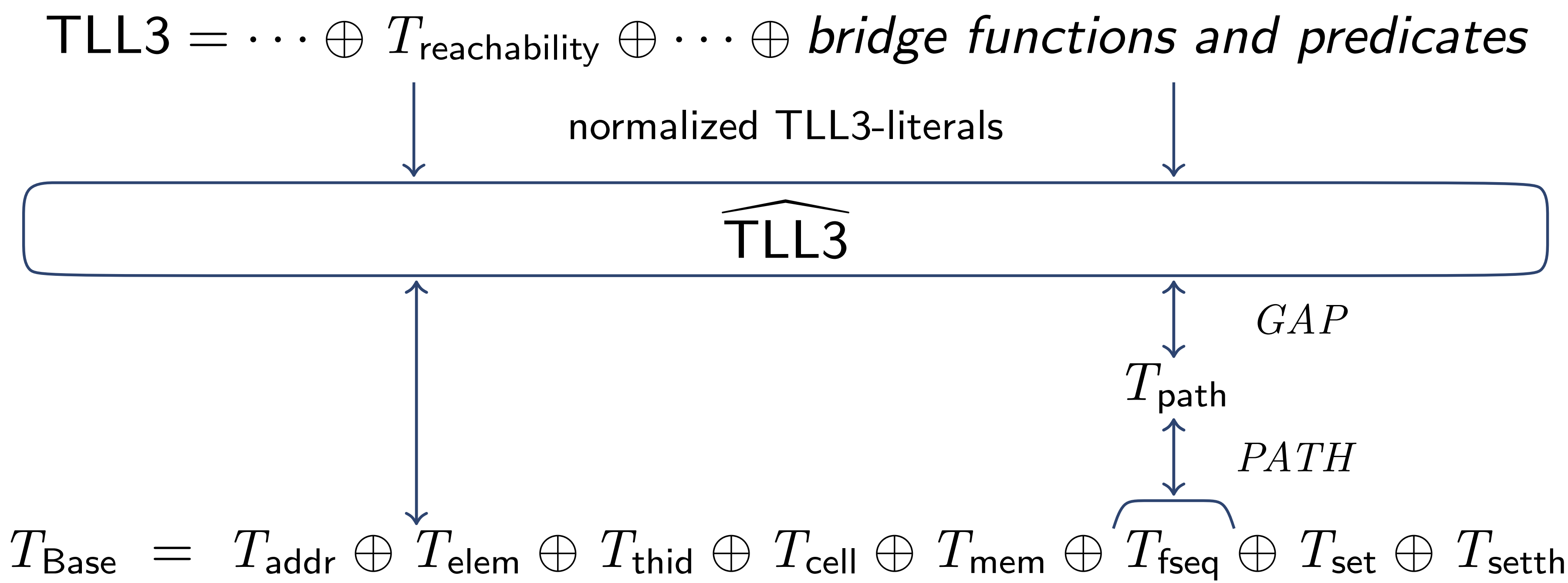
$PATH$

$$T_{\mathsf{Base}} \;=\; T_{\mathsf{addr}} \oplus T_{\mathsf{elem}} \oplus T_{\mathsf{thid}} \oplus T_{\mathsf{cell}} \oplus T_{\mathsf{mem}} \oplus T_{\mathsf{fseq}} \oplus T_{\mathsf{set}} \oplus T_{\mathsf{setth}}$$

A Combination-based Decision Procedure for TLL3

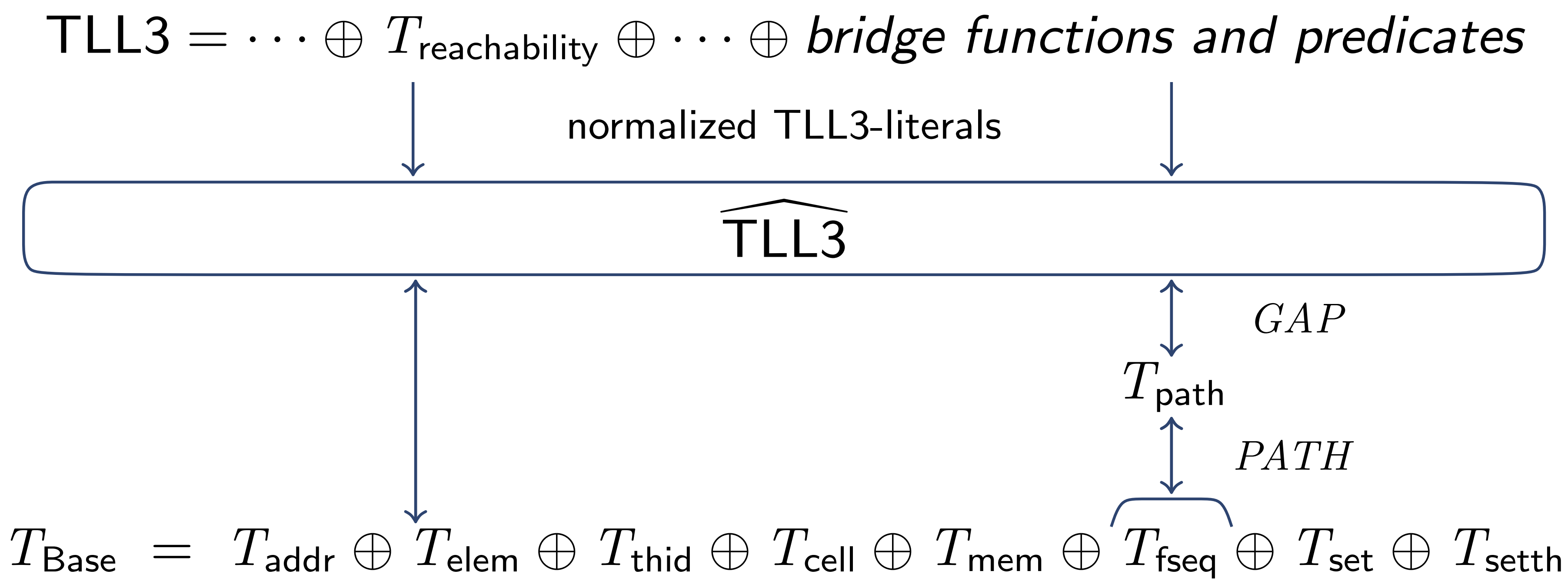


A Combination-based Decision Procedure for TLL3



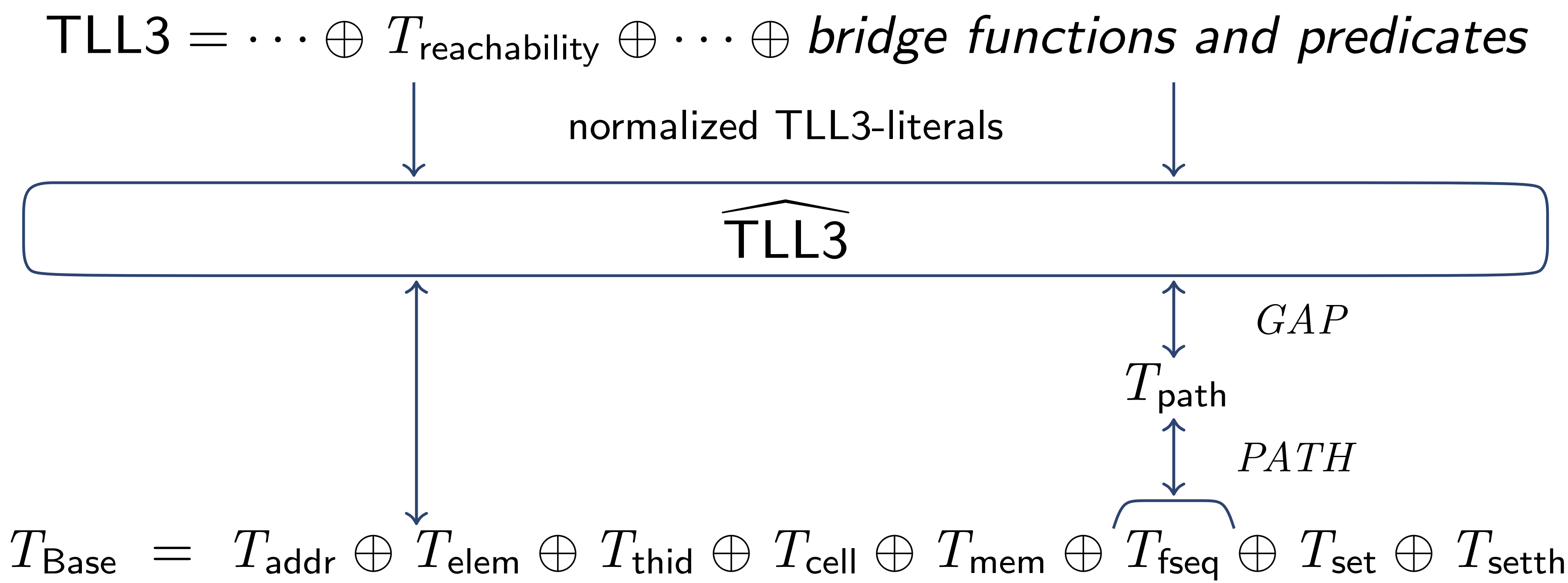
- **Unfolding** of definitions in $PATH$ and GAP

A Combination-based Decision Procedure for TLL3



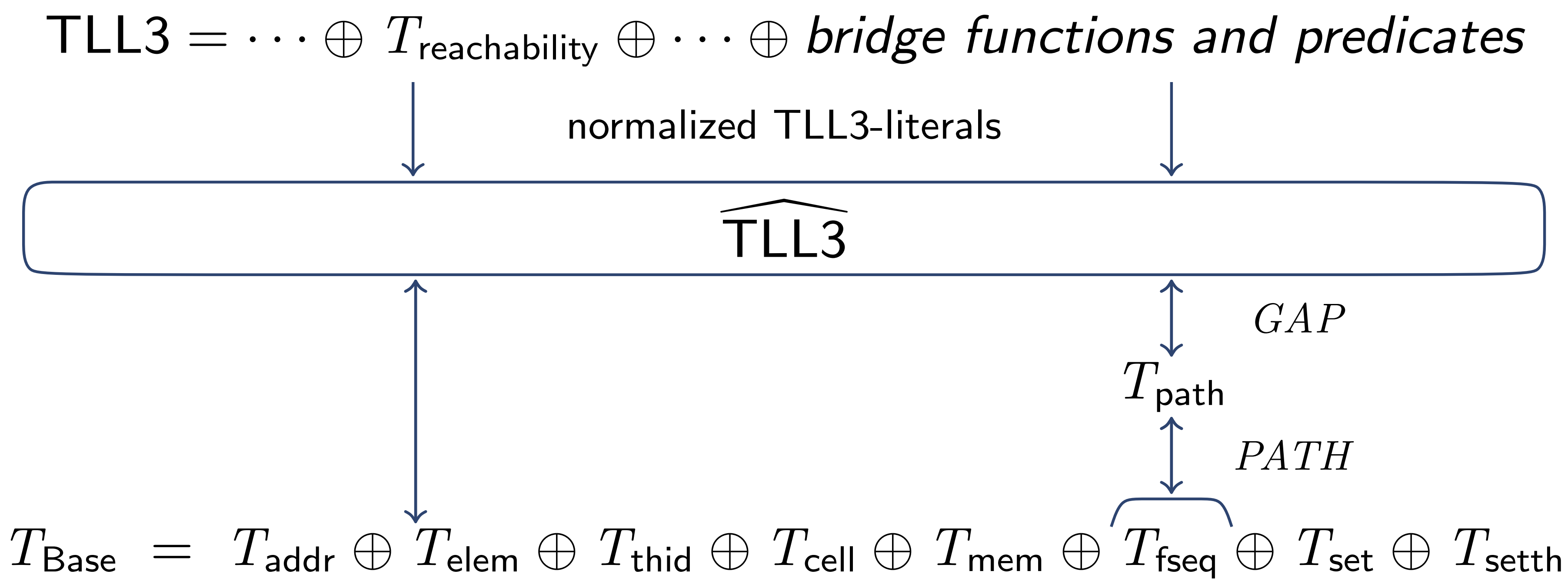
- **Unfolding** of definitions in $PATH$ and GAP
- *SMP* guarantees that all theories are **finite witnessable**

A Combination-based Decision Procedure for TLL3



- **Unfolding** of definitions in $PATH$ and GAP
- *SMP* guarantees that all theories are **finite witnessable**
- T_{cell}, T_{mem}, T_{set}, T_{setth} and T_{fseq} are all **stable infinite**

A Combination-based Decision Procedure for TLL3



- **Unfolding** of definitions in $PATH$ and GAP
- *SMP* guarantees that all theories are **finite witnessable**
- T_{cell}, T_{mem}, T_{set}, T_{setth} and T_{fseq} are all **stable infinite**
- These are **smooth** with respect to sorts addr, elem and thid

Conclusions

- We defined **TLL3**, a theory for concurrent single-linked lists
- We proved TLL3 **decidable**, by *Small Model Property*
- We provide a **combination-based decision procedure** for TLL3
- A step towards the assisted verification of **temporal properties** over **concurrent data-types**: VD + DP
- **Current and future** work:
  parametrized verification diagrams, DP for concurrent skiplists, concurrent hash-maps, concurrent Schorr-Waite
- Many possible **collaborations**:
  DPs as combination, SMTs, implementation