Assisted Verification of Invariance for Parametrized Systems

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Motivation

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$$S[N]: \quad T_1 \quad T_2 \quad \cdots \quad T_N$$

- Consider a **finite** but **unbounded** set of threads

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- Running the **same program** in **paralallel**
- Classical scenarios include:
  - Device drivers
  - Distributed algorithms
  - Concurrent datastructures
  - Robotic swarms
  - Biological Systems

Motivation

$S[N]:$

GOAL : **Prove parametrized invariance** $\varphi(\overline{v})$

$$S[N] \models \varphi(\overline{v}) \qquad \text{for all } N > 1$$

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Motivating Example: Ticket Mutex Algorithm

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global

$Int\ avail := 0$

$Set\langle Int\rangle\ bag := \emptyset$

procedure MUTEXC

$Int\ ticket$

begin

loop

nondet

$\left\langle \begin{array}{l} ticket := avail{+}{+} \\ bag.add(ticket) \end{array} \right\rangle$

await $(bag.min == ticket)$

critical

$bag.remove(ticket)$

end loop

end procedure

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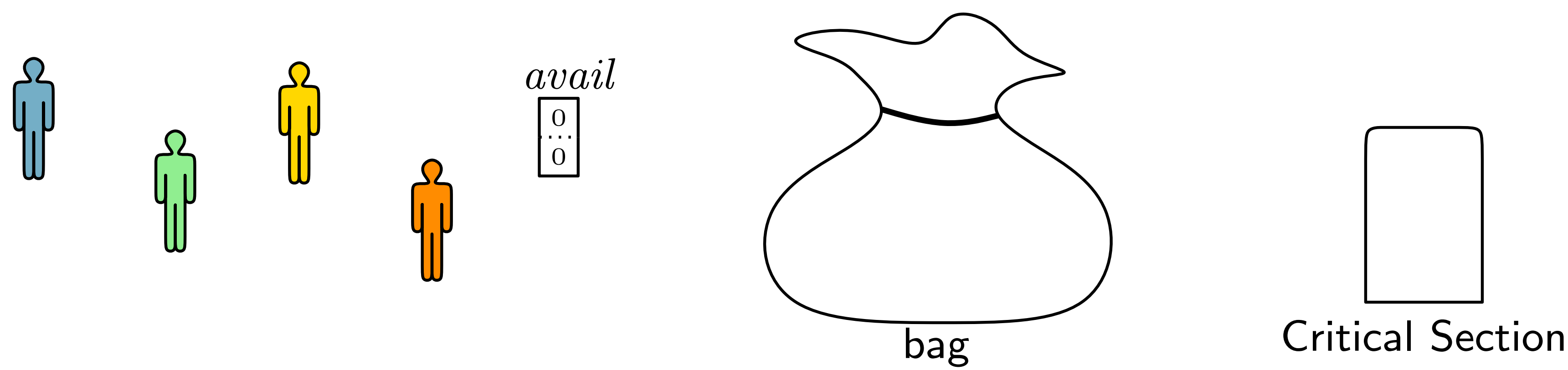

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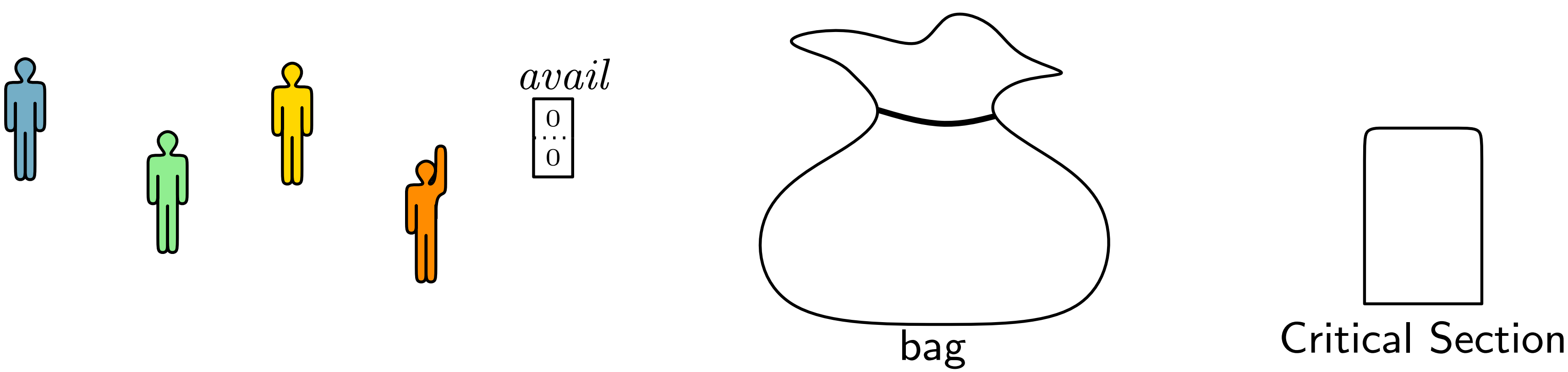

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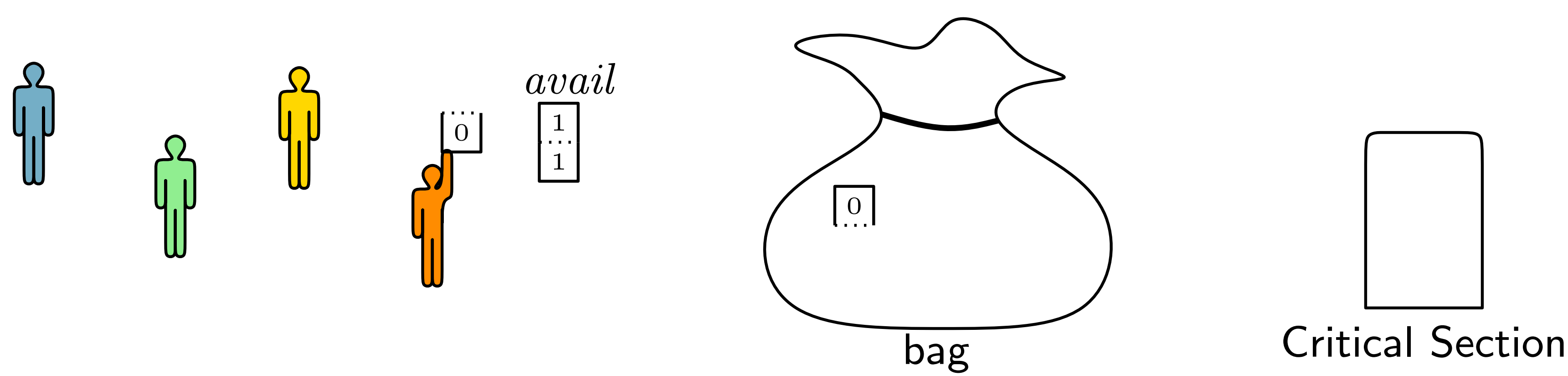

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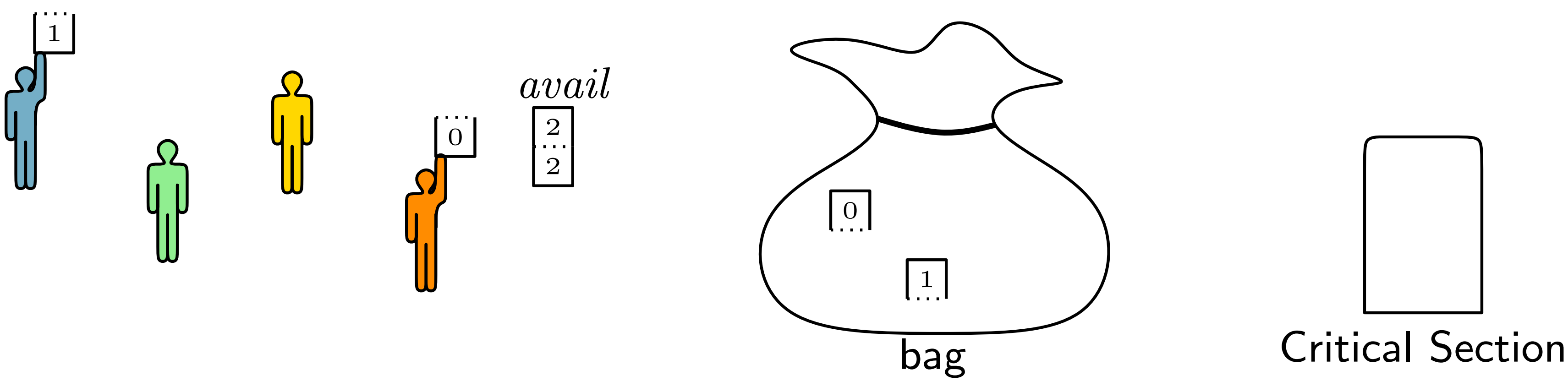

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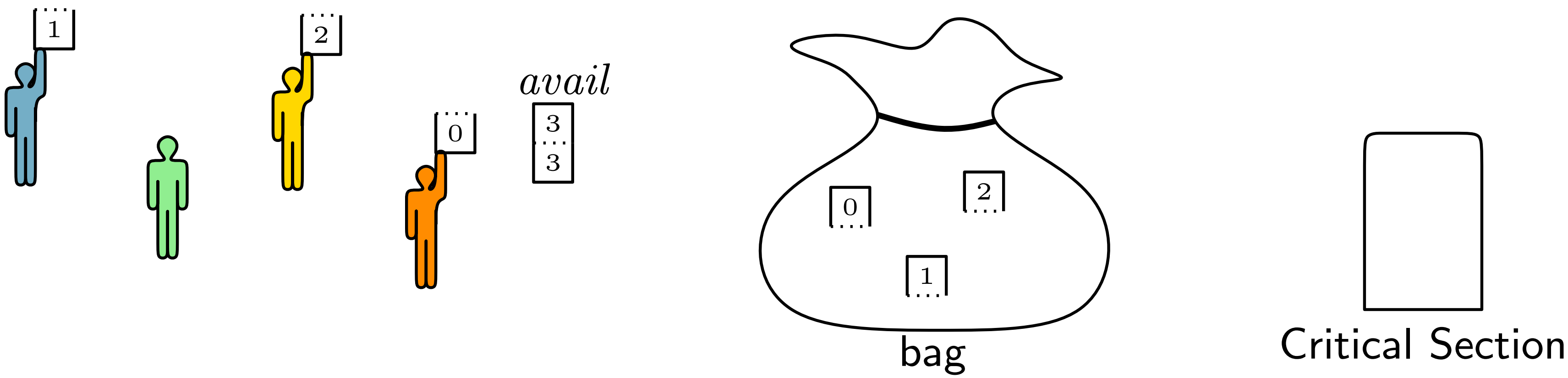

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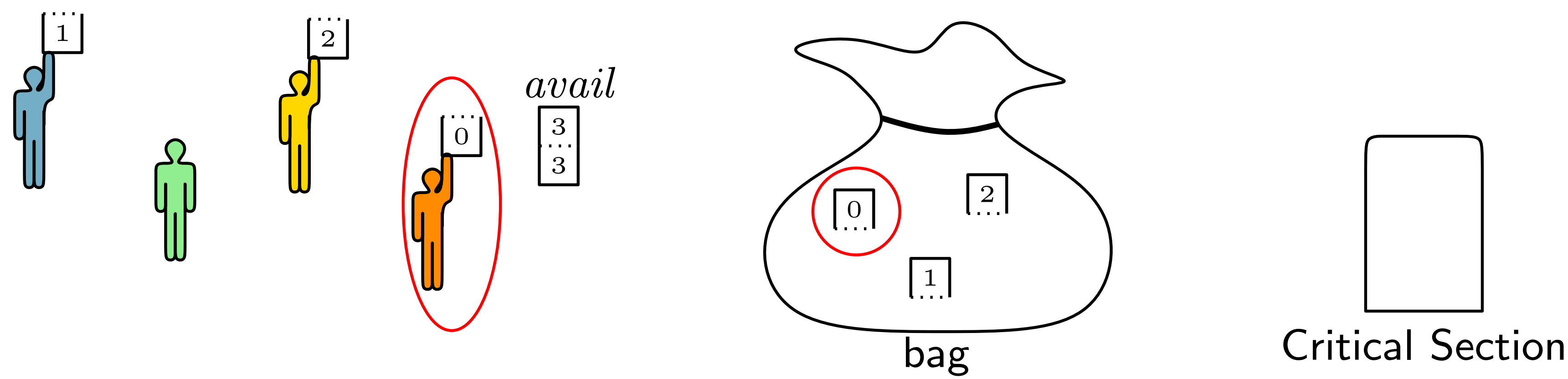

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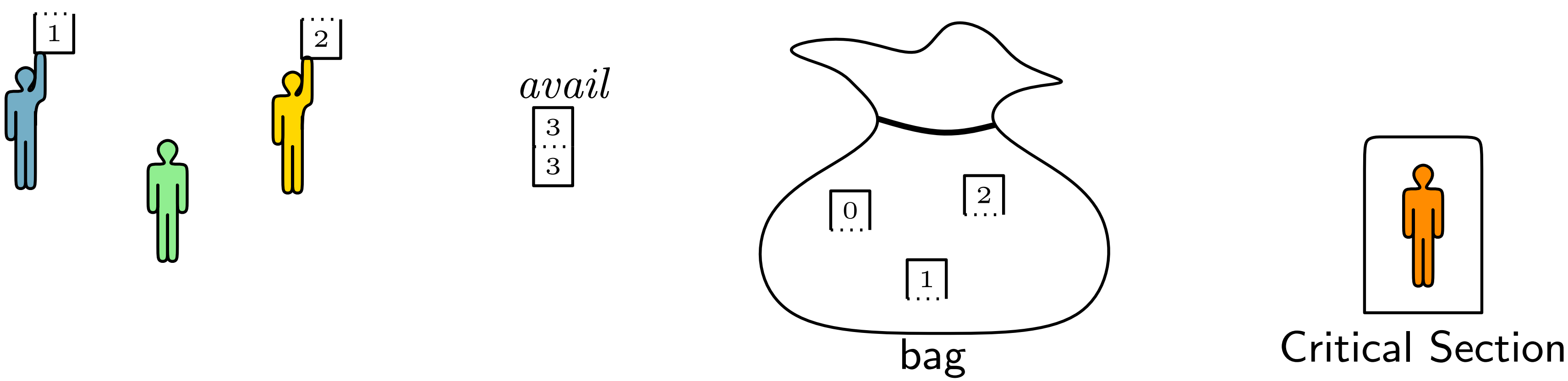

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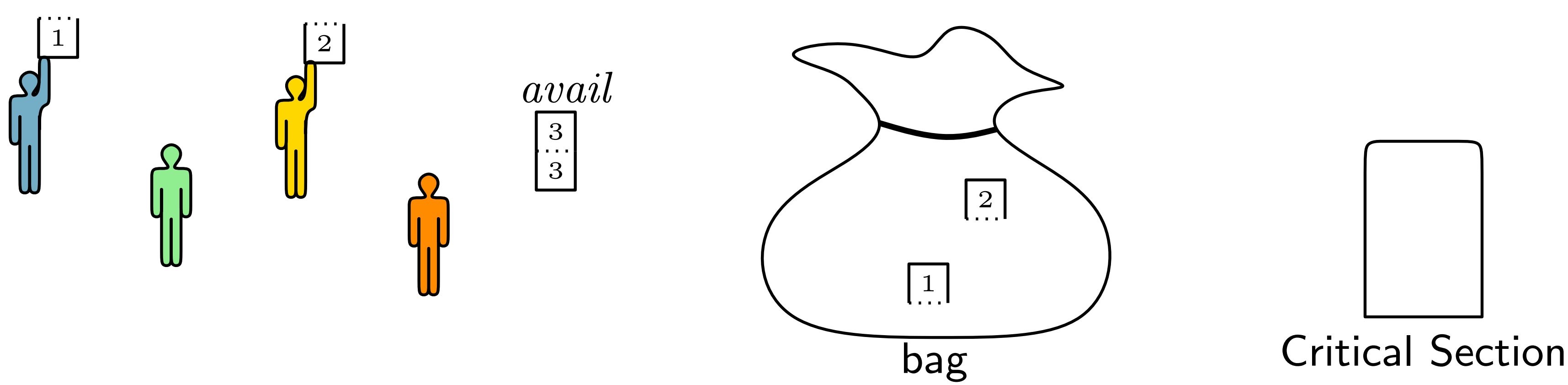

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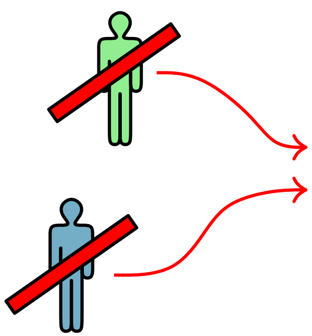

$$\mathsf{mutex}(i, j) \mathrel{\hat{=}} \Box\big[i \neq j \rightarrow \neg(critical(i) \wedge critical(j))\big]$$

General Deductive Verification [Manna-Pnueli '95]

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Problem

Unbounded number of VCs

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2 Threads
T_1, T_2

- Initiation: **1 VC**

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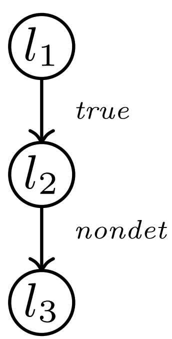

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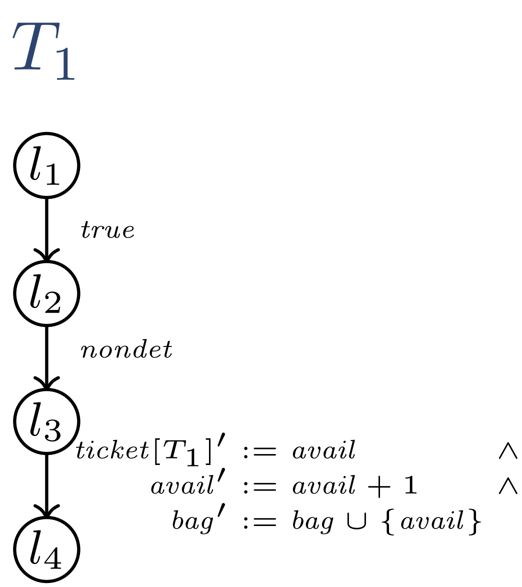

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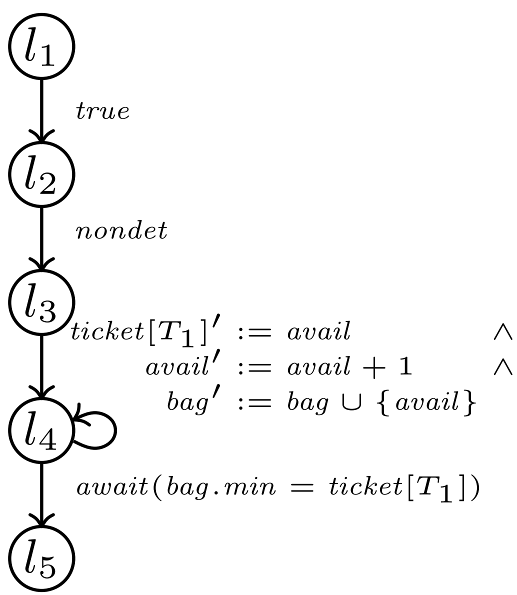

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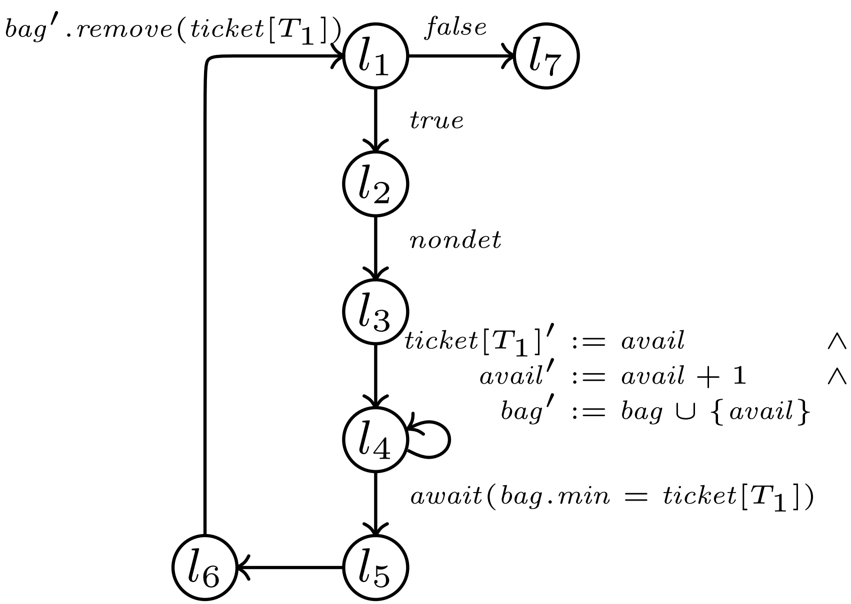

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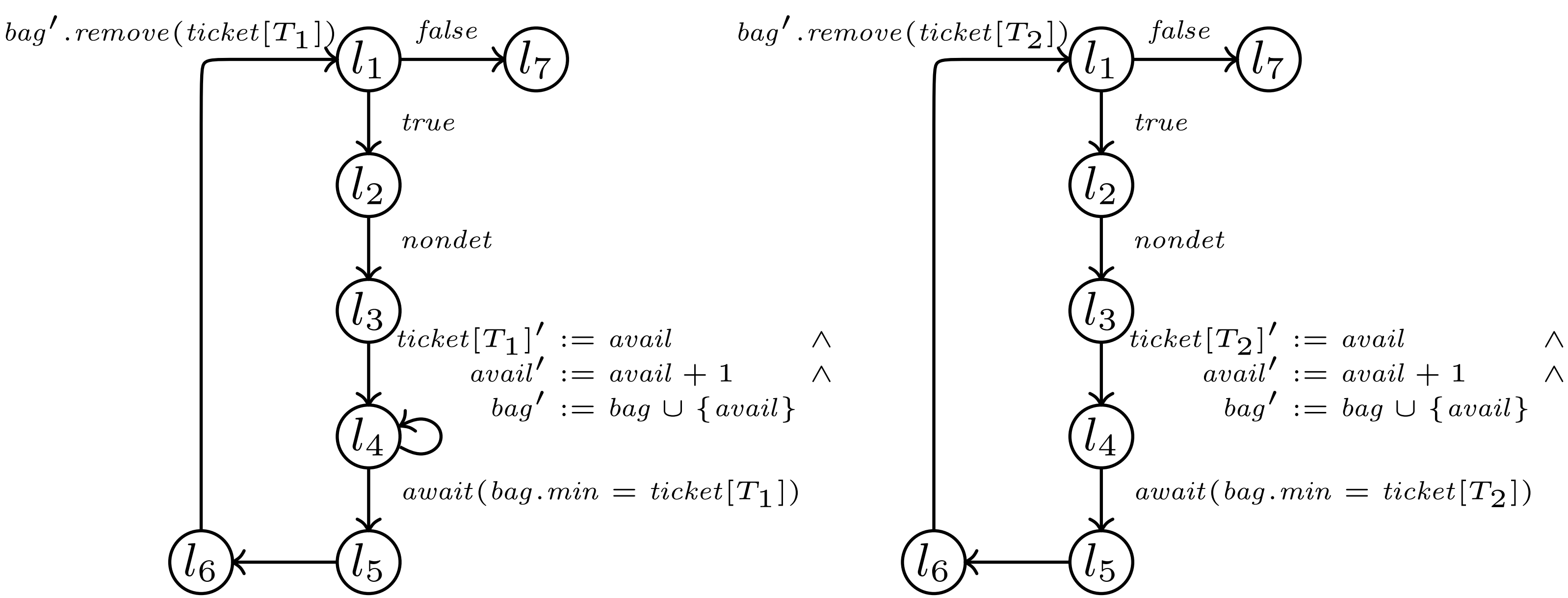
- Consecution: **18 VC**

$T_1 \quad \| \quad T_2$



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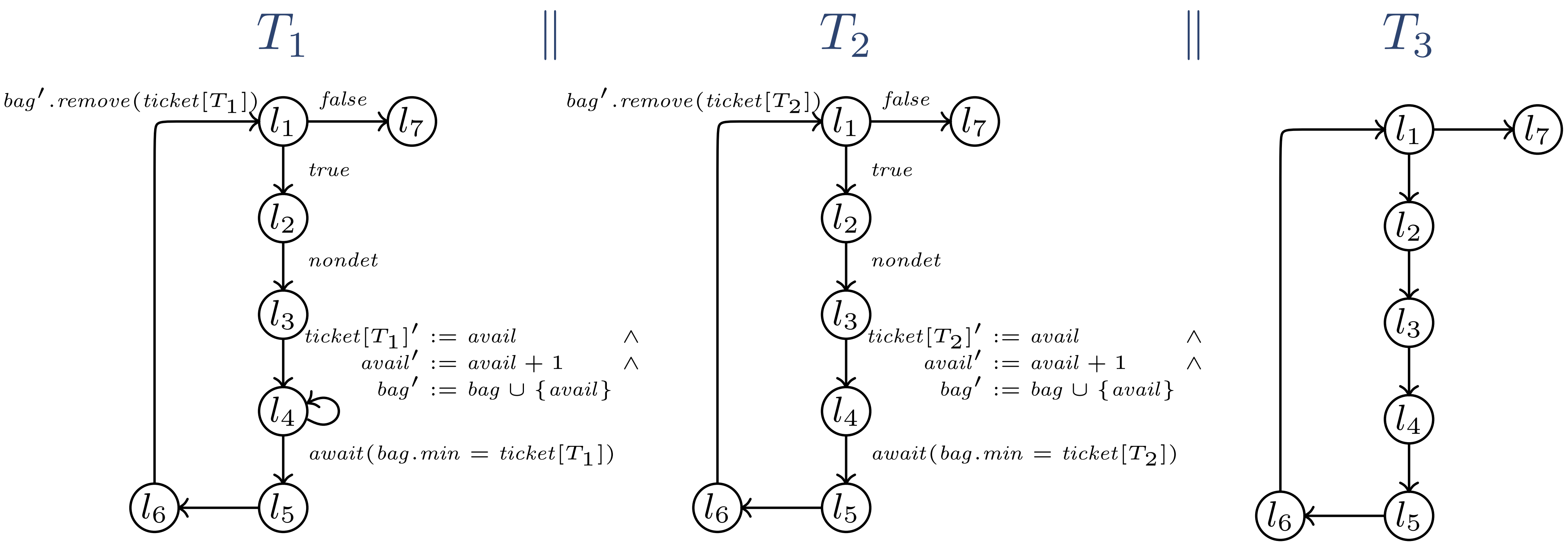

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- Consecution: ~~**18 VC**~~ **27 VC**

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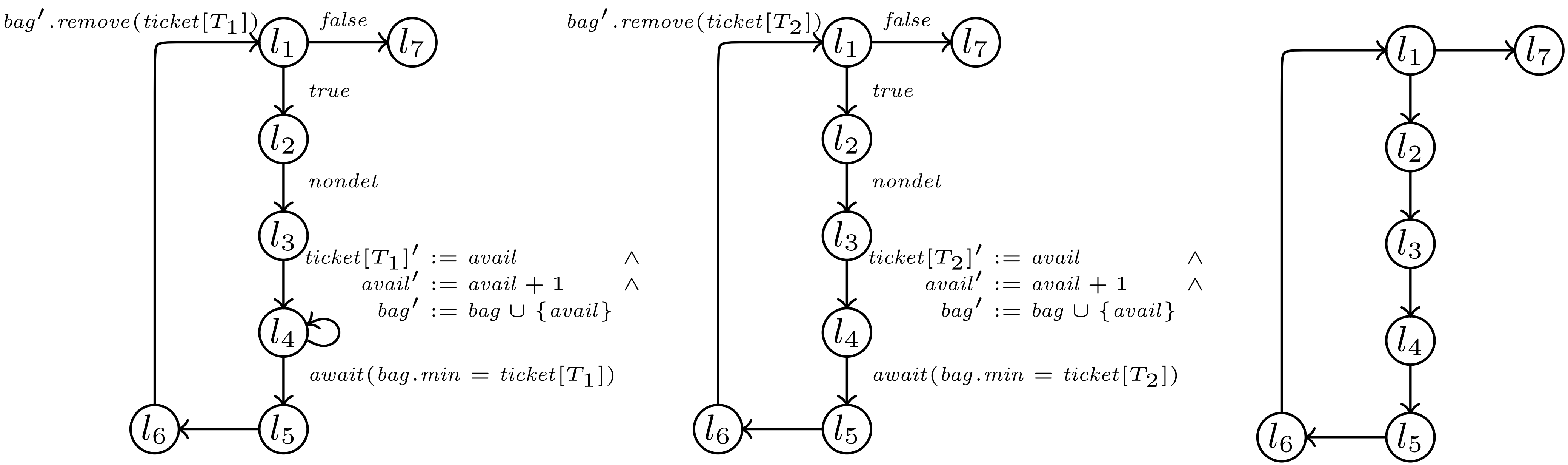

Parametrized Invariance (p-inv)

- **Bounded** number of VC, based on **program** and **specification**

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To show that $\mathcal{S}$ satisfies $\varphi(\overline{v})$:

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Initiation (I)

Self-consecution (SC)

Other-consecution (OC)

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Initiation

Self-consecution

Other-consecution

- For our example: mutex(i, j)

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Initiation (I), **Self-consecution** (SC), **Other-consecution** (OC)

- For our example: $\text{mutex}(i, j)$ **#VC : 1**

$$(\text{ I }) \quad \Theta(i, j) \rightarrow \text{mutex}$$

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- For our example: $\text{mutex}(i, j)$ **#VC : 1 + 18**

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Other-consecution

Independently on #threads in the system

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$$

LEAP: A Temporal Deductive Prover

LEAP

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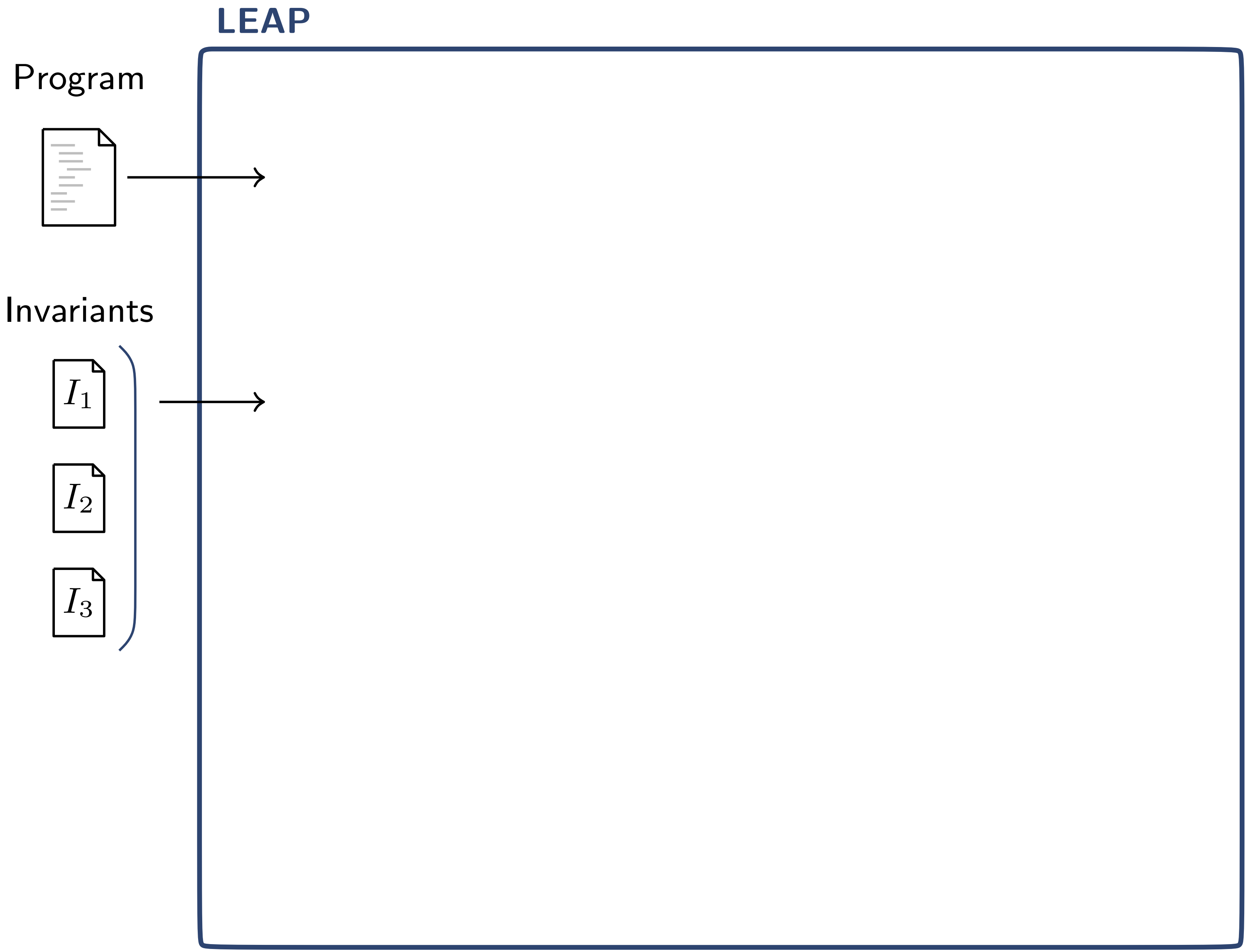

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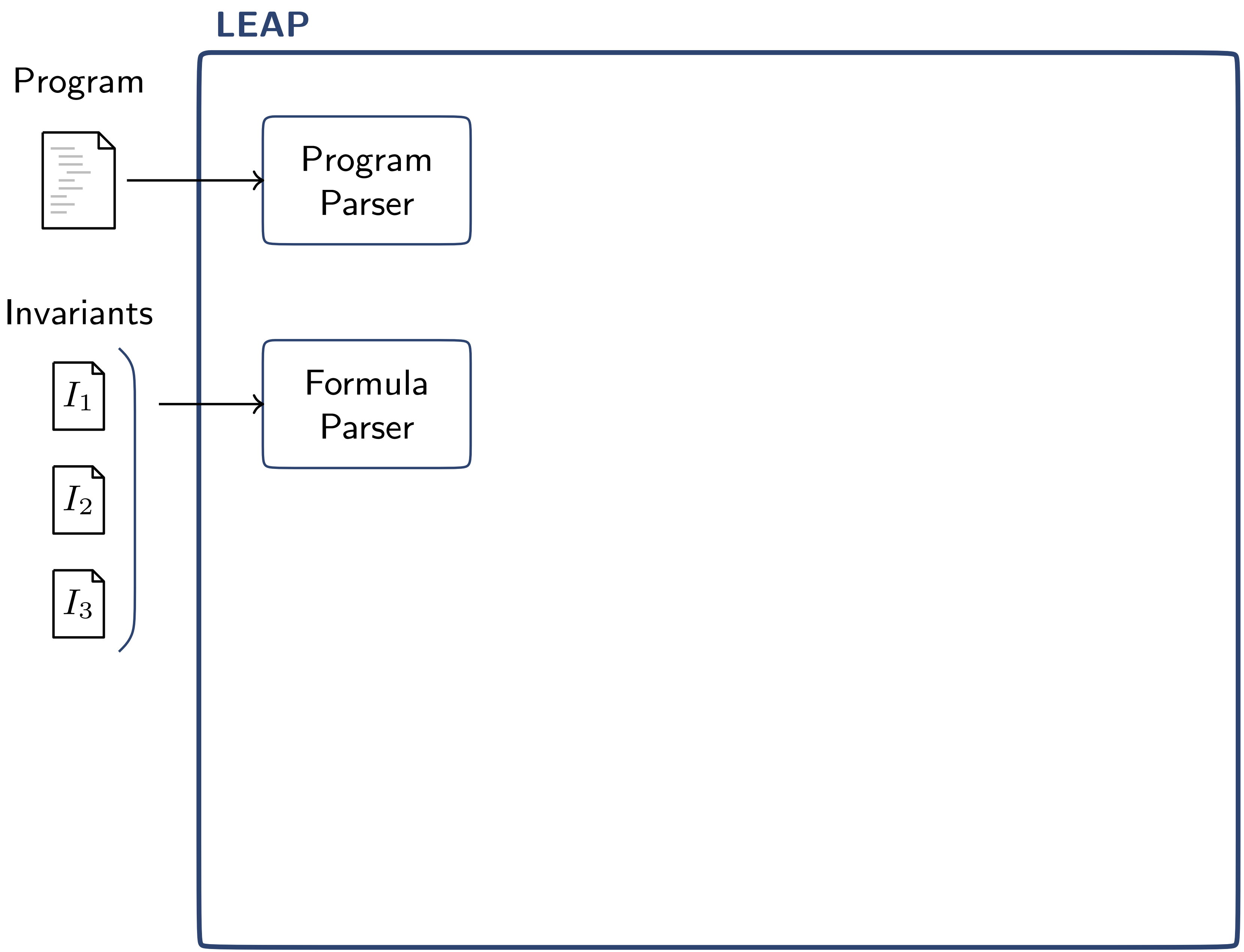

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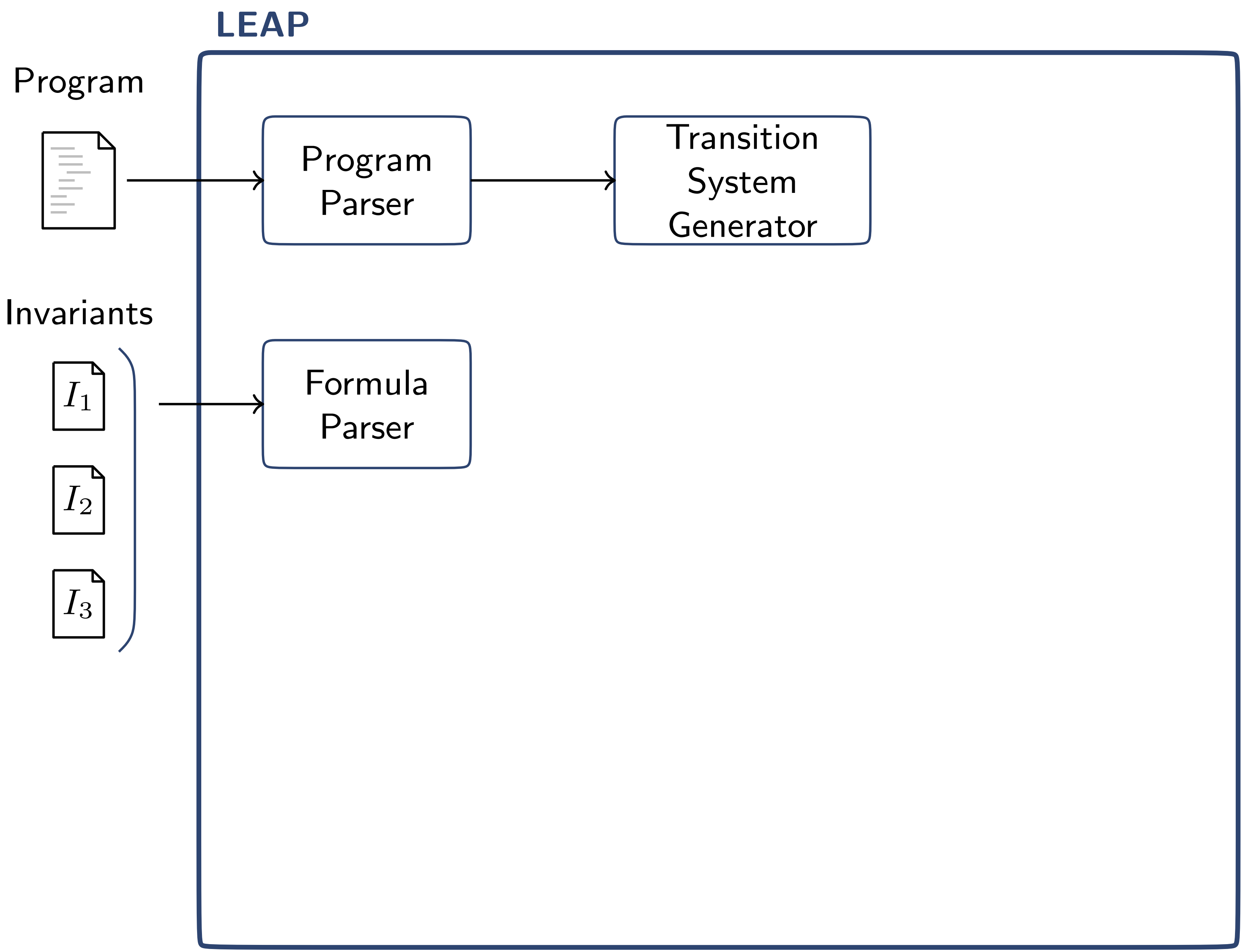

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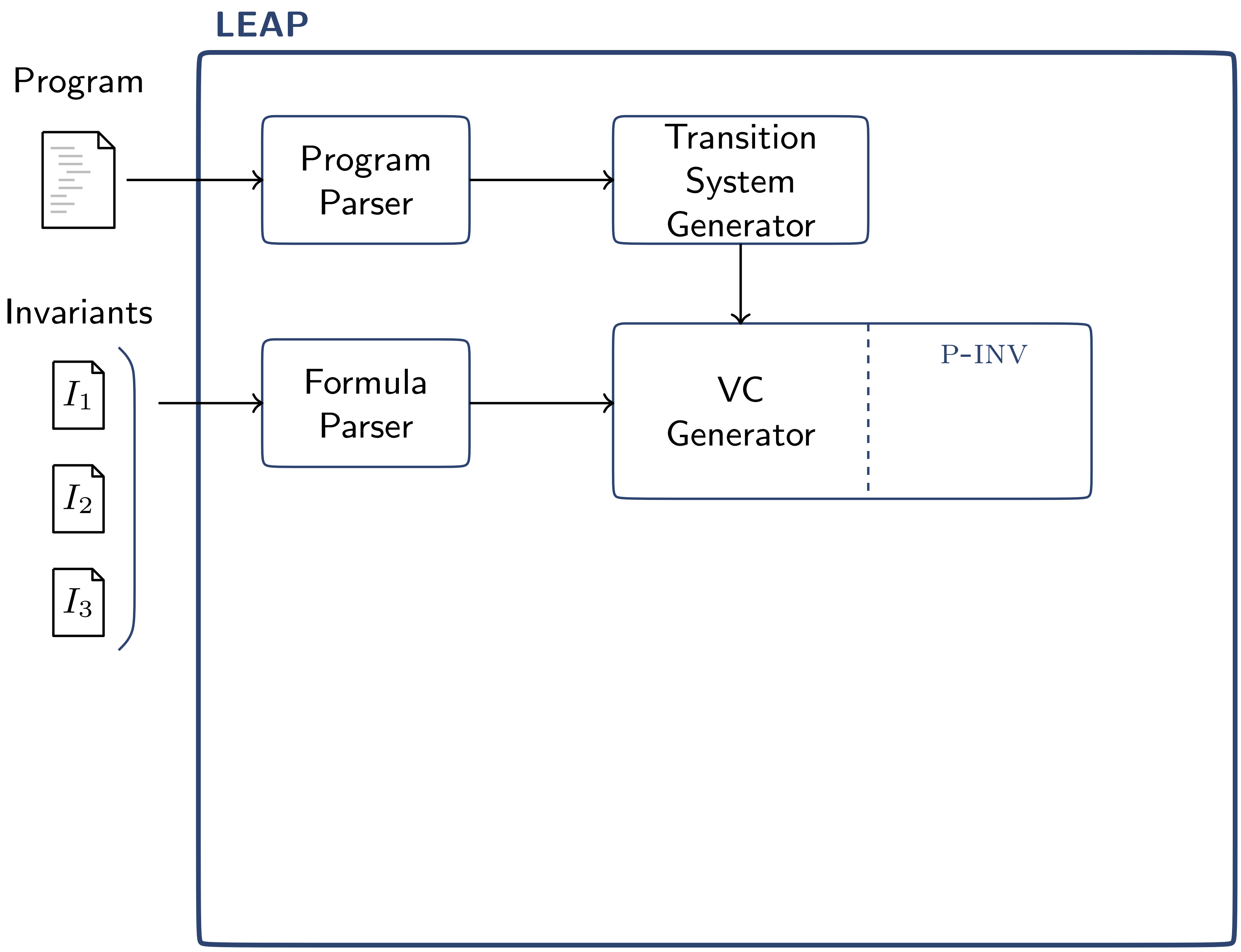

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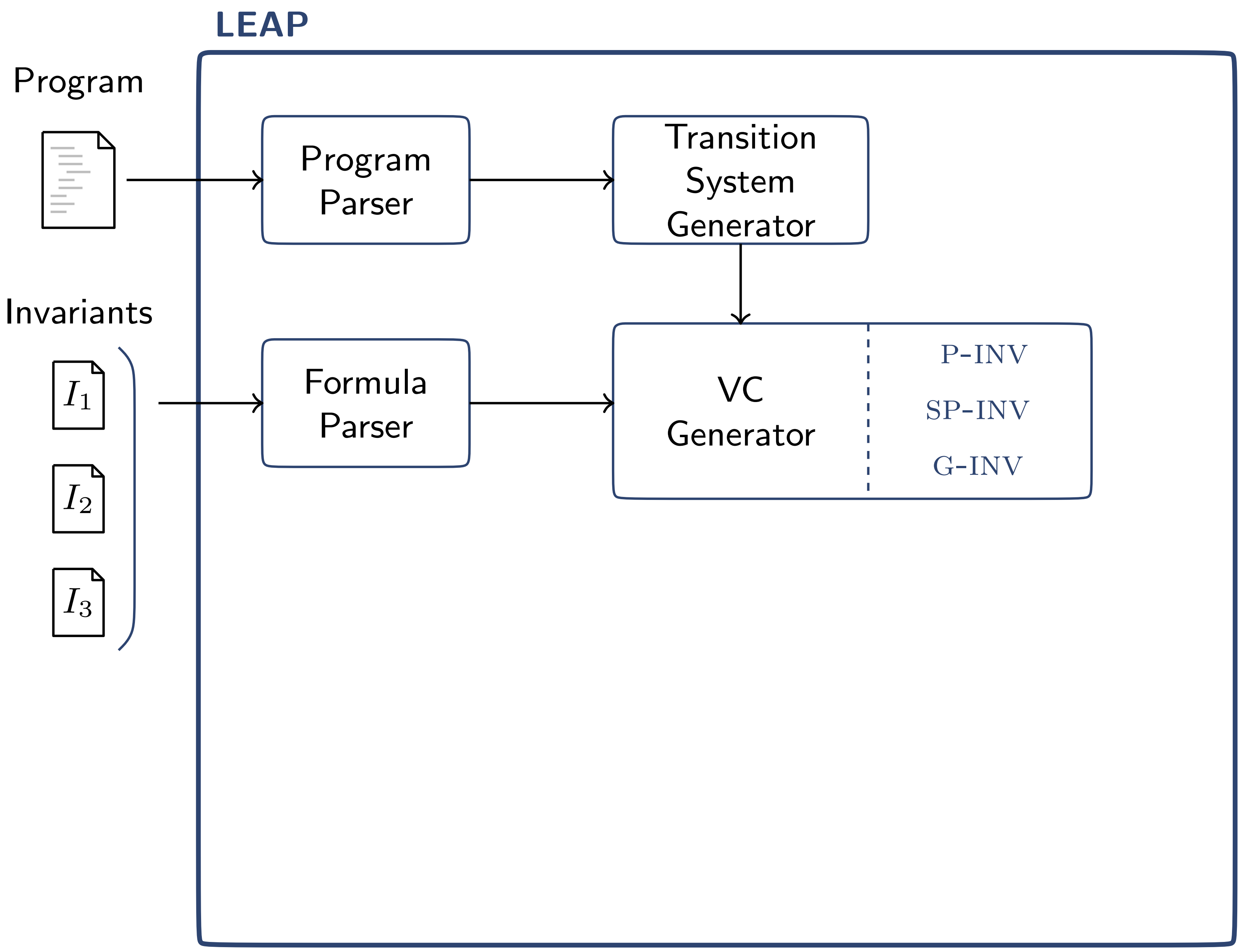

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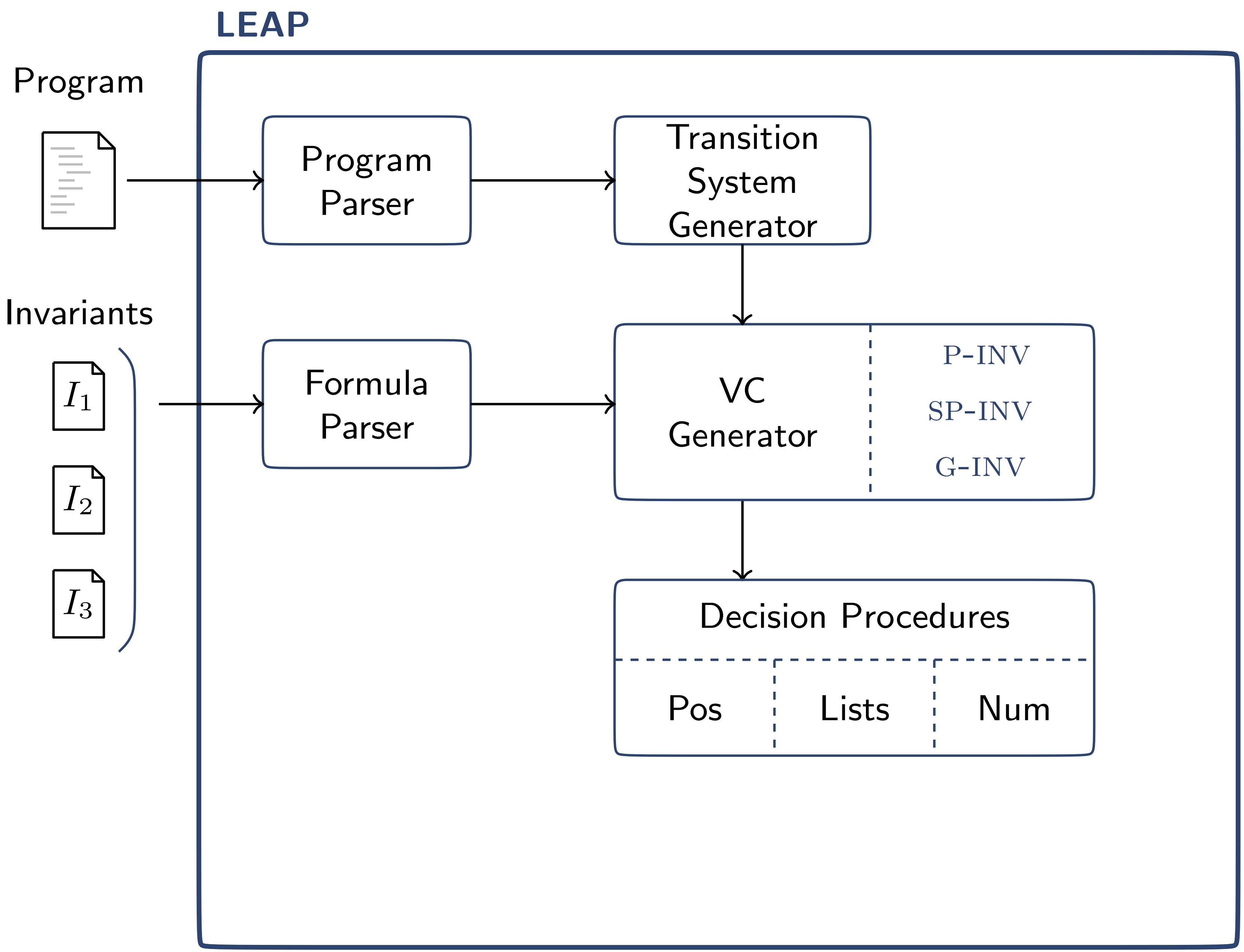

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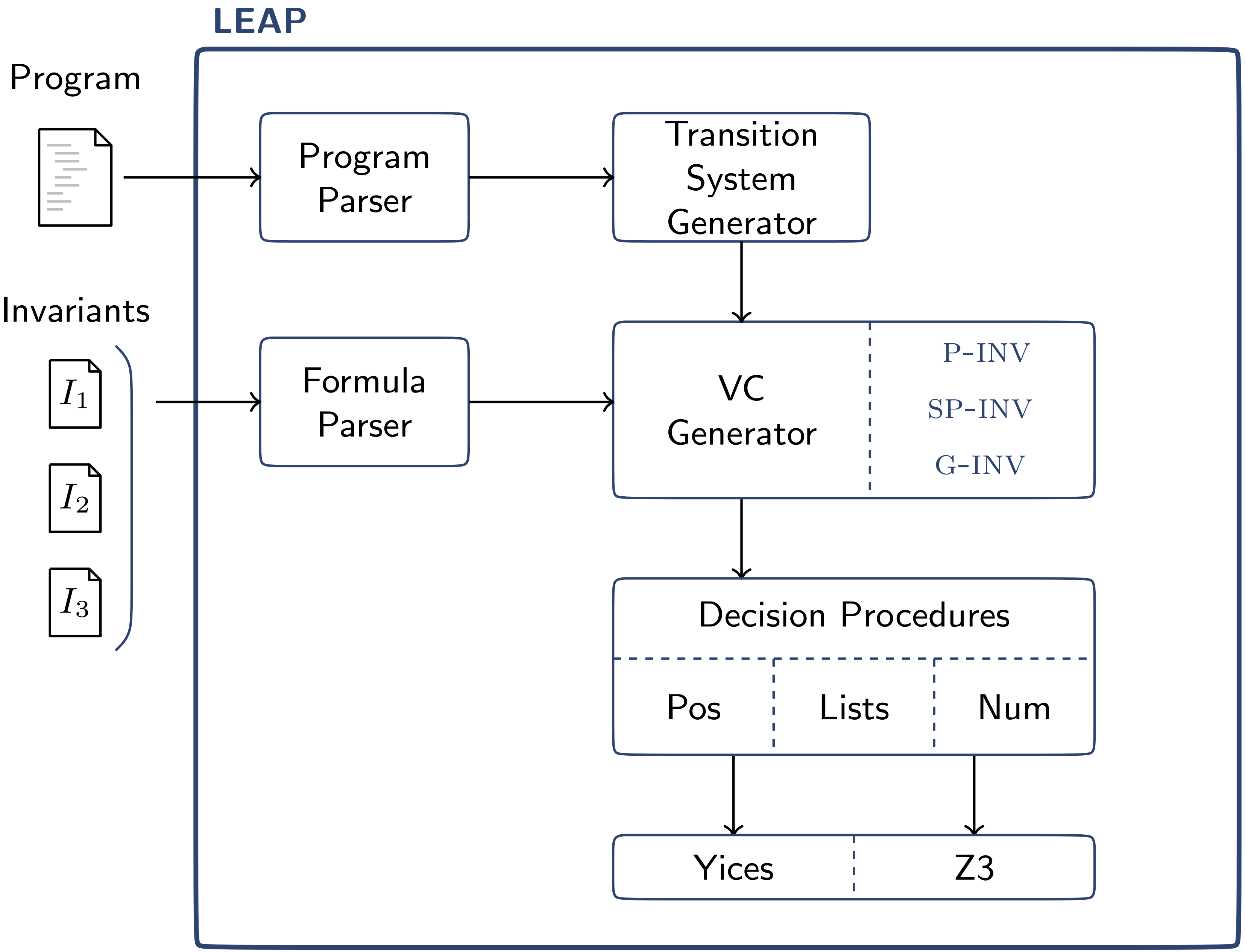

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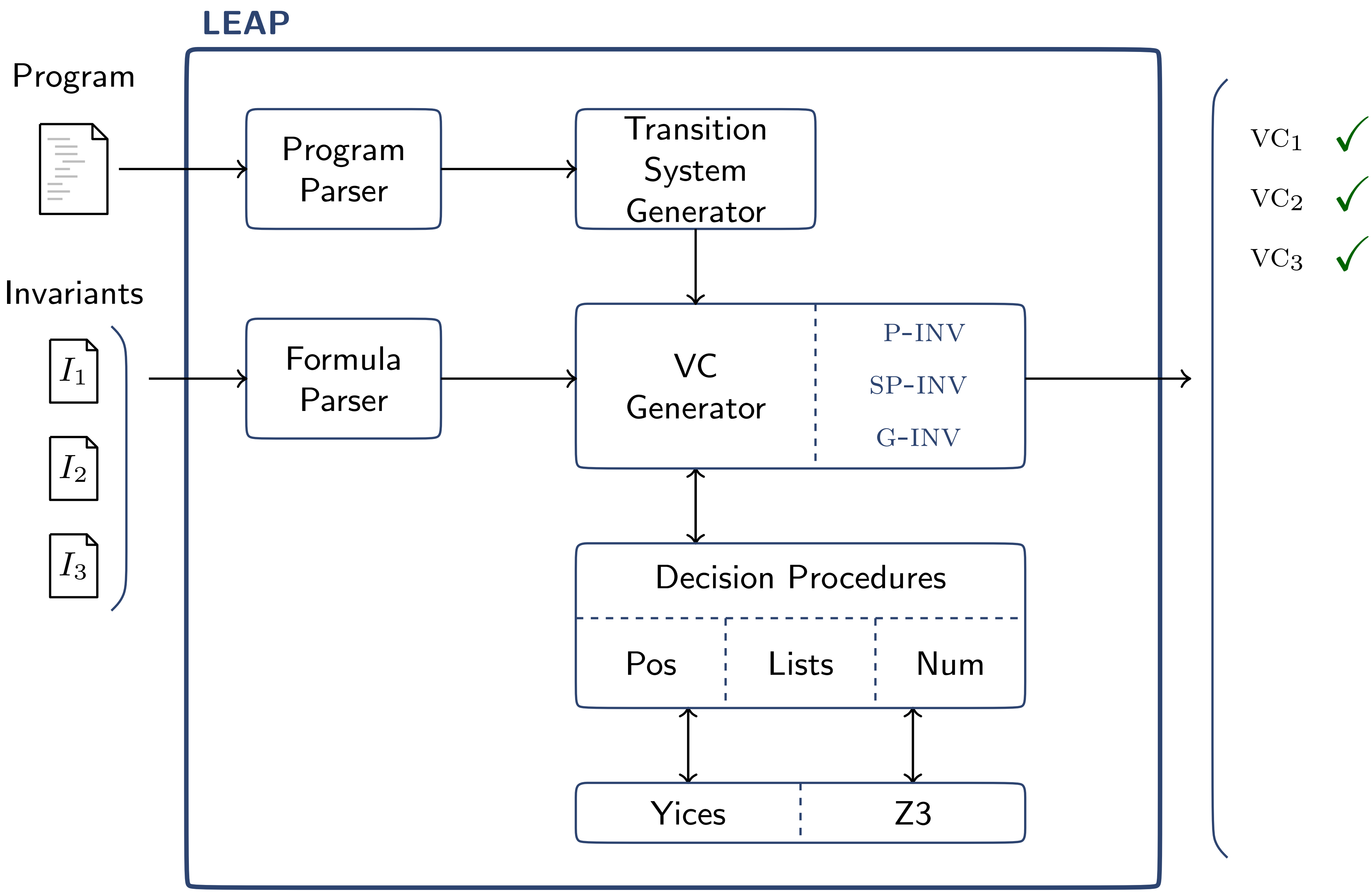

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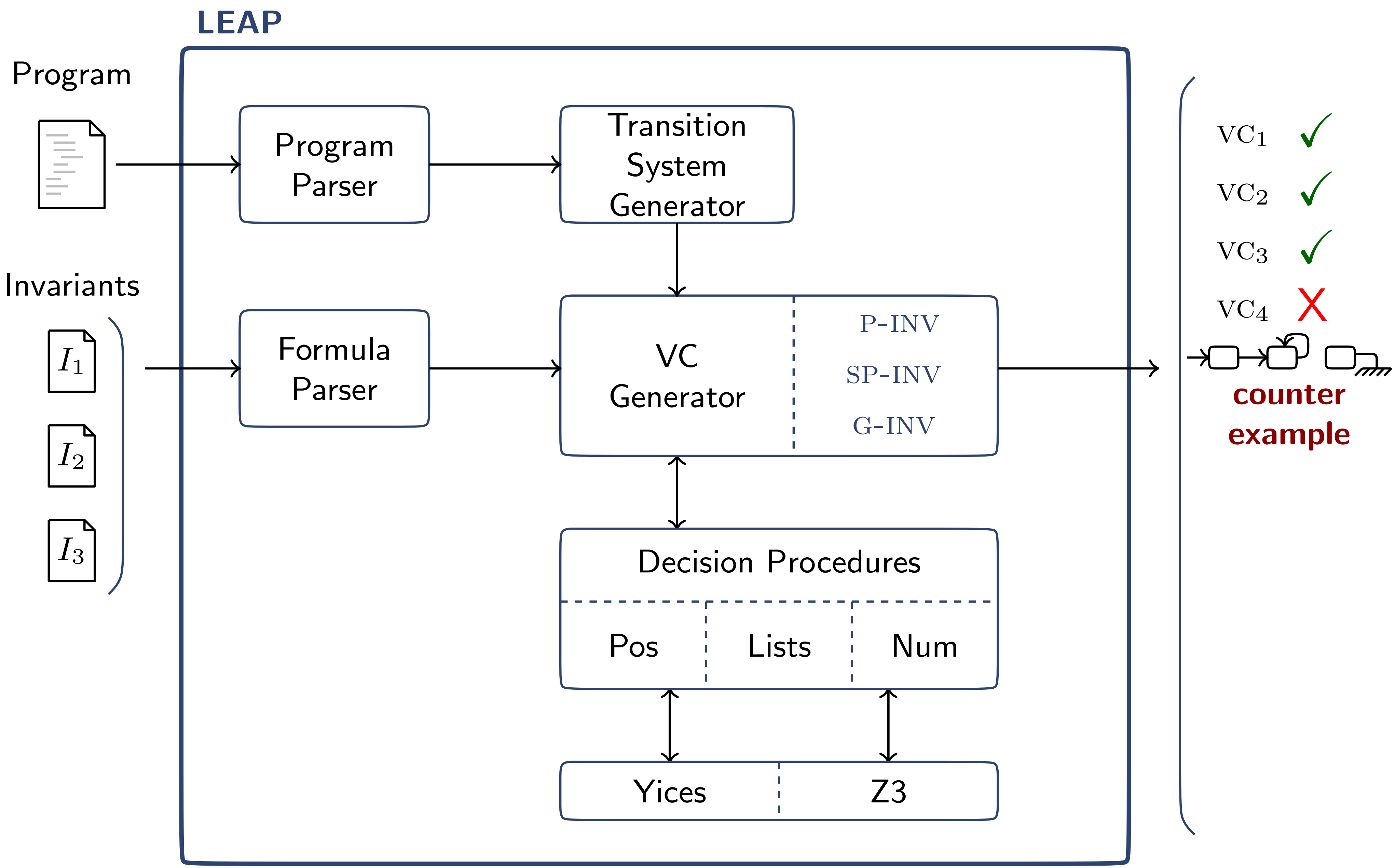

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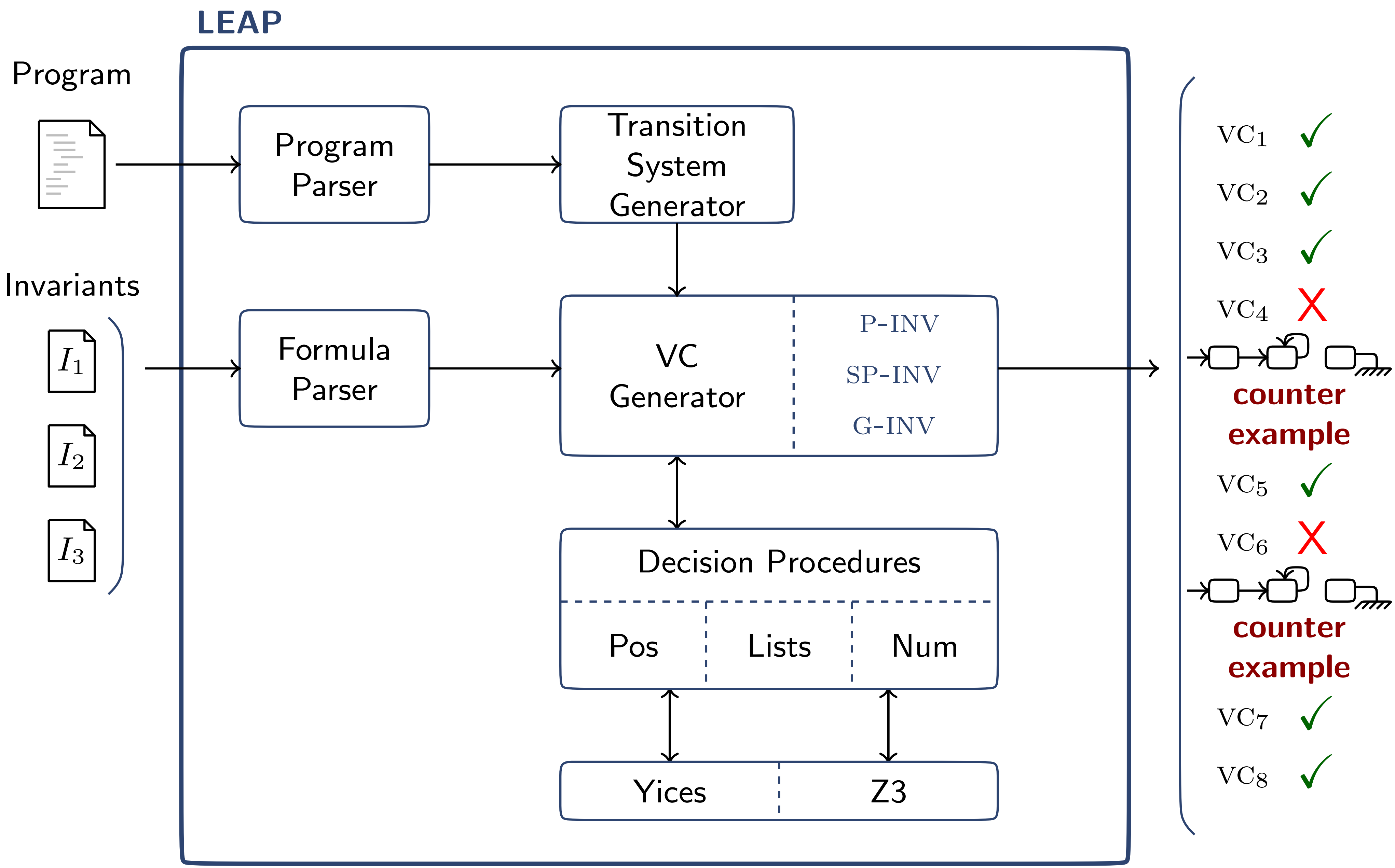

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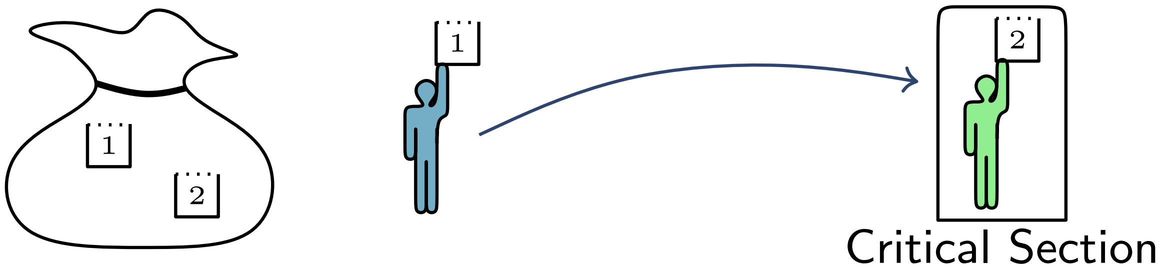

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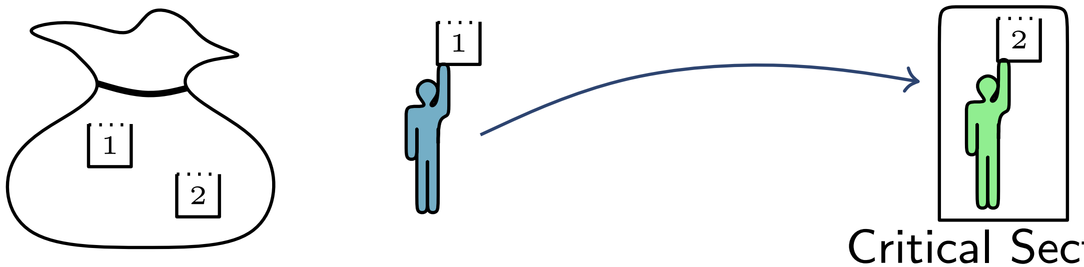

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$$\text{minticket}(i) \quad \hat{=} \quad \Box\left[critical(i) \rightarrow min(bag) = ticket[i]\right]$$

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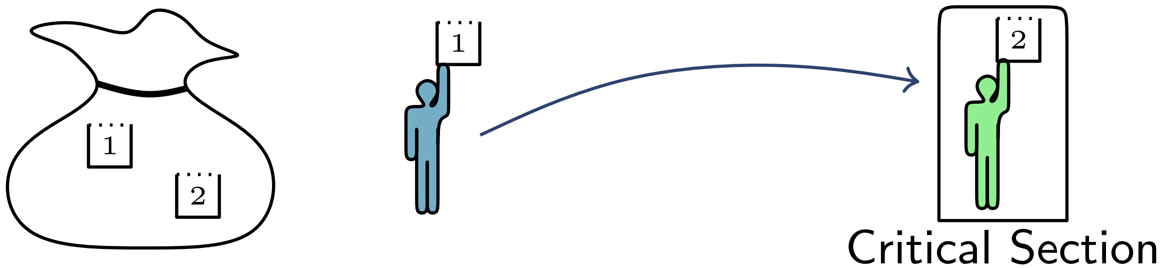

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> We now require a **new rule** for **invariant support**

Parametrized Invariance with Support (sp-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi(\overline{v})$. Find $\psi(\overline{w})$ with:

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\begin{array}{lllcll}
(\ \mathsf{S}\ ) & & \Box\psi & & & \\
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strenghtening

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Instantiate the assumptions for **self** and **others**

$\psi \rhd (A \rightarrow B)$ whether $\left[\left(\bigwedge_{\sigma \in S} \psi_\sigma \wedge A\right) \rightarrow B\right]$

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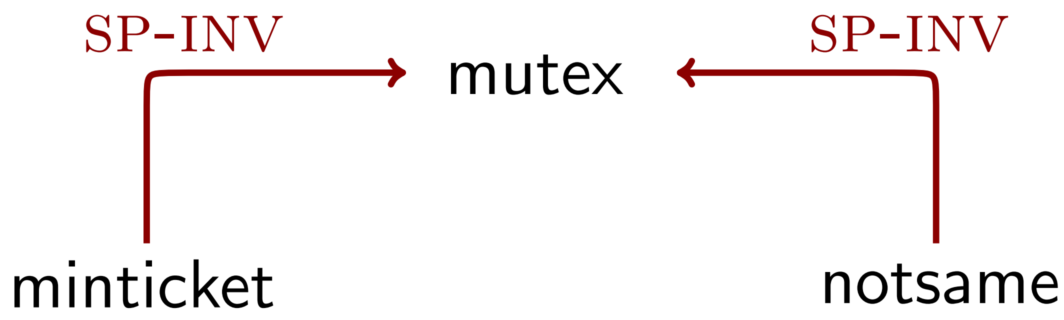

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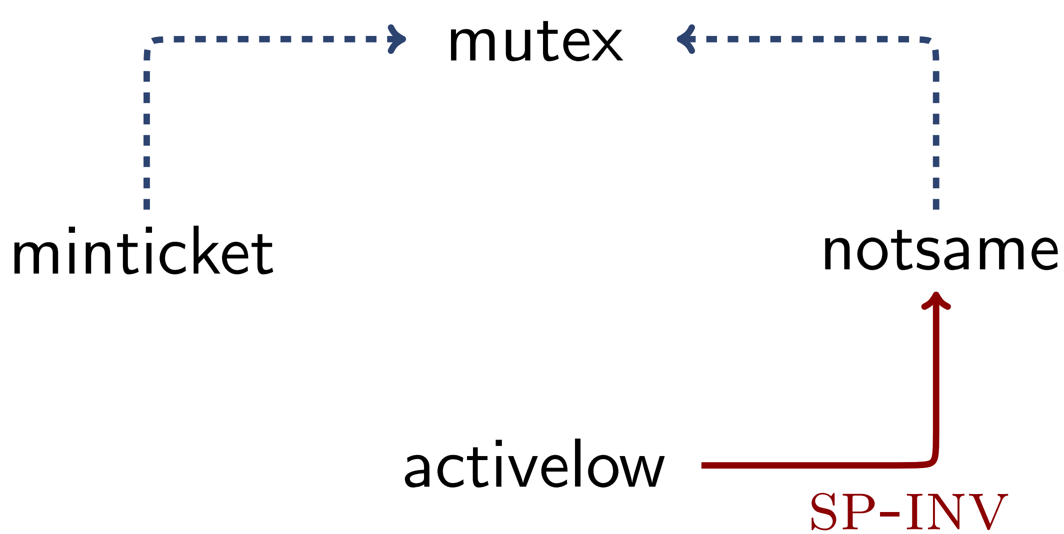

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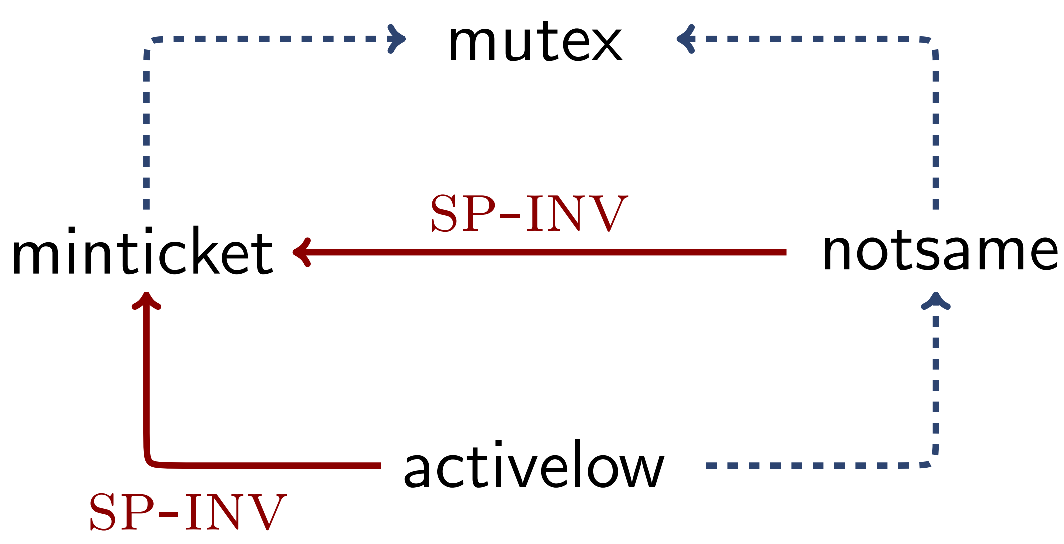

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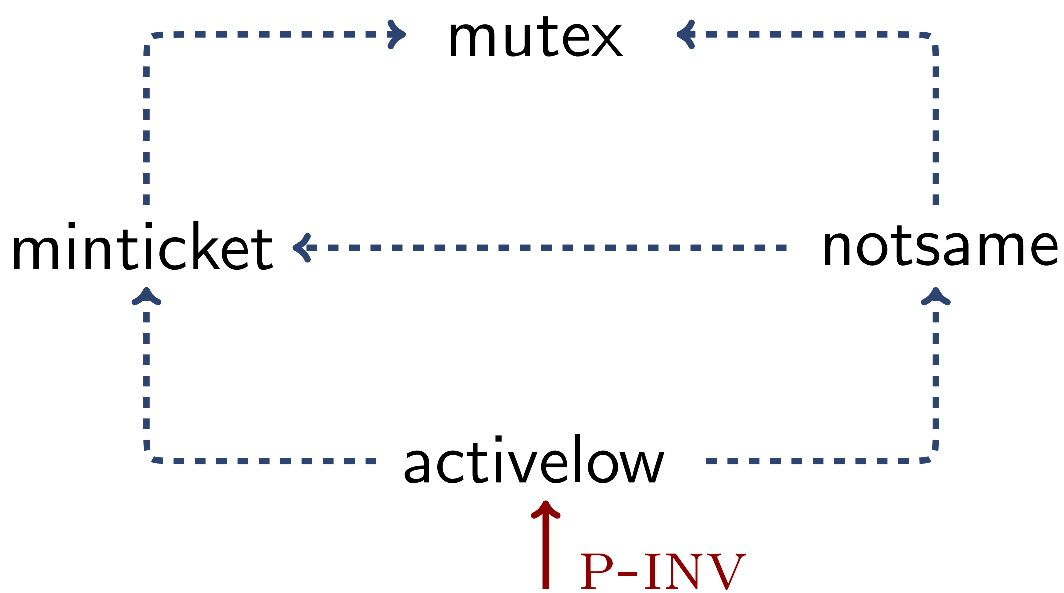

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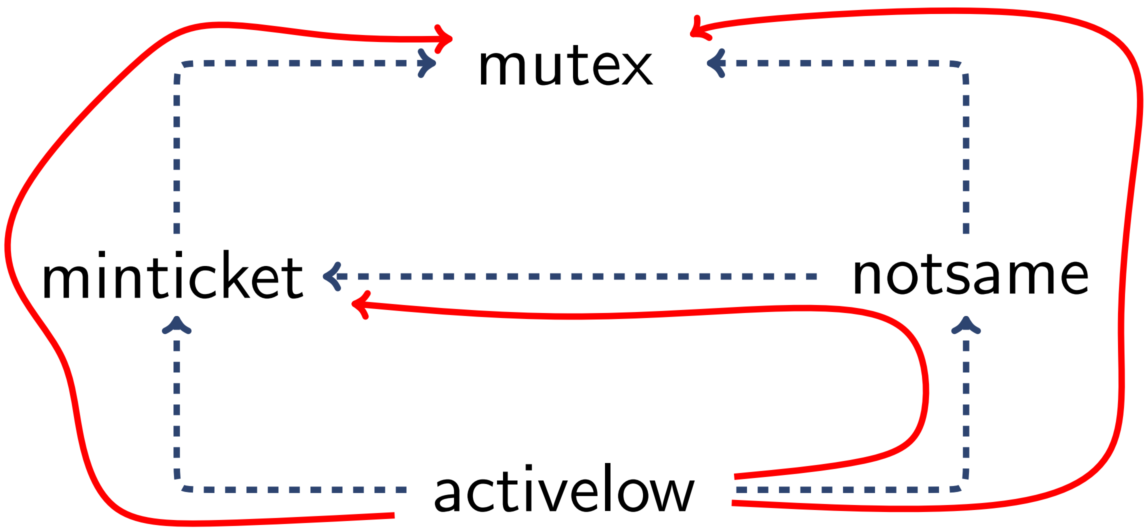


In this case, the proof is a **DAG**

DEMO: Mutex (2nd attempt)

- **LEAP format**

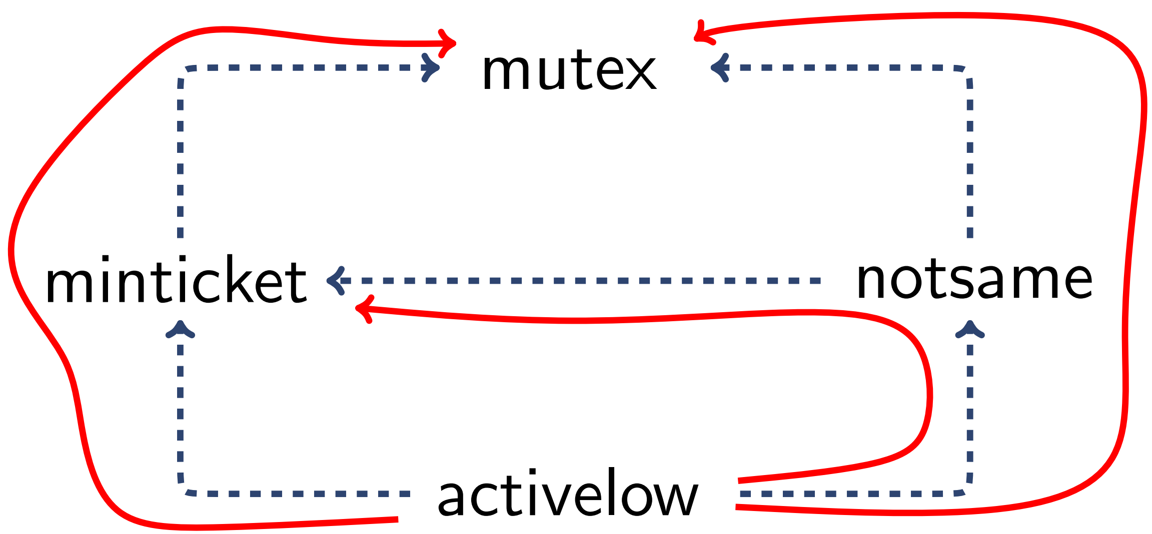
```
-> activeLow

-> notSame [3:SC:activeLow]

-> minTicket [3:activeLow; 6:OC:activeLow,notSame]

-> mutex [4:SC:notSame, minTicket]
```

- How does the **full proof** look?



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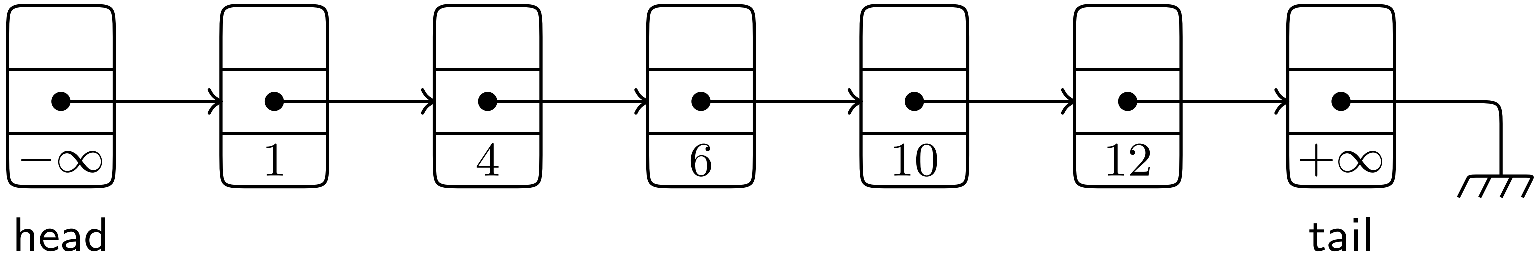
Case Study 2: Lock-Coupling Lists

- A concurrent implementation of sets



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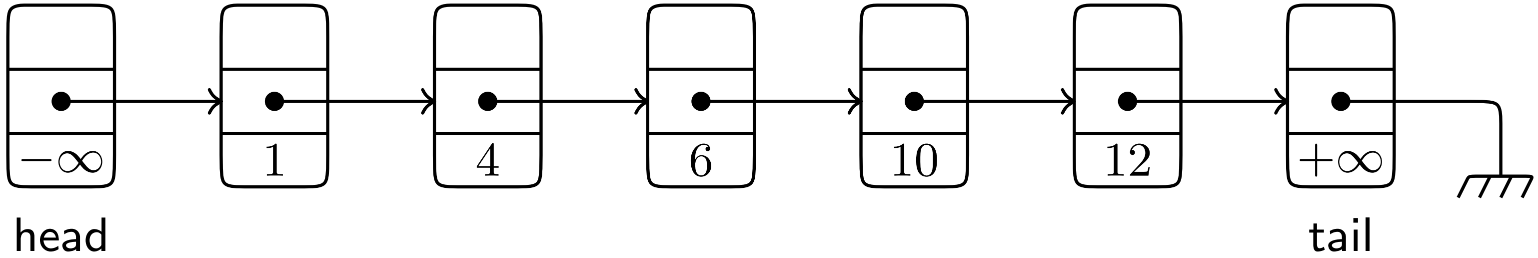
- Operations such as SEARCH, INSERT and REMOVE



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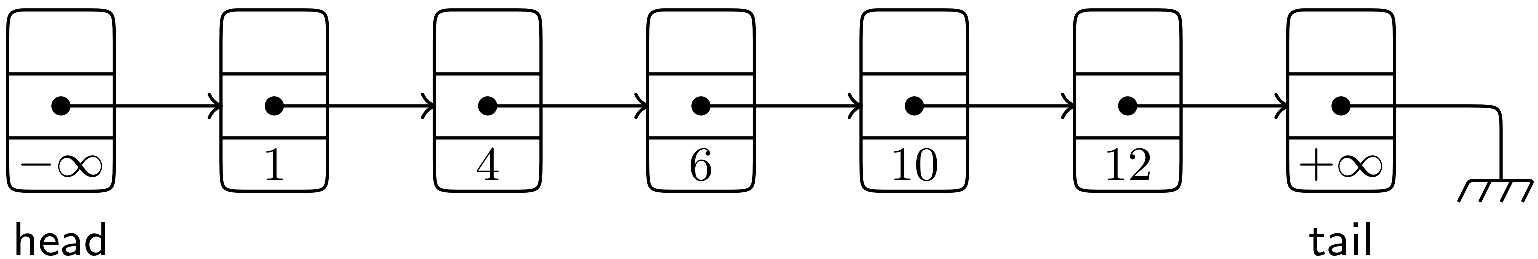
SEARCH(9)



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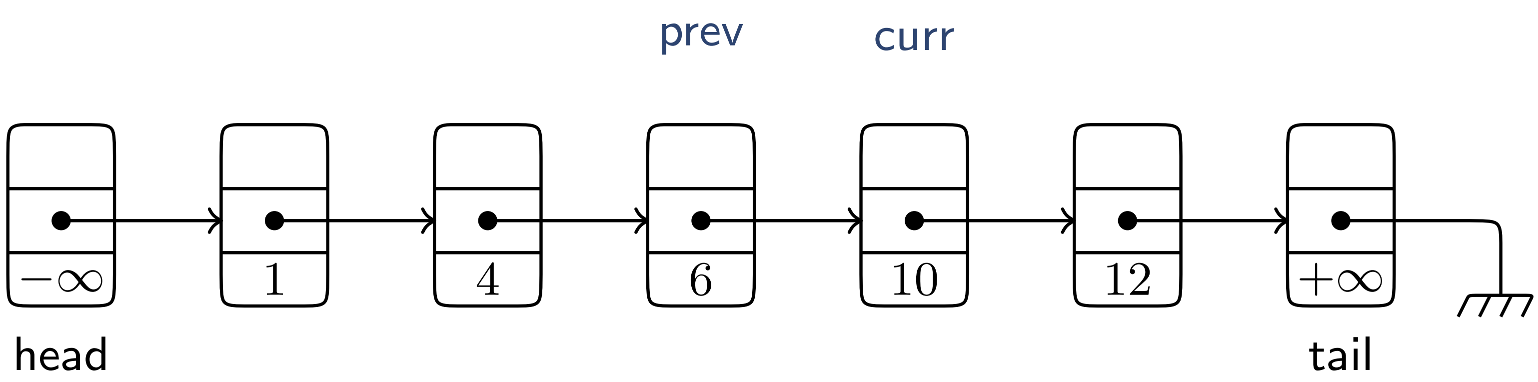

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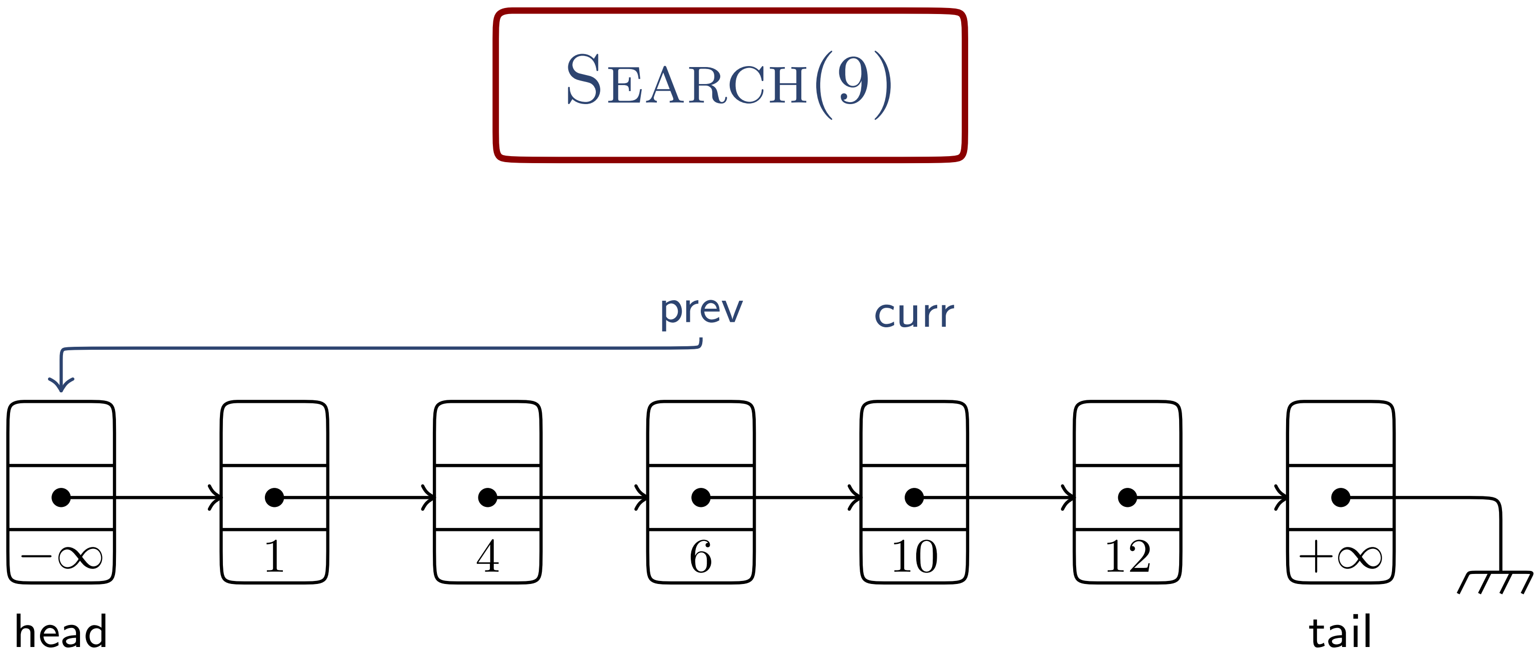

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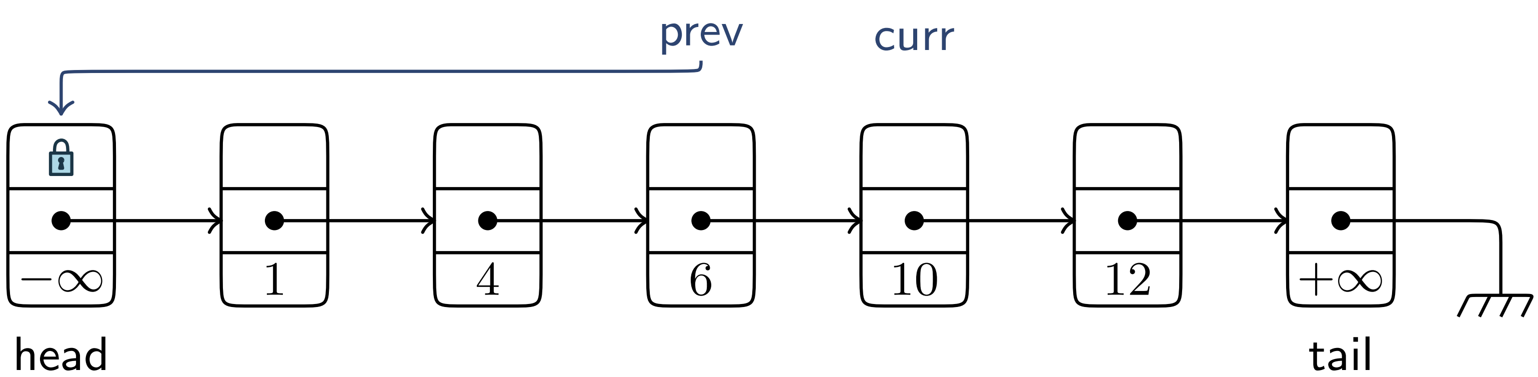

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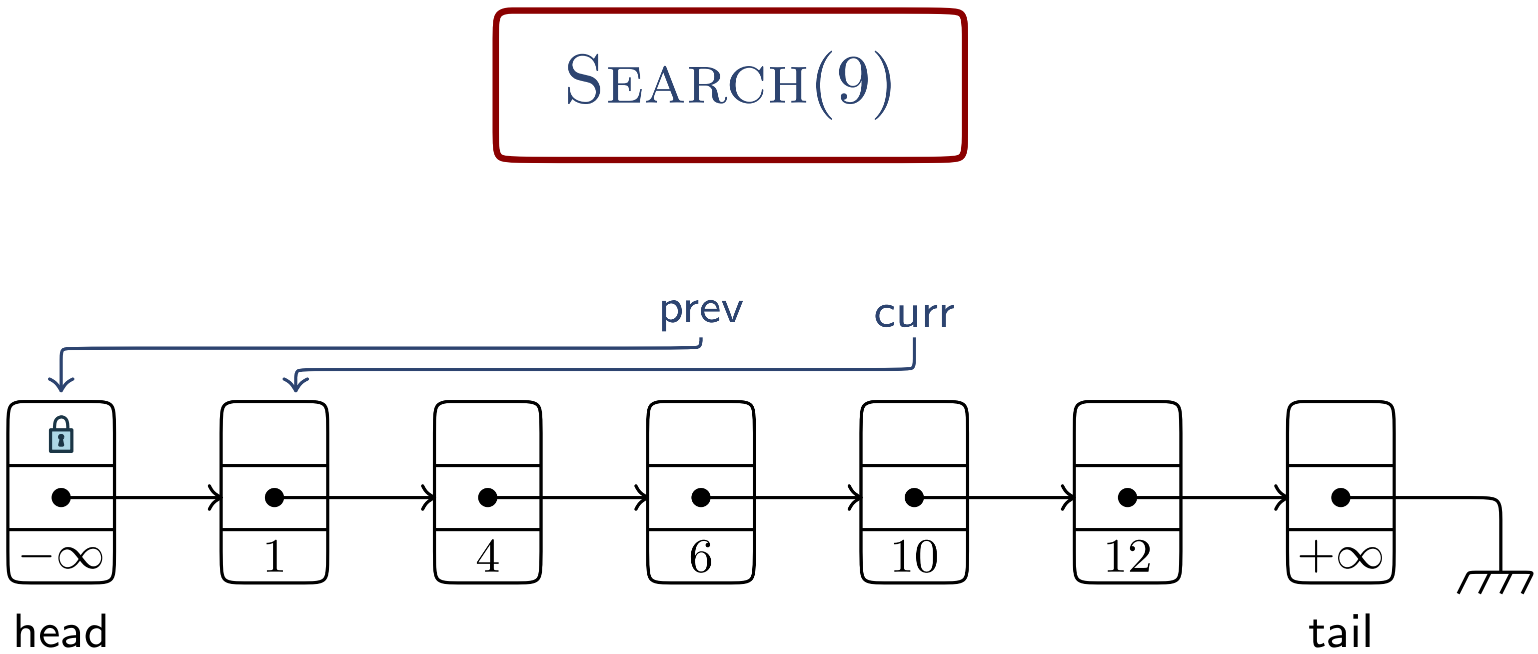

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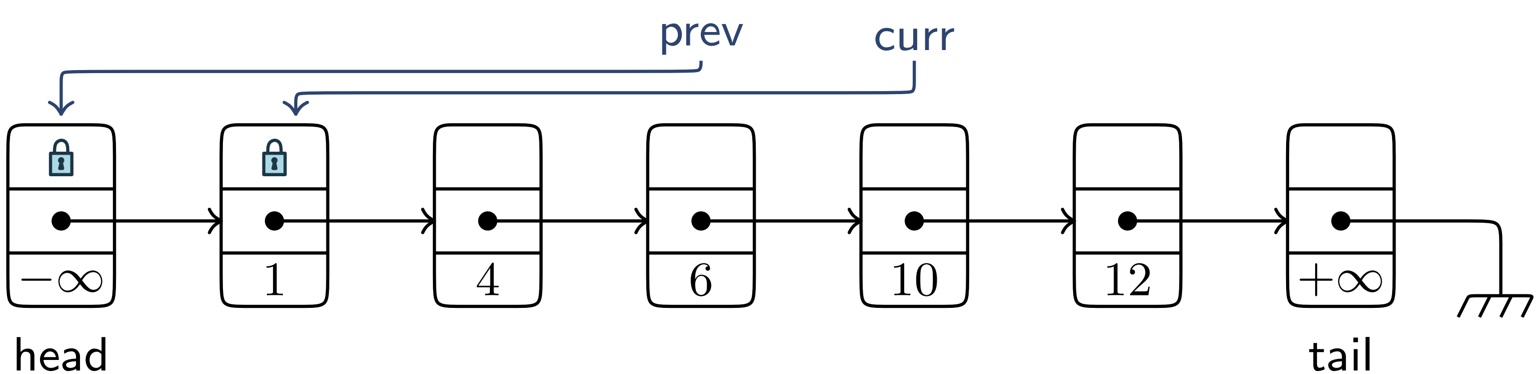

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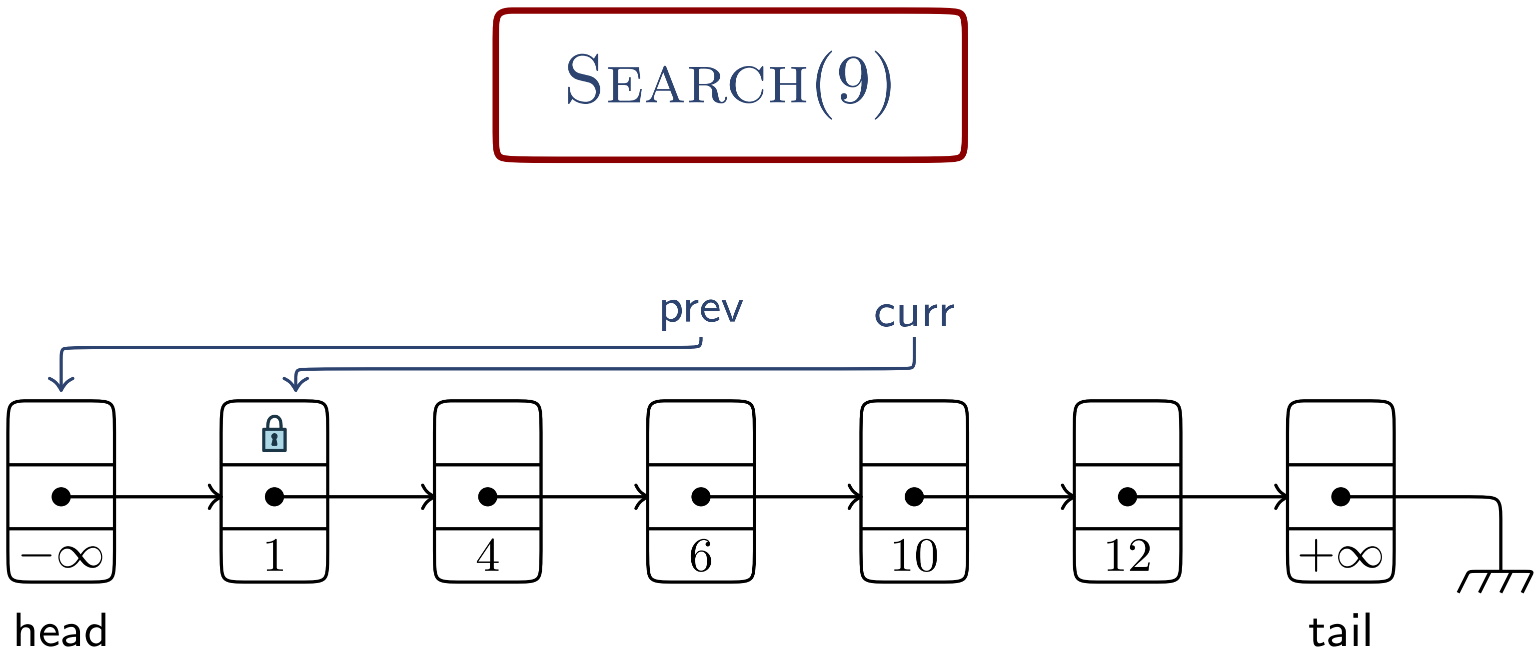

Case Study 2: Lock-Coupling Lists

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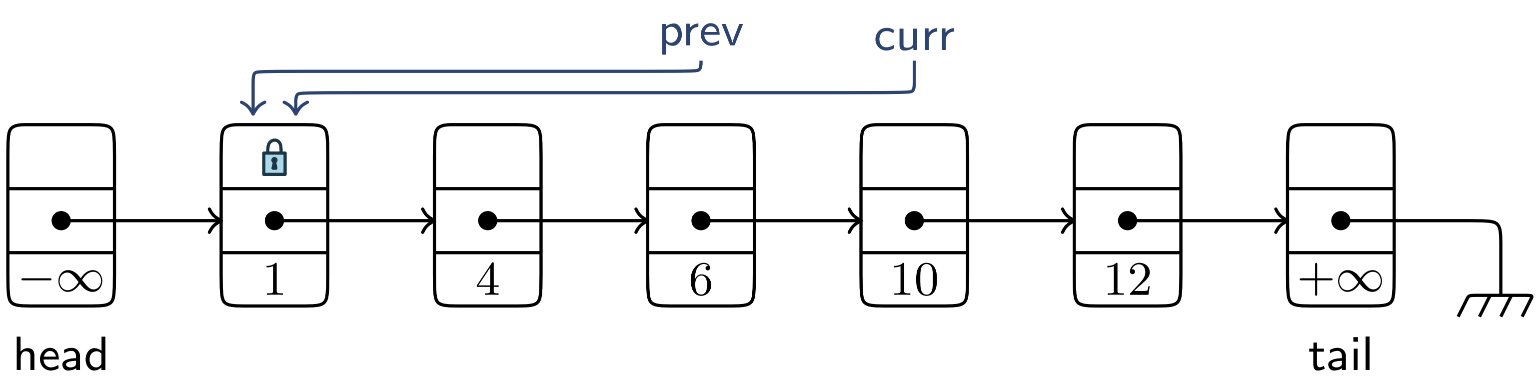
SEARCH(9)



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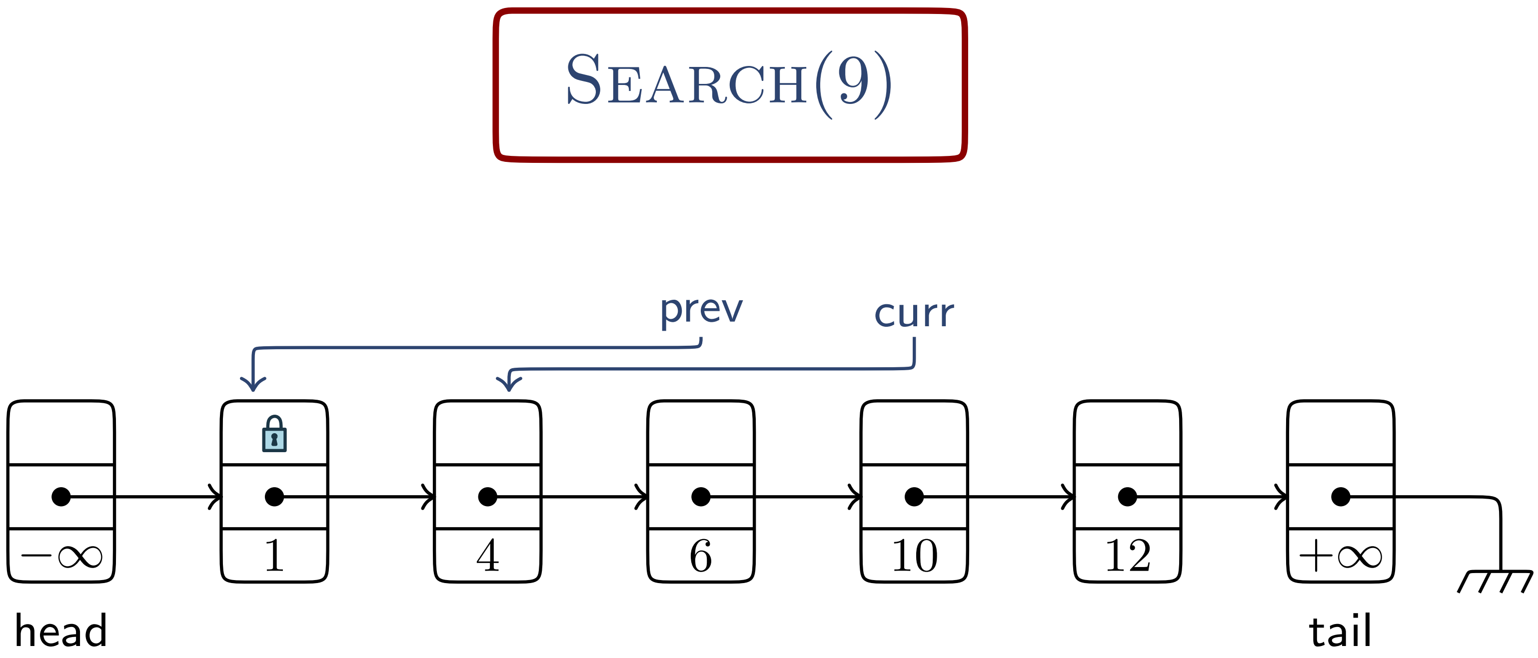

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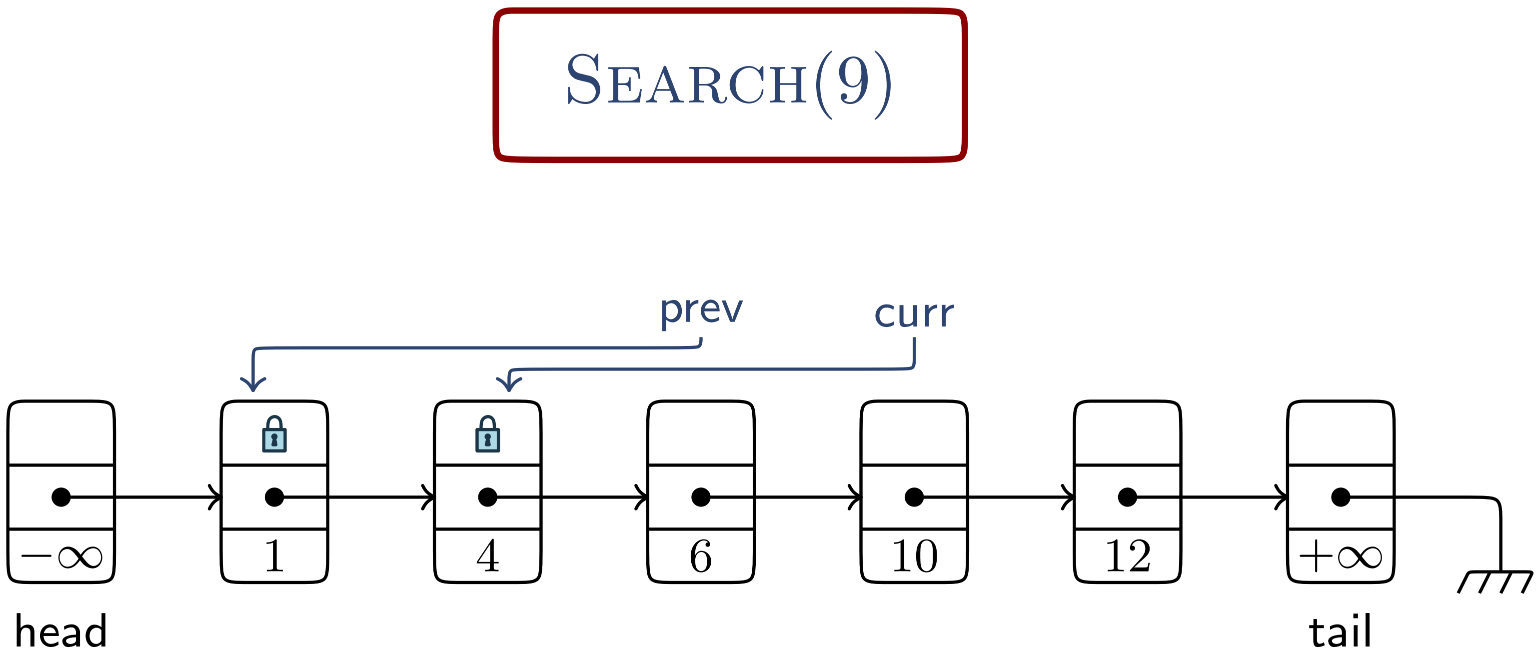

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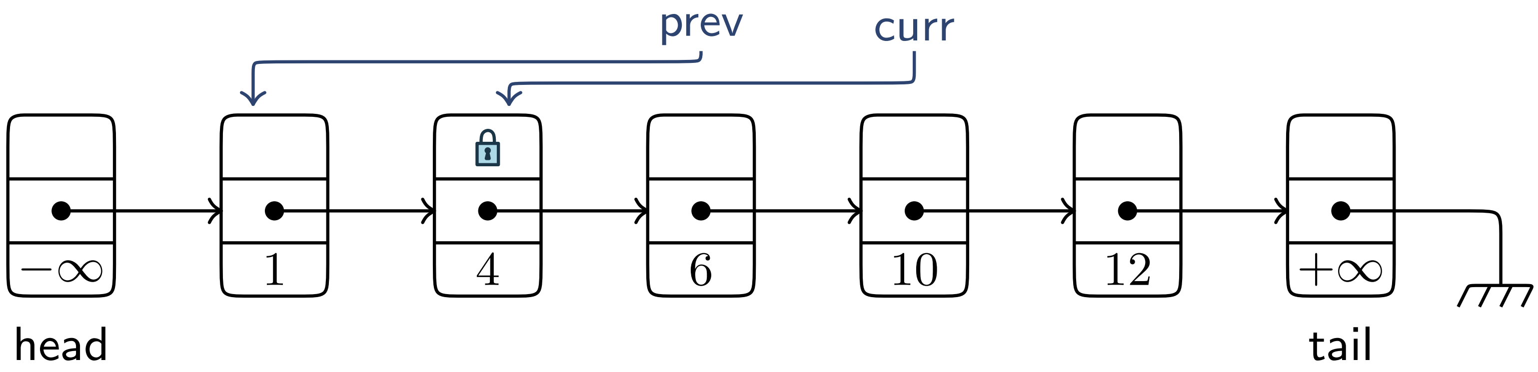

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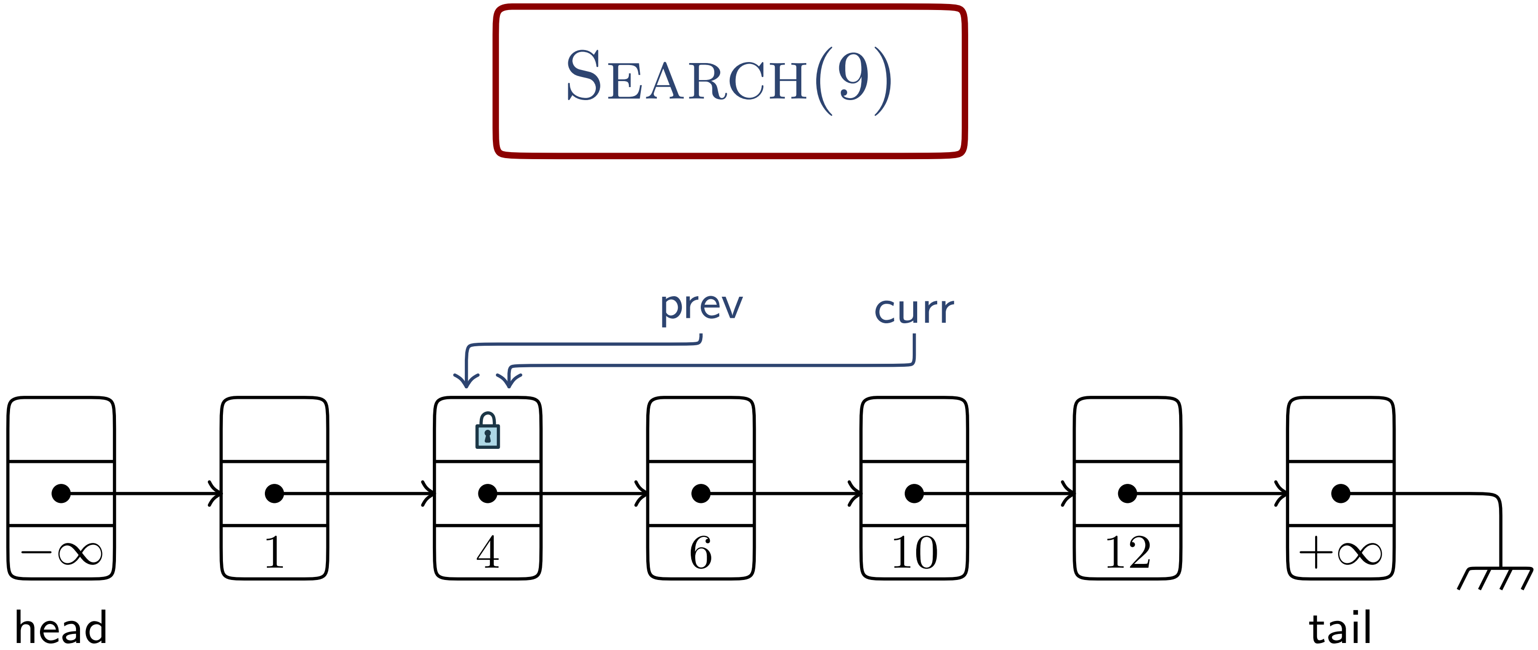

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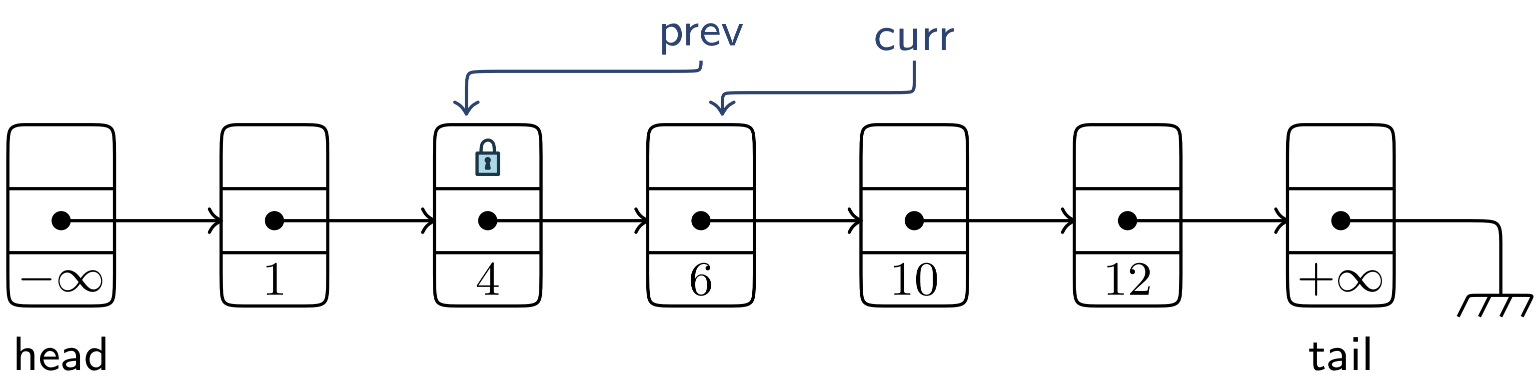

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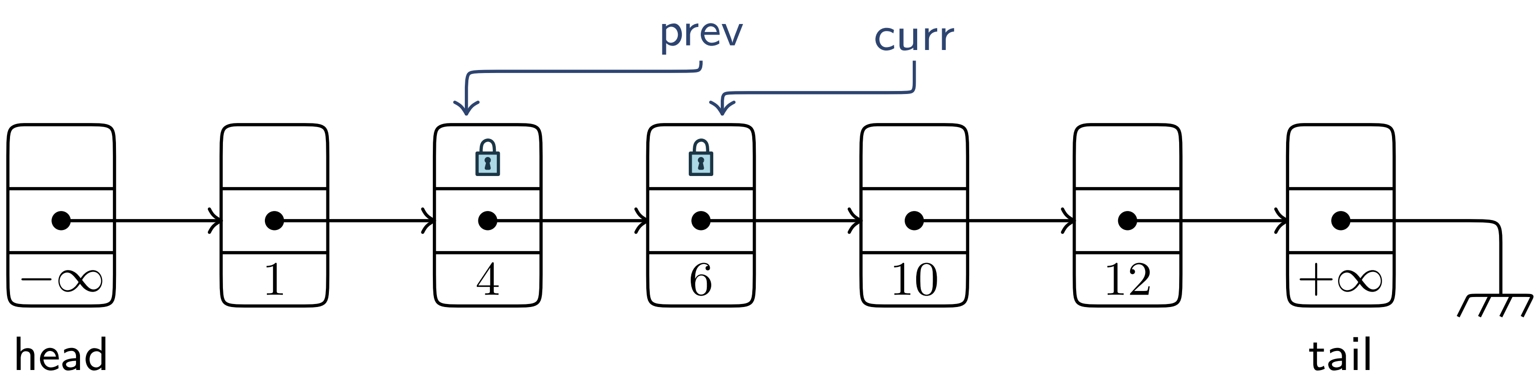

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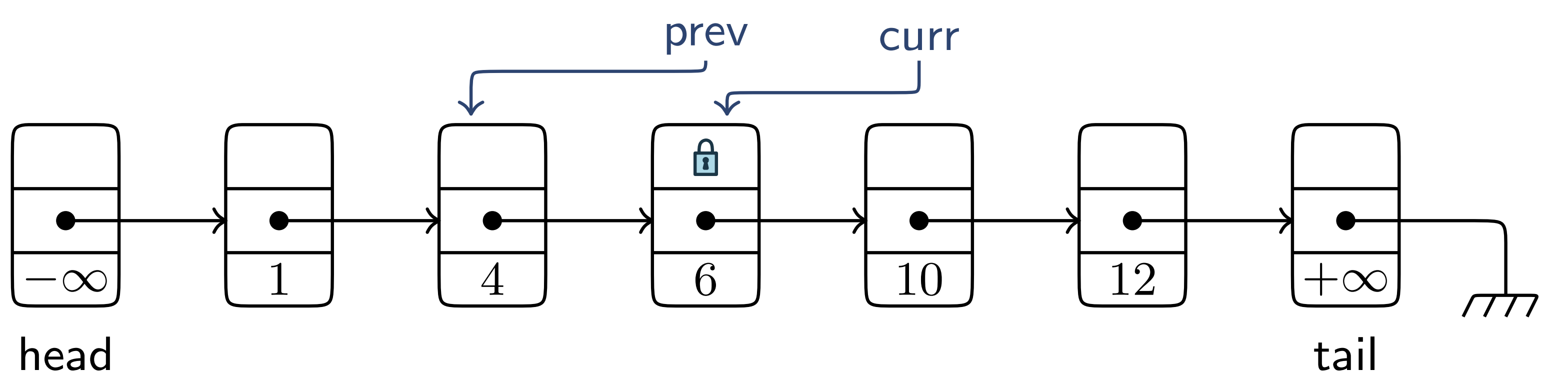

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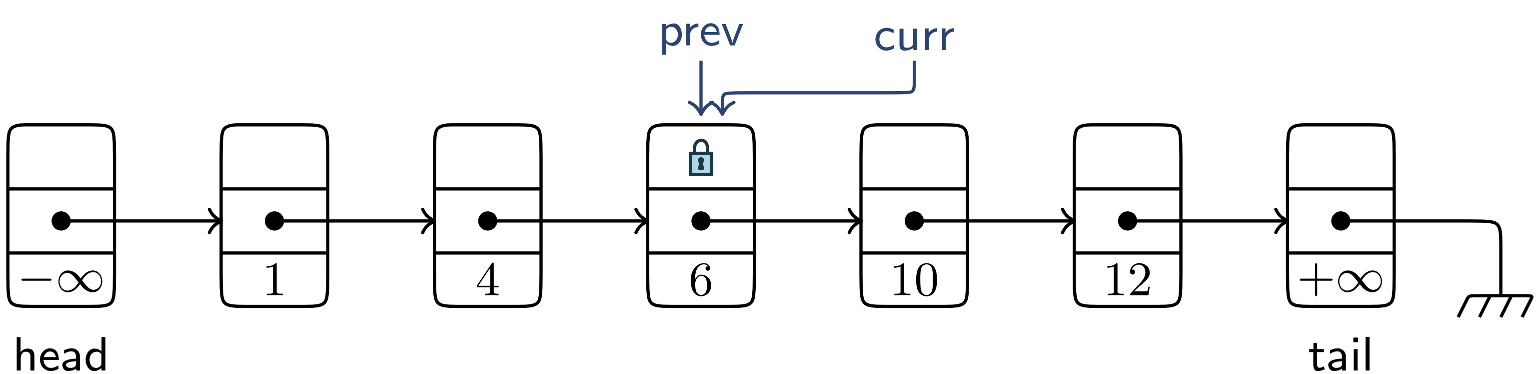

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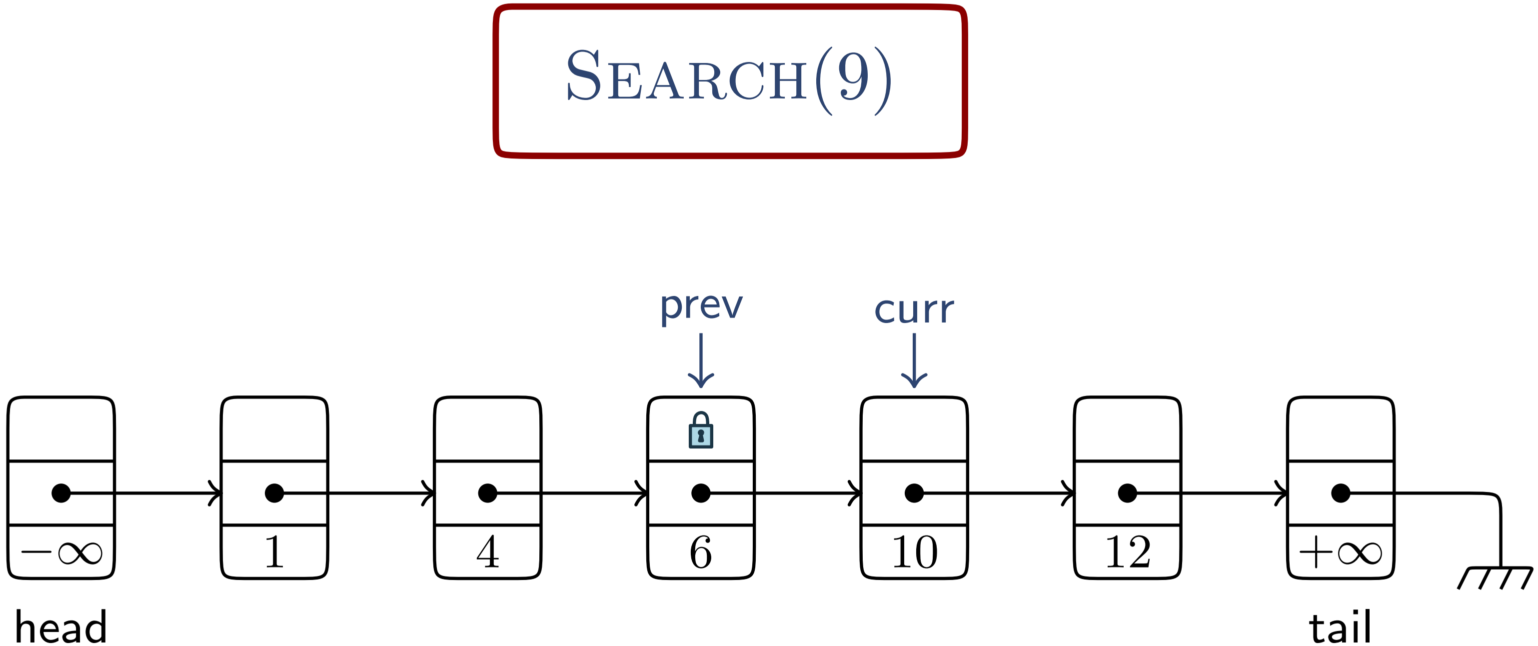

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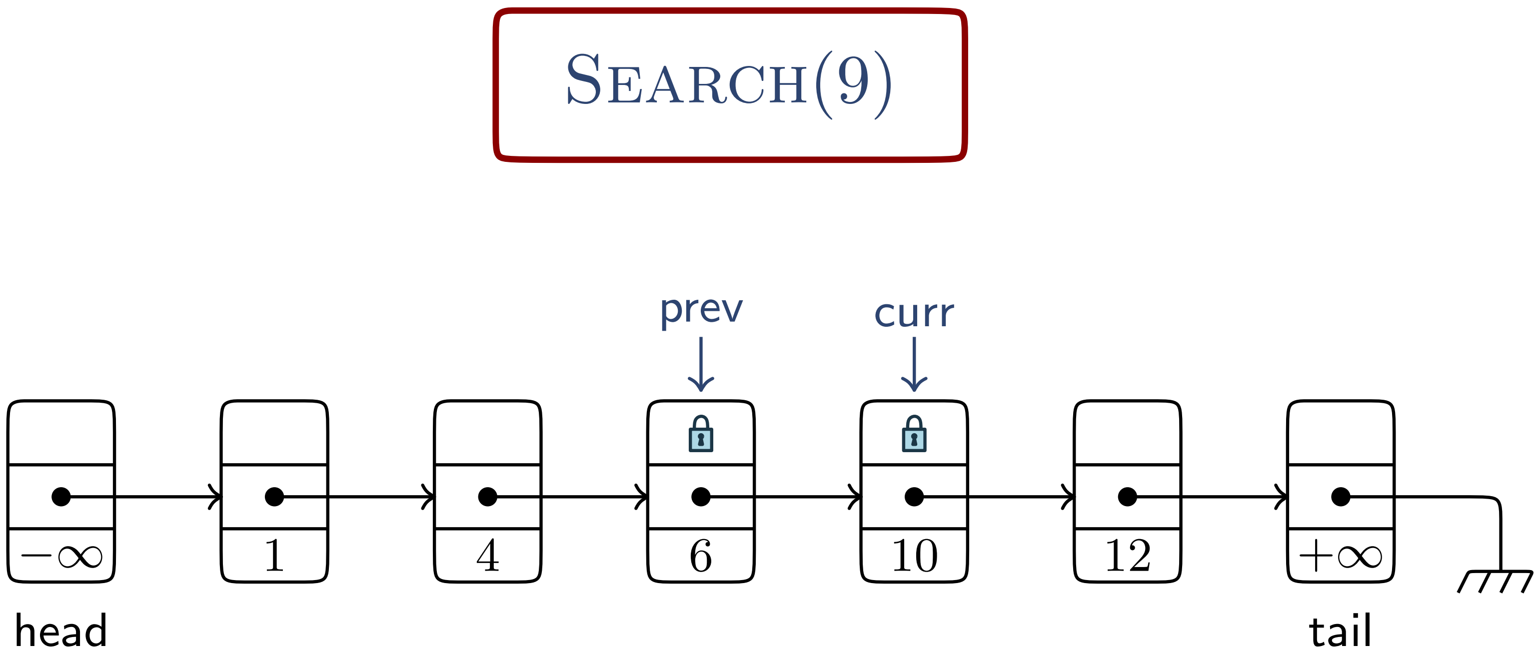

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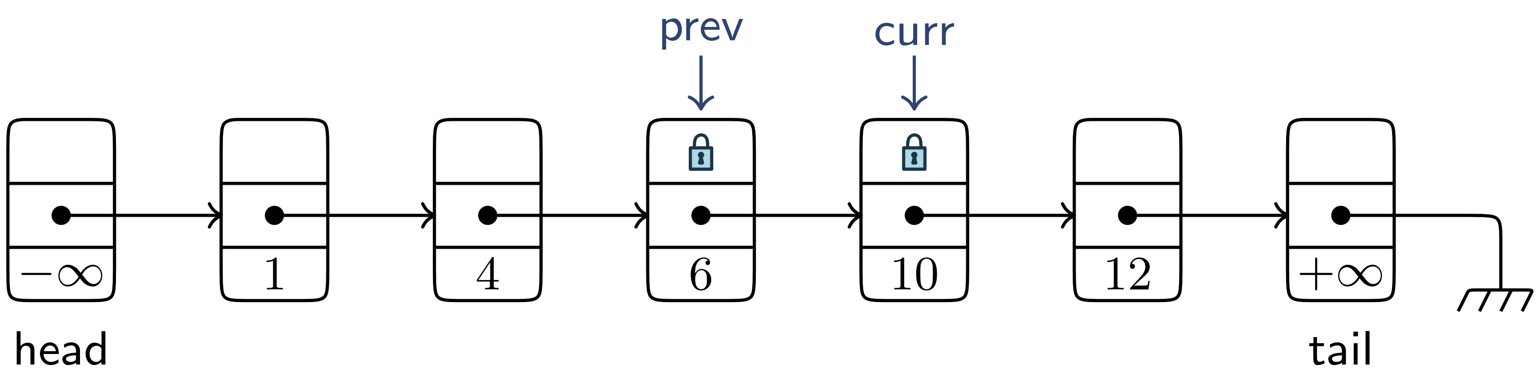

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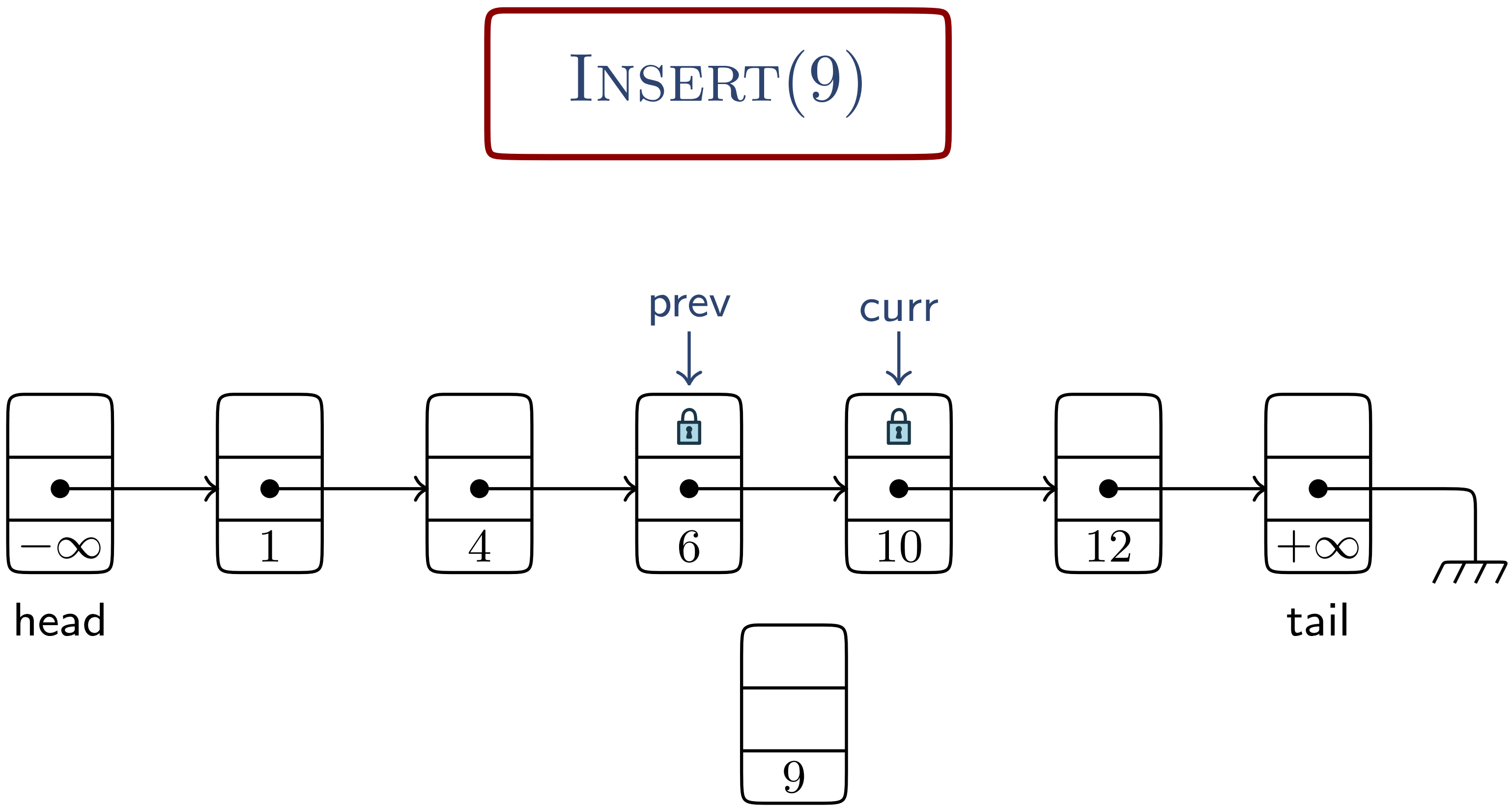

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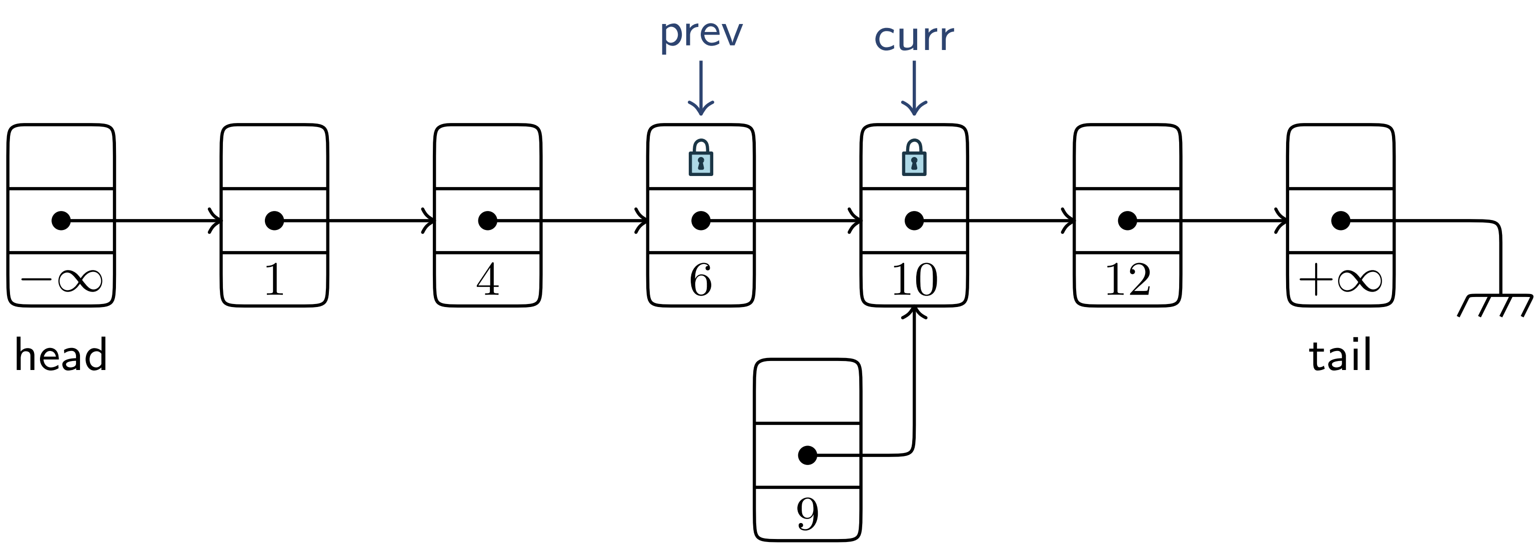

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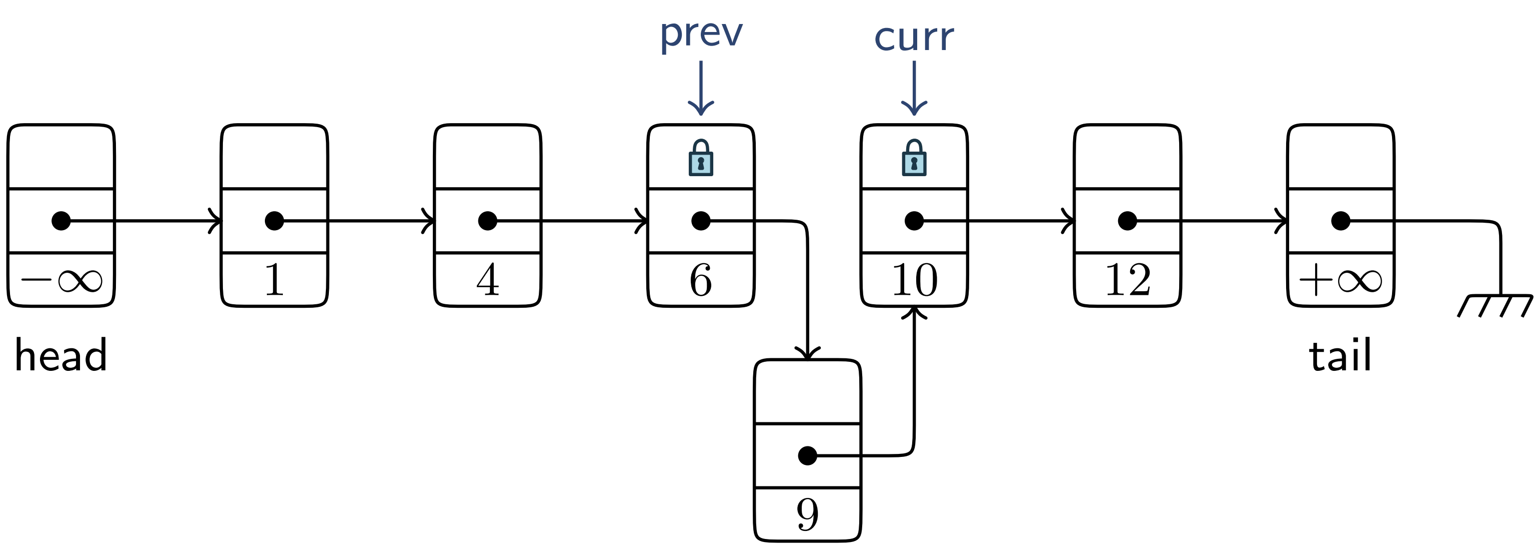

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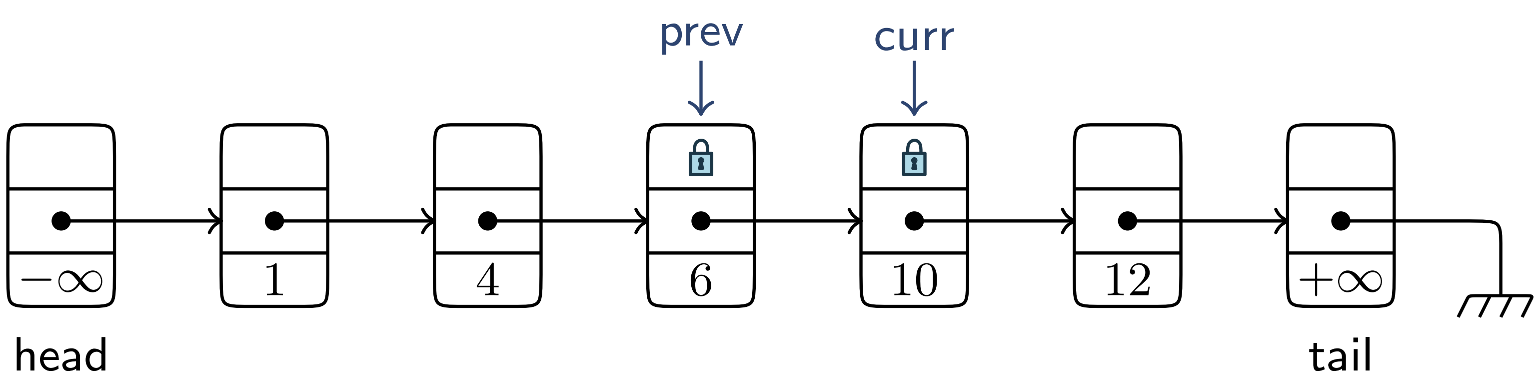
REMOVE(10)



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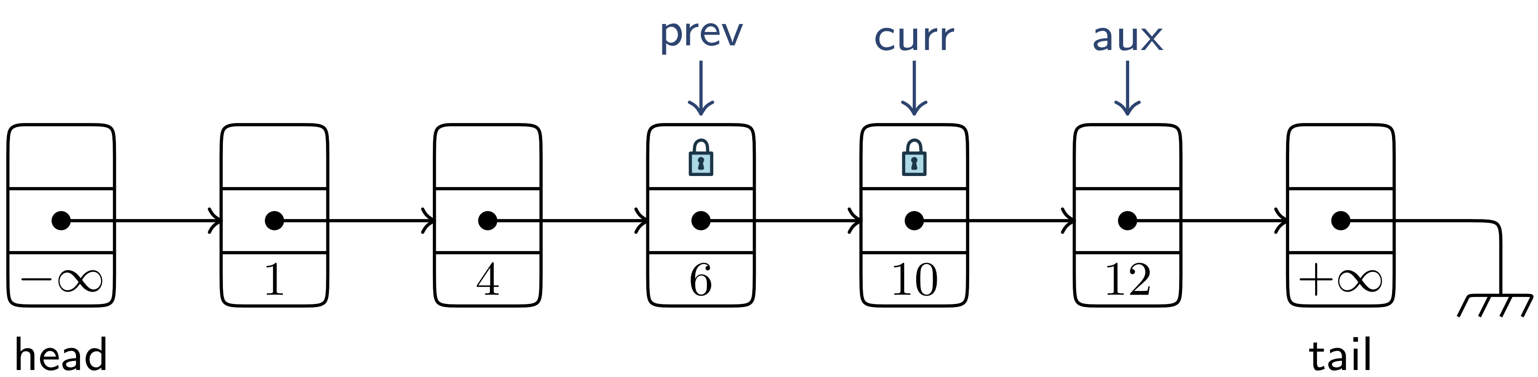
REMOVE(10)



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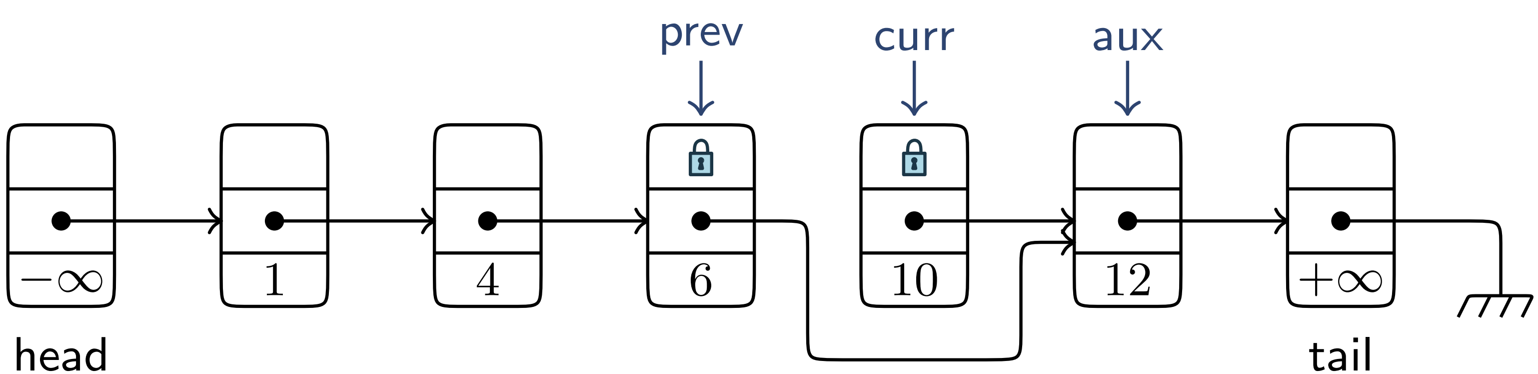
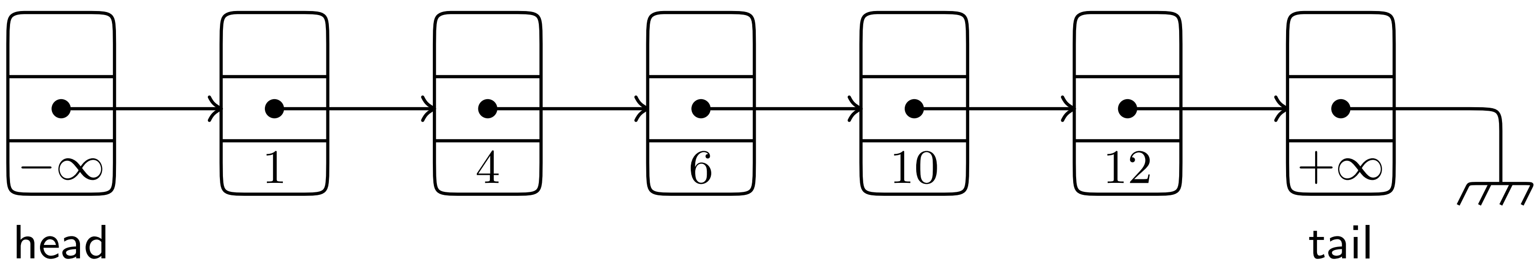


Case Study 2: Lock-Coupling Lists

- We would like to verify list preservation (i.e., $\Box$list)

$$\text{list} \triangleq \left(\qquad \right)$$



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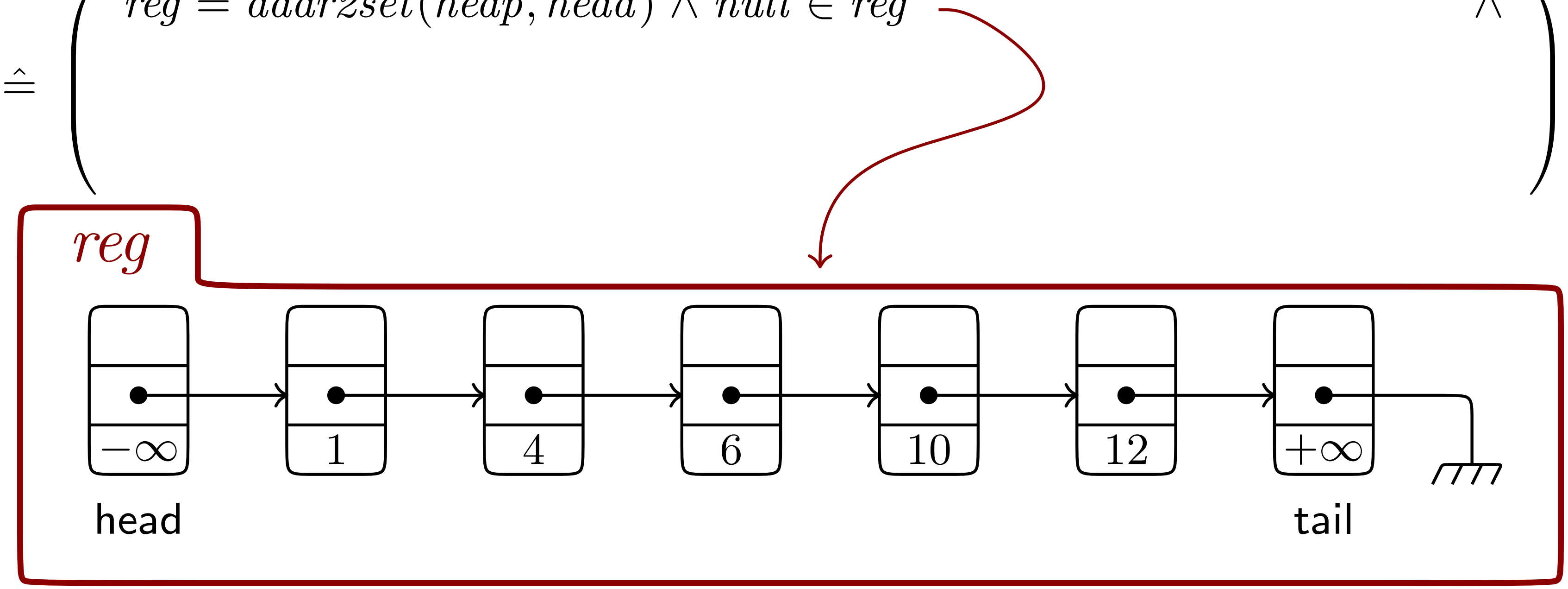

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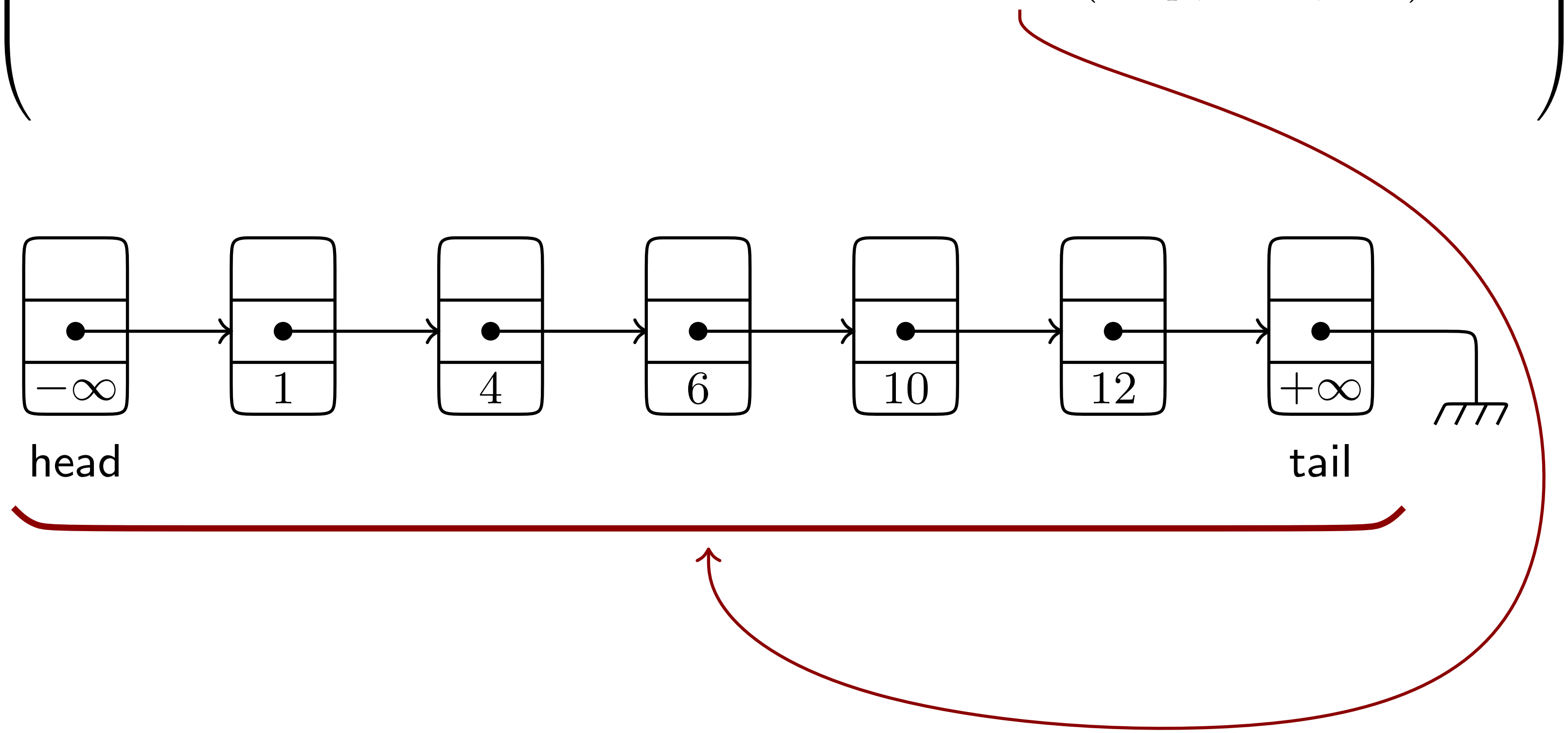

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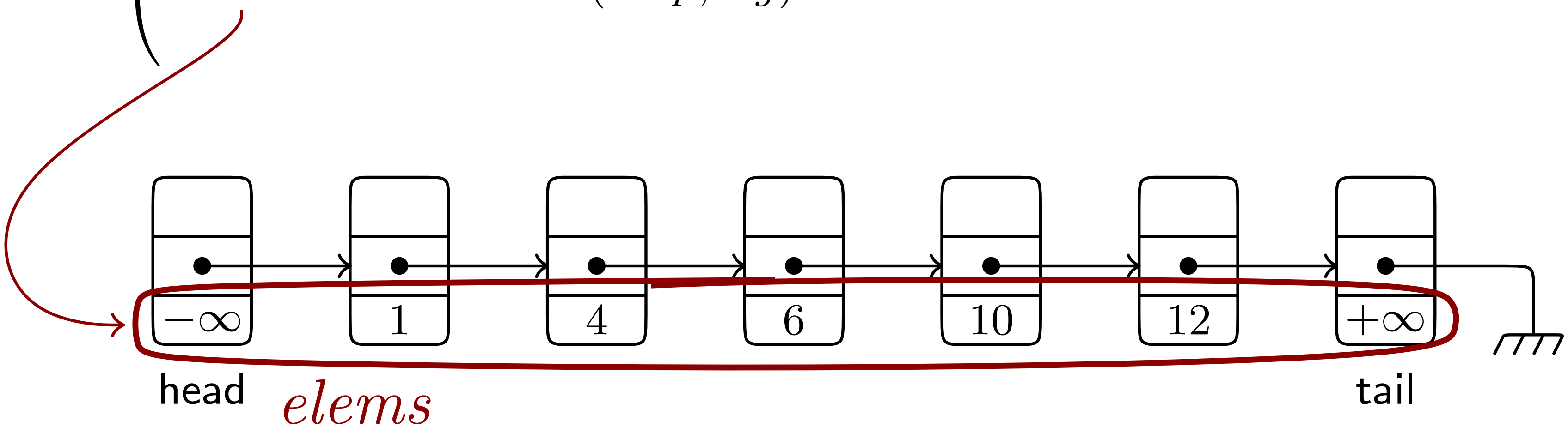

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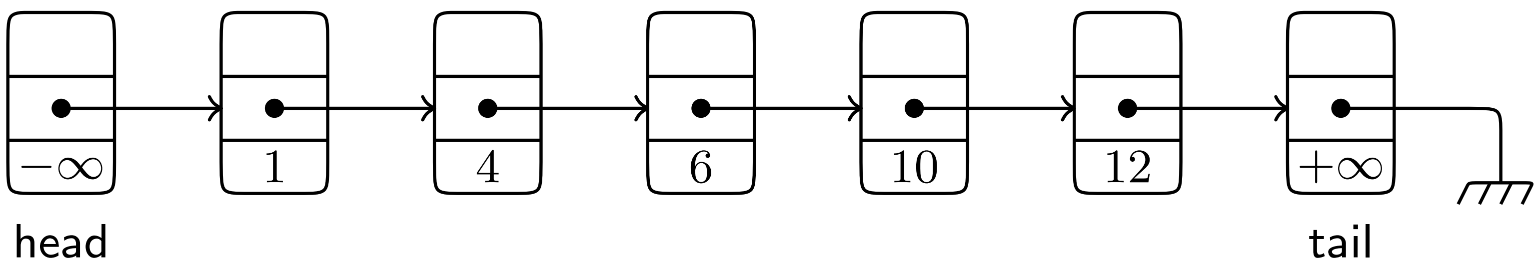

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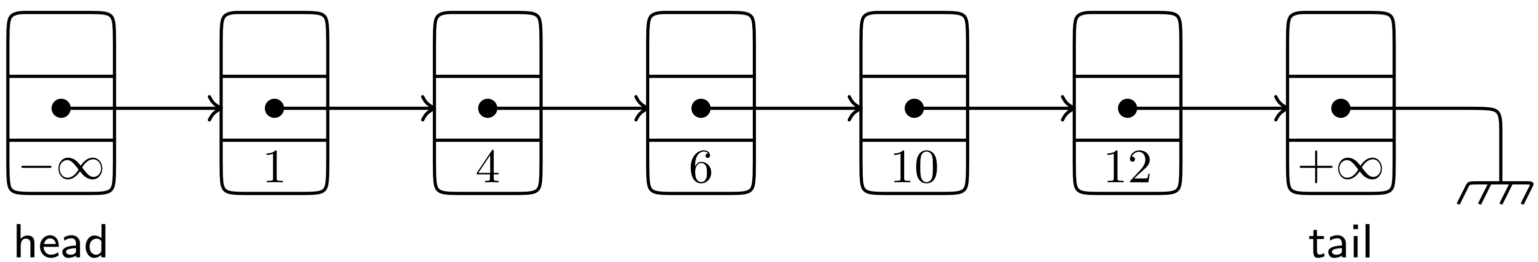

- We declare some **auxiliary invariants**

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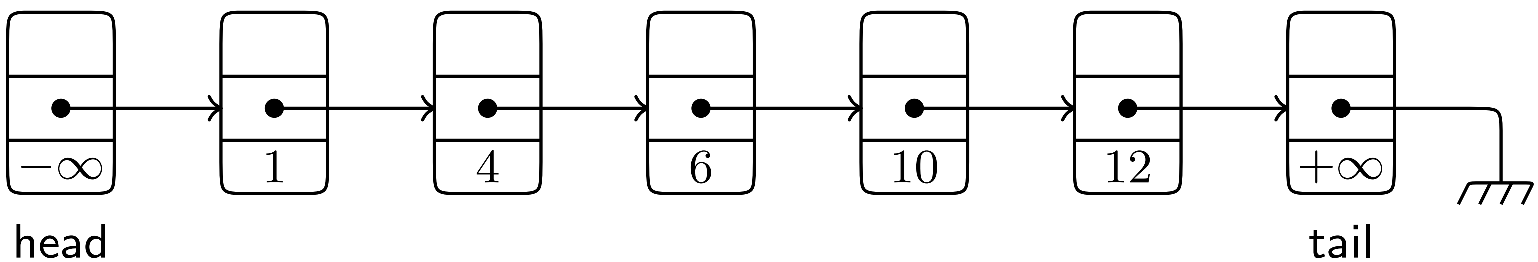

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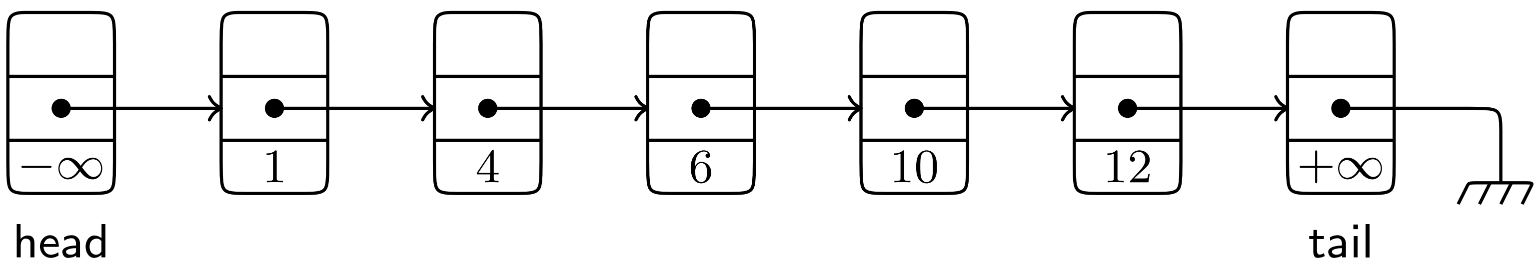

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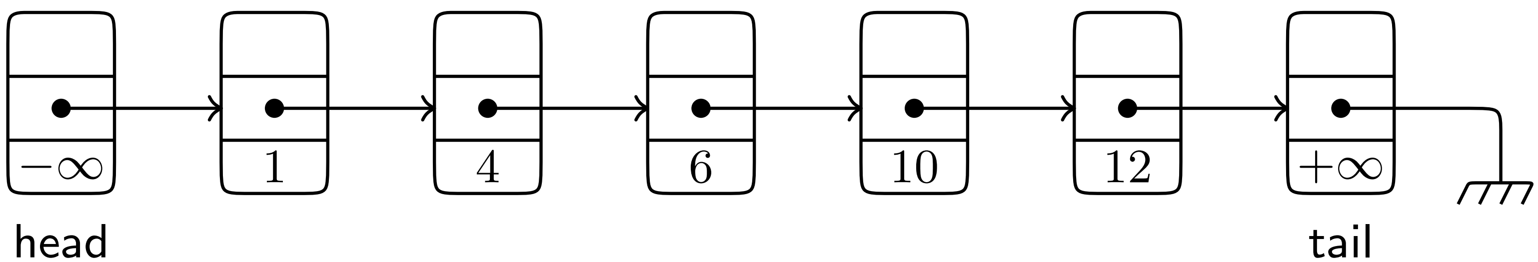

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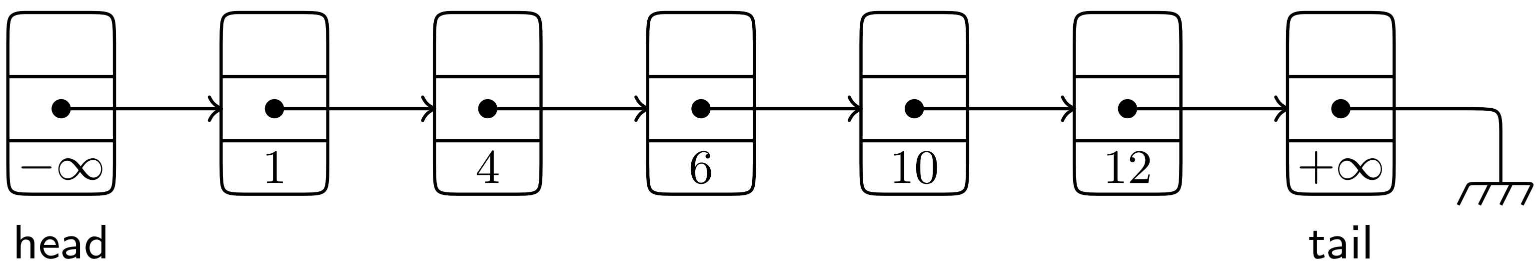

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order ⇝ "order is preserved"

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region ⇝ "nodes within region"

disj ⇝ "new nodes are disjoint from others"

DEMO: lock-coupling lists (1st attempt)

- Let's try to verify region...

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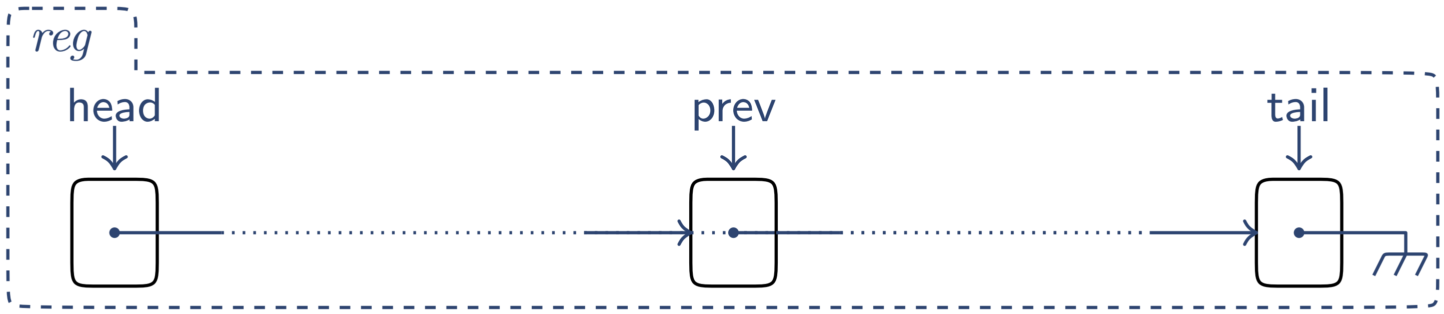
- ... **fails** on transition 28 (i.e., $prev := curr$)



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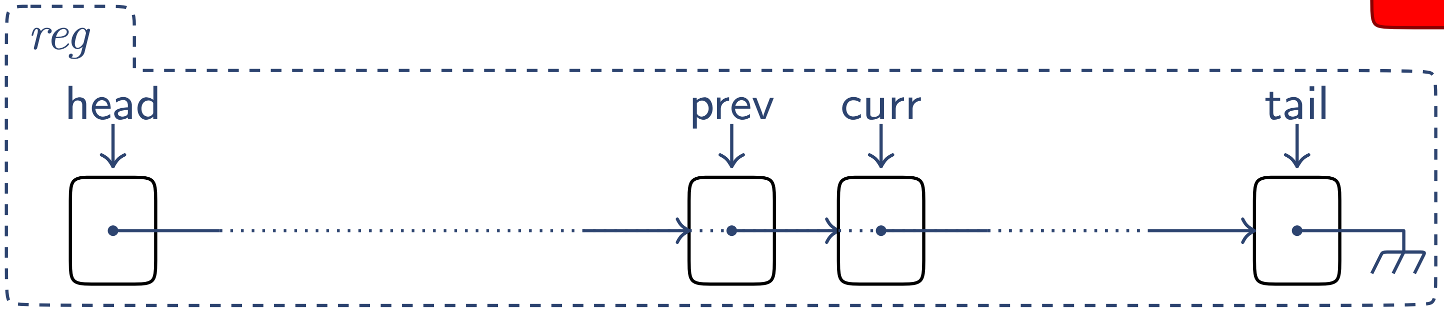
Need **next** as support



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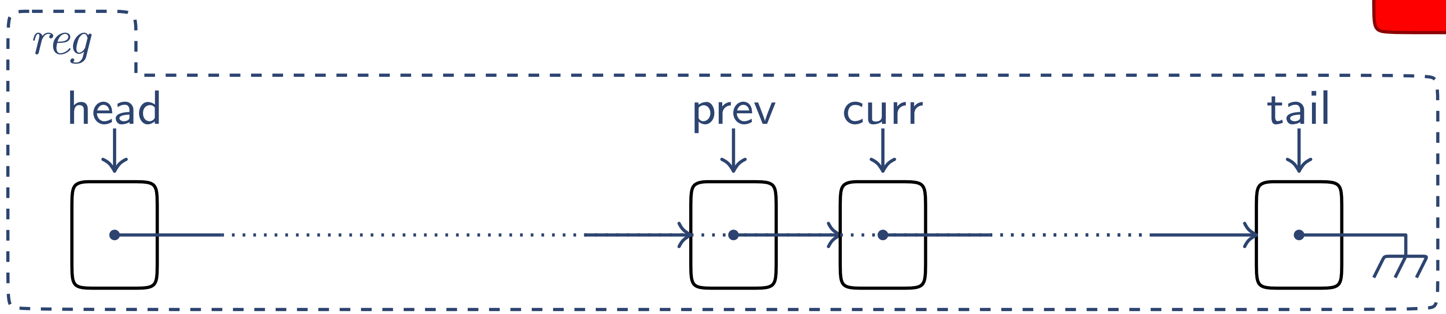

- And when verifying next...

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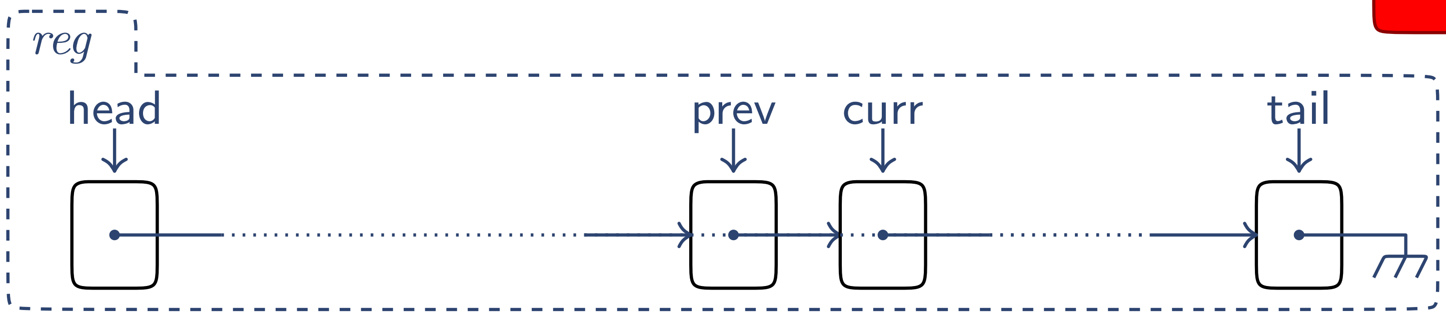

- And when verifying next…
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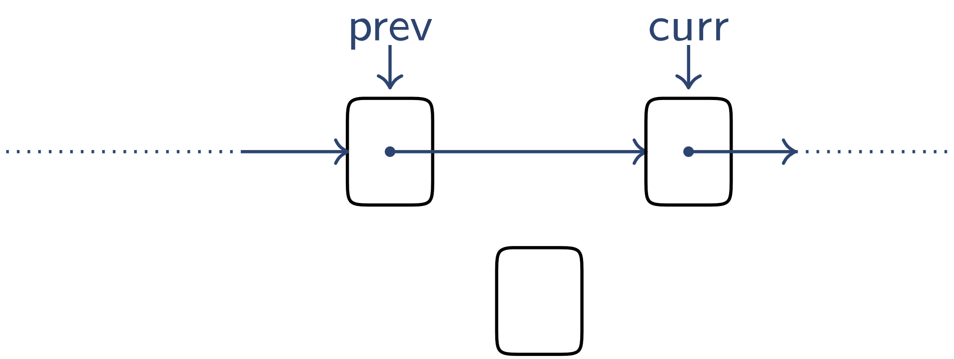

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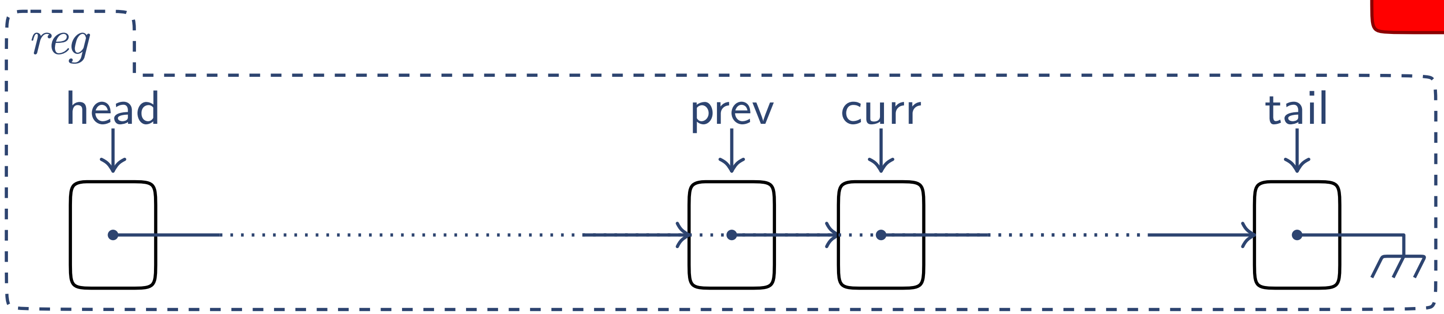

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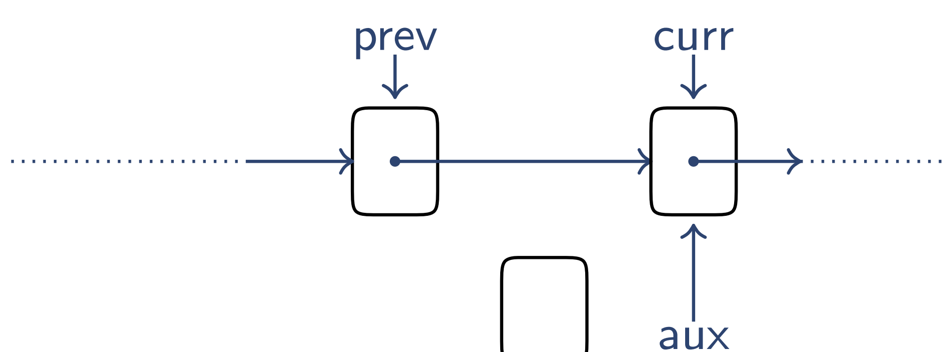

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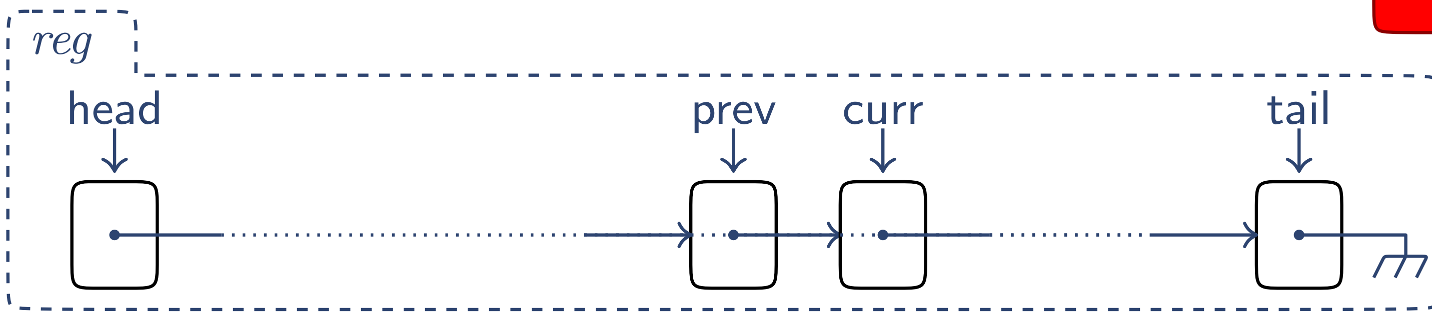

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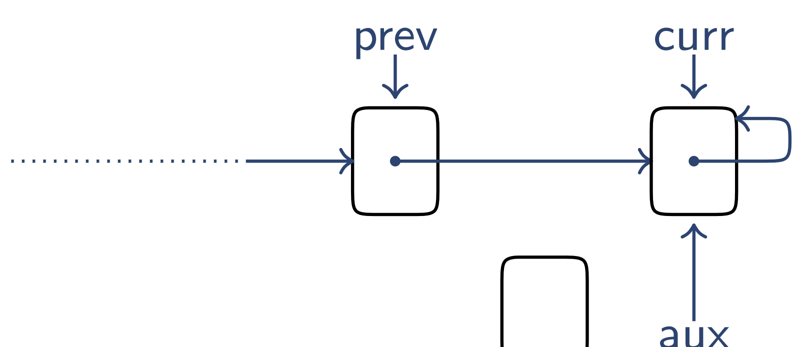

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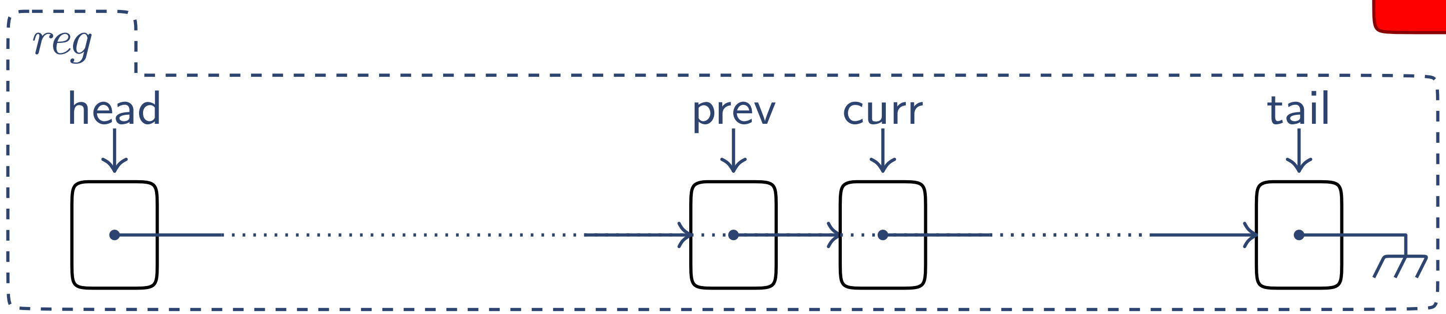

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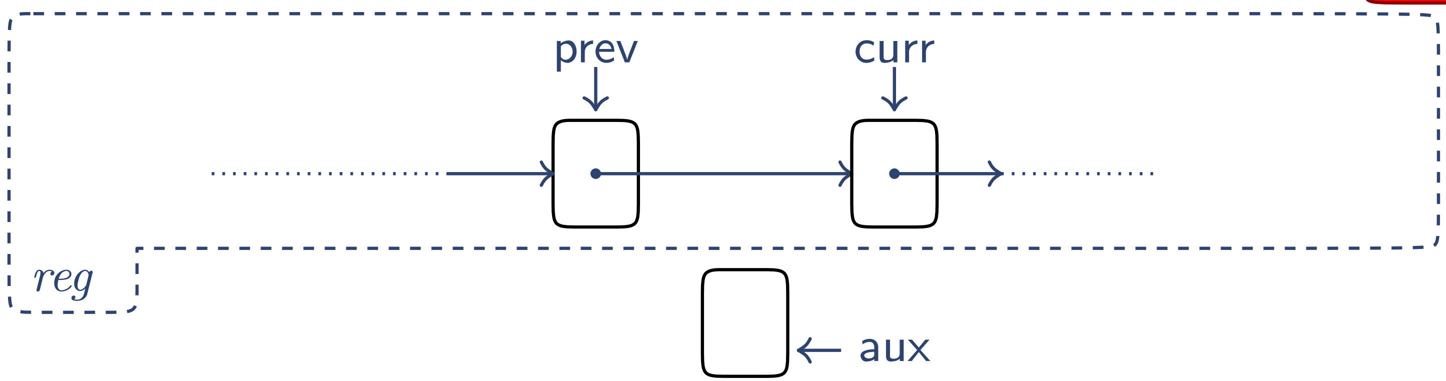

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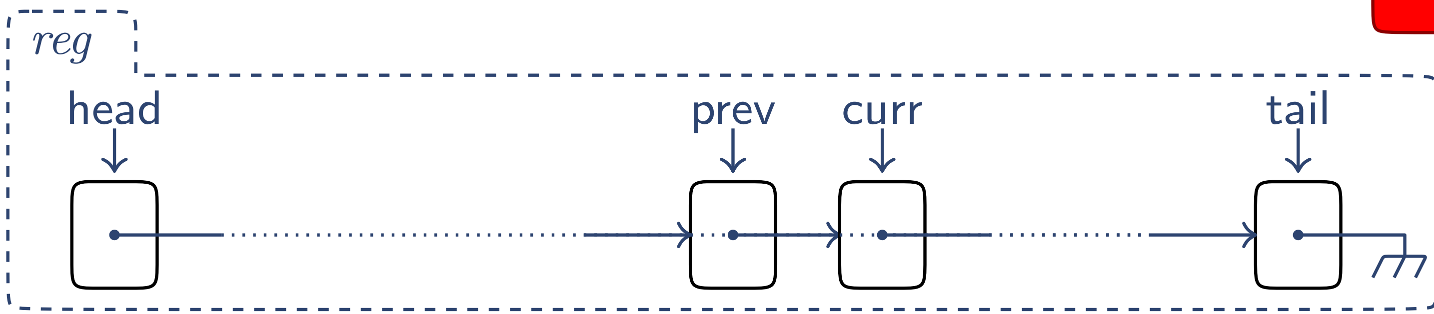

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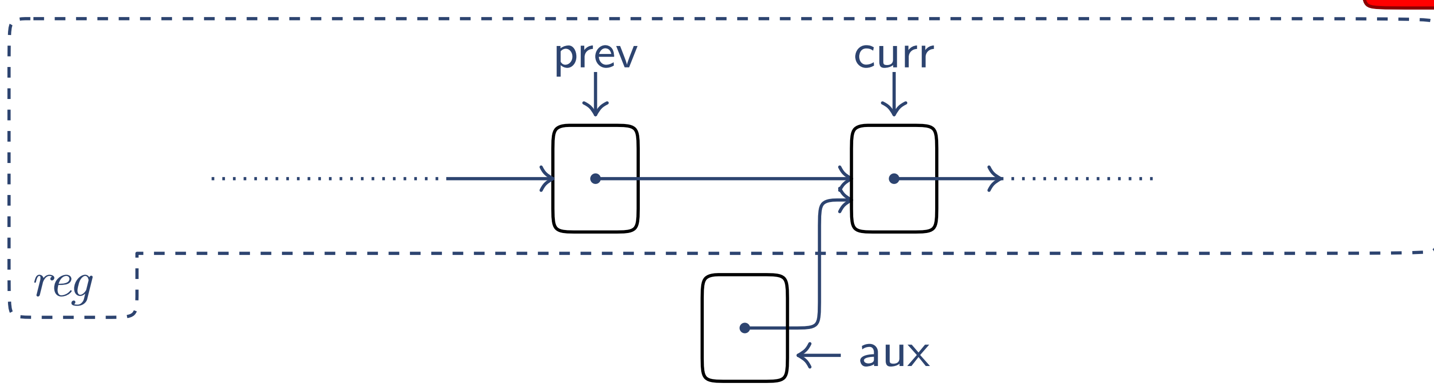

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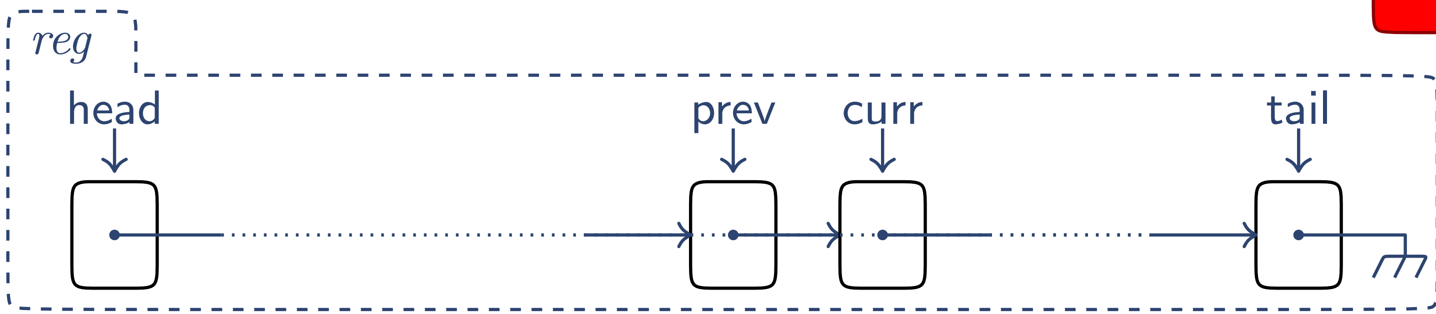

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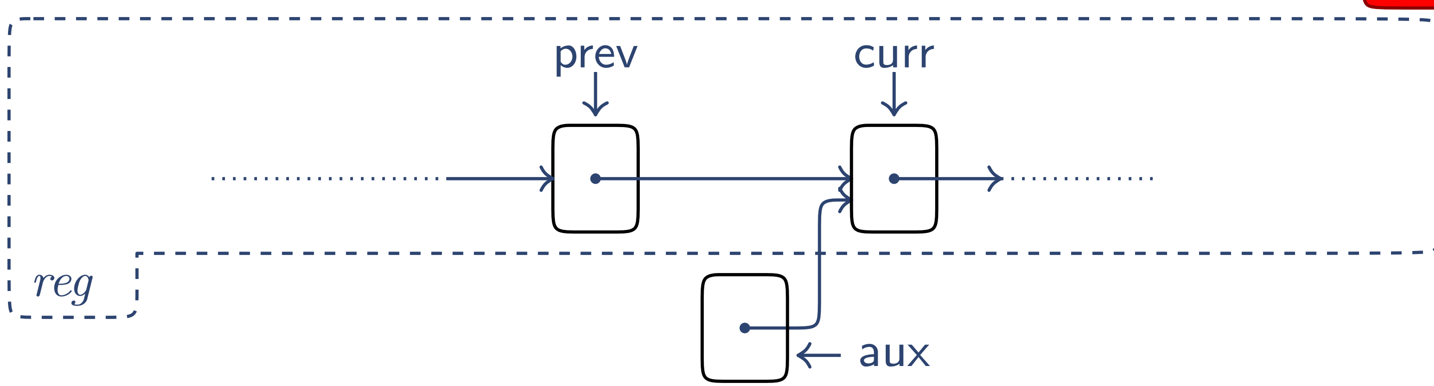

We have **circularity**!

~~SP-INV~~

Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

(I)		Θ	$\rightarrow$	$\varphi_i \wedge \varphi_j$	
($\mathsf{SC_i}$)	$\varphi_i, \varphi_j \rhd$	$\tau^{(t)}$	$\rightarrow$	φ_i'	forall τ, forall $t \in \overline{v}$
($\mathsf{SC_j}$)	$\varphi_i, \varphi_j \rhd$	$\tau^{(t)}$	$\rightarrow$	φ_j'	forall τ, forall $t \in \overline{w}$
($\mathsf{OC_i}$)	$\varphi_i, \varphi_j \rhd$	$\bigwedge_{x \in \overline{v}} k \neq x \wedge \tau^{(k)}$	$\rightarrow$	φ_i'	forall τ, fresh $k \notin \overline{v}$
($\mathsf{OC_j}$)	$\varphi_i, \varphi_j \rhd$	$\bigwedge_{x \in \overline{w}} k \neq x \wedge \tau^{(k)}$	$\rightarrow$	φ_j'	forall τ, fresh $k \notin \overline{w}$

$\Box\varphi_i \wedge \Box\varphi_j$

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initiation

$$
\begin{array}{llcccl}
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(\ \mathsf{SC_i}\ ) & \varphi_i, \varphi_j \rhd & \tau^{(t)} & \rightarrow & \varphi_i' & \text{forall } \tau, \text{ forall } t \in \overline{v} \\
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(\ \mathsf{OC_i}\ ) & \varphi_i, \varphi_j \rhd & \bigwedge_{x \in \overline{v}} k \neq x \wedge \tau^{(k)} & \rightarrow & \varphi_i' & \text{forall } \tau, \text{ fresh } k \notin \overline{v} \\
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\hline
 & & \Box\varphi_i \wedge \Box\varphi_j & & &
\end{array}
$$

Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

initiation

self-consecution

(I)		Θ	$\rightarrow$	$\varphi_i \wedge \varphi_j$	
(SC$_i$)	$\varphi_i, \varphi_j \rhd$	$\tau^{(t)}$	$\rightarrow$	φ_i'	forall τ, forall $t \in \overline{v}$
(SC$_j$)	$\varphi_i, \varphi_j \rhd$	$\tau^{(t)}$	$\rightarrow$	φ_j'	forall τ, forall $t \in \overline{w}$
(OC$_i$)	$\varphi_i, \varphi_j \rhd$	$\bigwedge_{x \in \overline{v}} k \neq x \wedge \tau^{(k)}$	$\rightarrow$	φ_i'	forall τ, fresh $k \notin \overline{v}$
(OC$_j$)	$\varphi_i, \varphi_j \rhd$	$\bigwedge_{x \in \overline{w}} k \neq x \wedge \tau^{(k)}$	$\rightarrow$	φ_j'	forall τ, fresh $k \notin \overline{w}$

$$\Box\varphi_i \wedge \Box\varphi_j$$

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others-consecution

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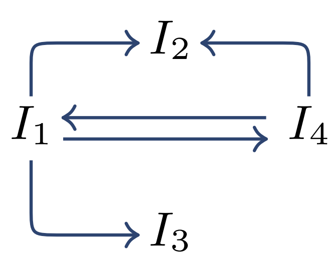
$$\Box\varphi_i \wedge \Box\varphi_j$$

self-consecution ($\mathsf{SC_i}$, $\mathsf{SC_j}$)

others-consecution ($\mathsf{OC_i}$, $\mathsf{OC_j}$)

- A **generalization** of G-INV is a **proof graph**



Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\Box\varphi_i(\overline{v}) \wedge \Box\varphi_j(\overline{w})$:

$$
\begin{array}{llrcll}
(\ \mathsf{I}\ ) & & \Theta & \rightarrow & \varphi_i \wedge \varphi_j & \\
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\hline
\multicolumn{6}{c}{\Box\varphi_i \wedge \Box\varphi_j}
\end{array}
$$

initiation (I)

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others-consecution ($\mathsf{OC_i}$, $\mathsf{OC_j}$)

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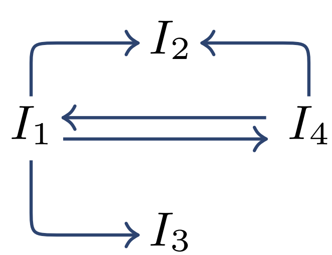

Theorem

Every node is invariant if every node is either:

- an inductive invariant, or
- has an incident edge and all VCs are valid

Parametrized Invariance with Graph Support (g-inv)

To show that $\mathcal{S}$ satisfies $\square\varphi_i(\overline{v}) \wedge \square\varphi_j(\overline{w})$:

$$
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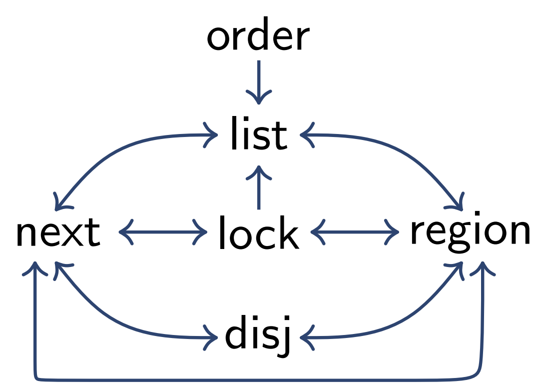
\hline
\multicolumn{6}{c}{\square\varphi_i \wedge \square\varphi_j}
\end{array}
$$

initiation

self-consecution

others-consecution



Theorem

Every node is invariant if every node is either:

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Experimental Results

	formula info		#solved vc		FS+TA	FS+RS	Graph
	index	#vc	pos	dp	time(s)	time(s)	time(s)
list	0	61	19	42	146.1	34.9	3.9
order	1	121	38	83	107.6	91.4	5.3
lock	1	121	57	64	39.7	8.9	1.8
next	1	121	38	83	166.7	18.4	3.4
region	1	121	95	26	15.7	4.1	1.3
disj	2	181	177	4	80.6	5.1	0.6
mutex	2	28	26	2	0.2	0.2	0.1
minticket	1	19	18	1	0.2	0.2	0.1
notsame	2	28	25	3	0.2	0.1	0.1
activelow	1	19	17	2	0.1	0.1	0.1

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Mutual exclusion algorithm: 94 VCs / 0.4 sec.

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	formula info		#solved vc		FS+TA time(s)	FS+RS time(s)	Graph time(s)
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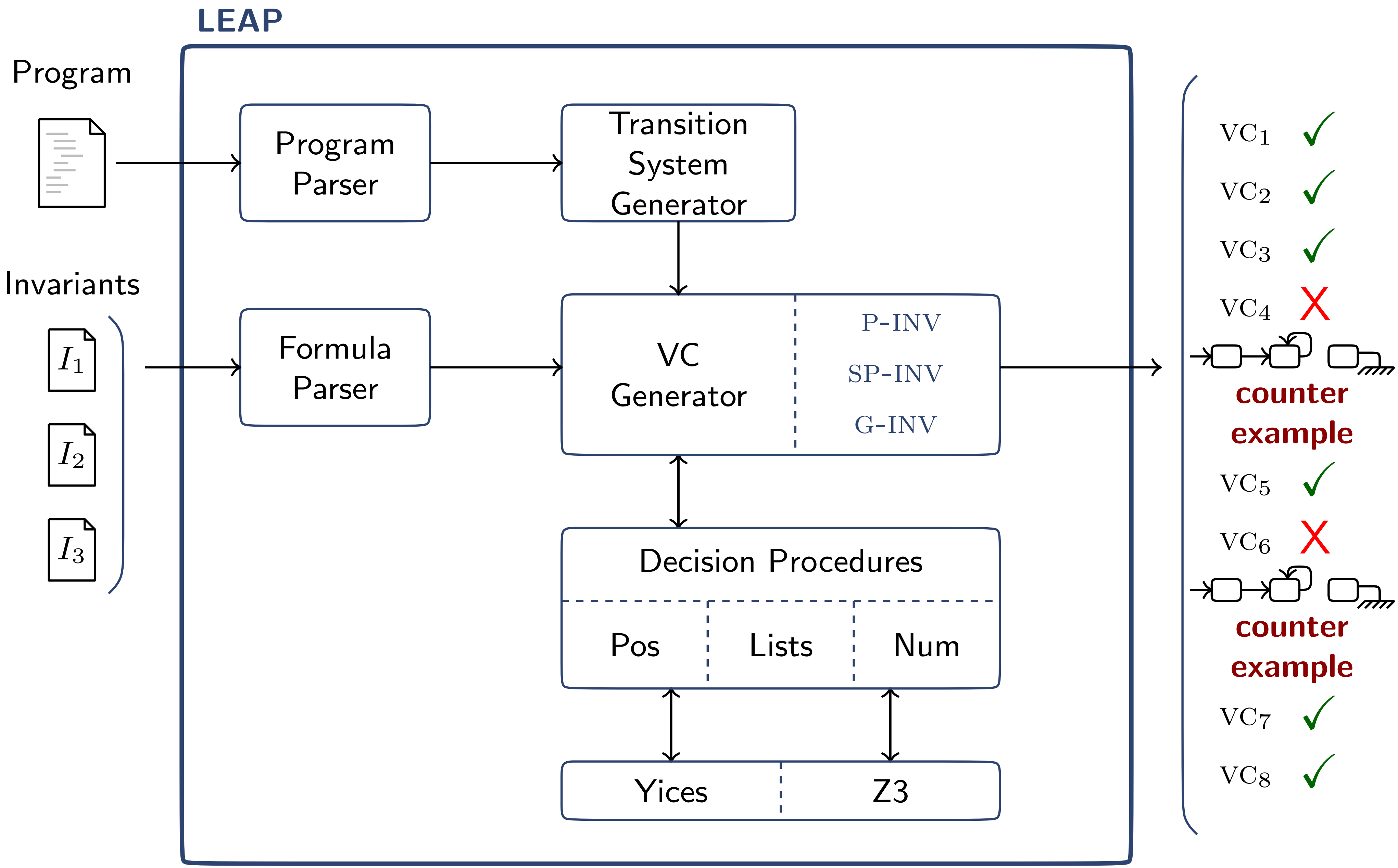
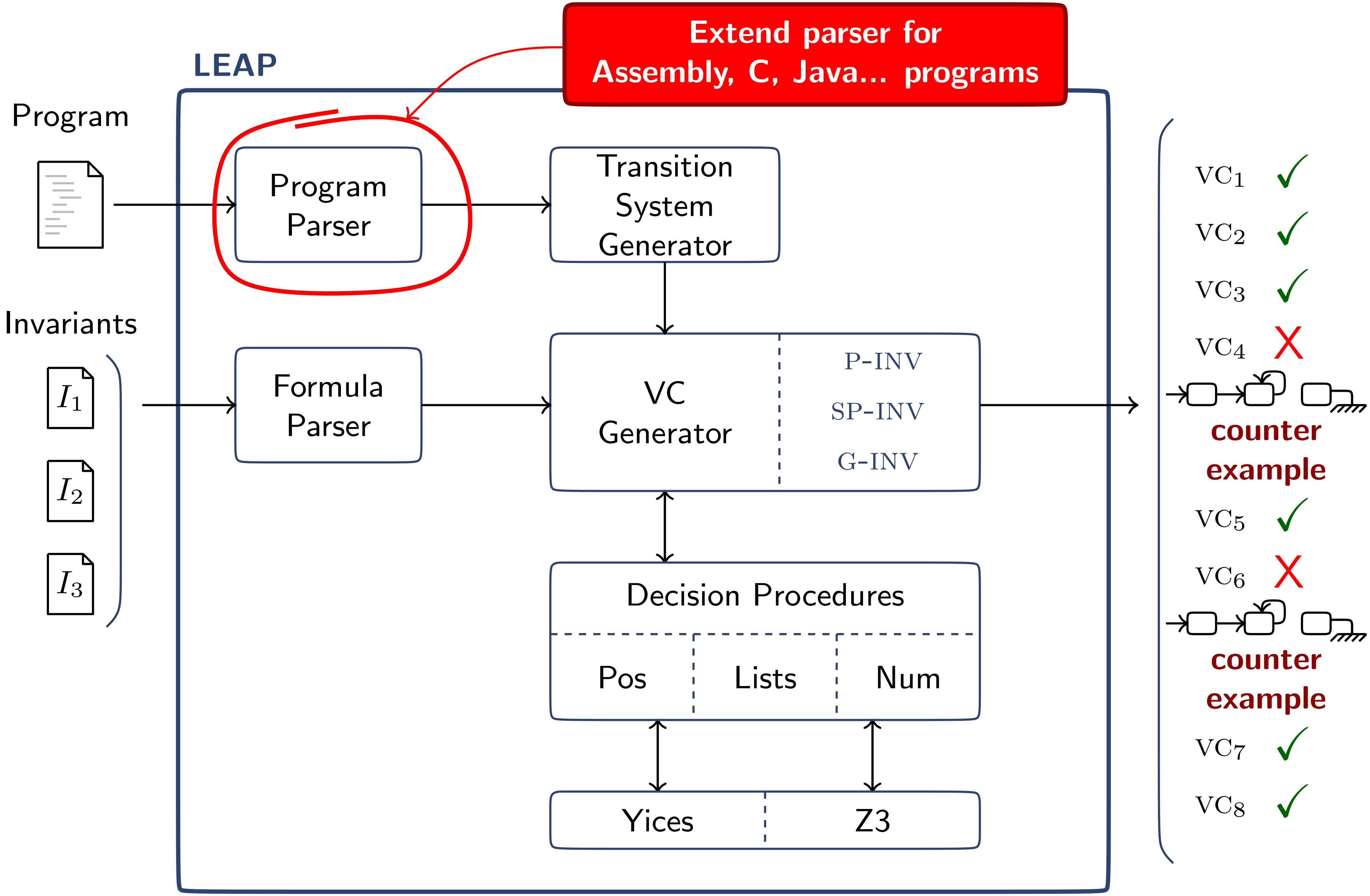
Lock-coupling Lists: 726 VCs / 16.3 sec.

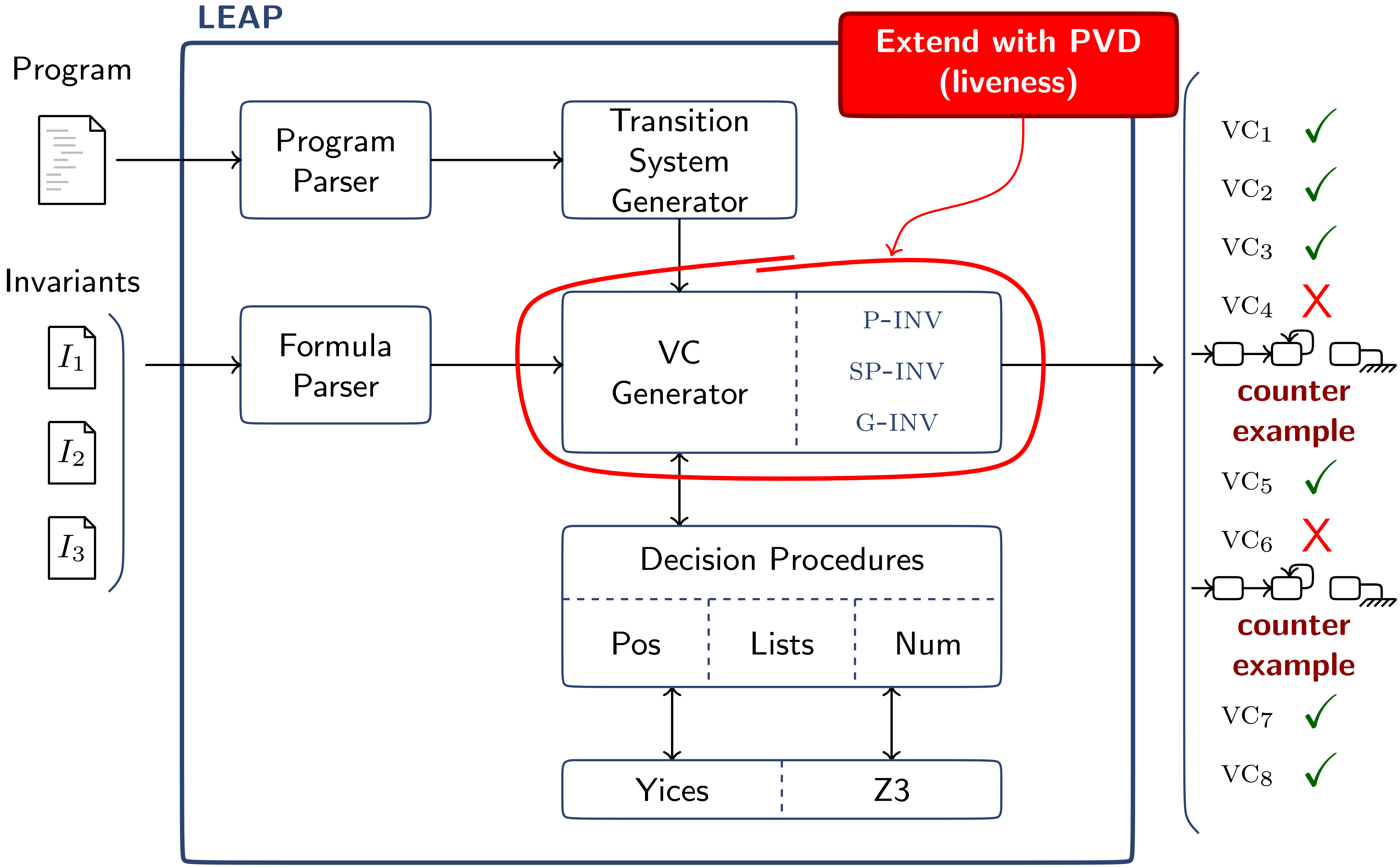
Future Ideas



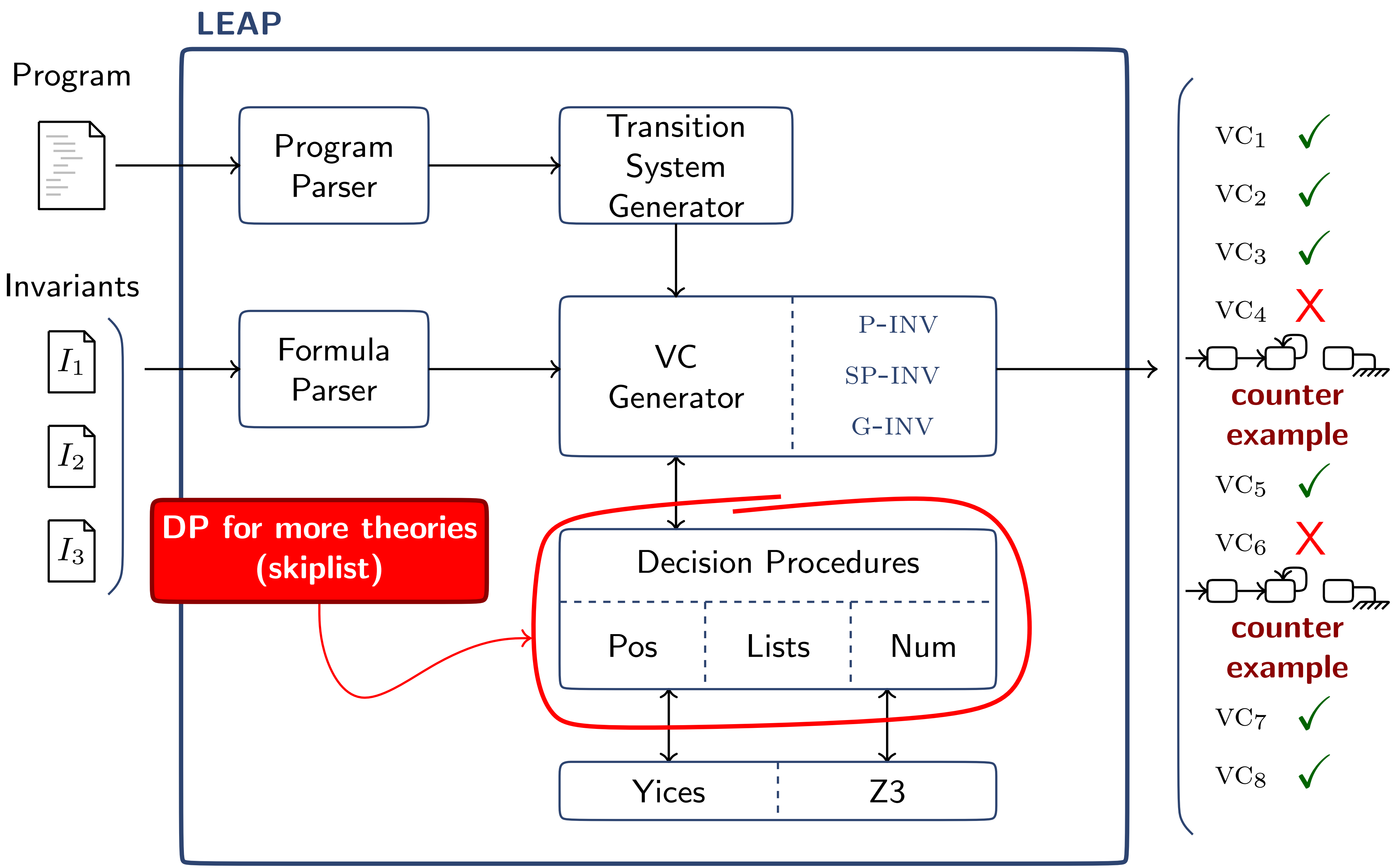
Future Ideas



Future Ideas

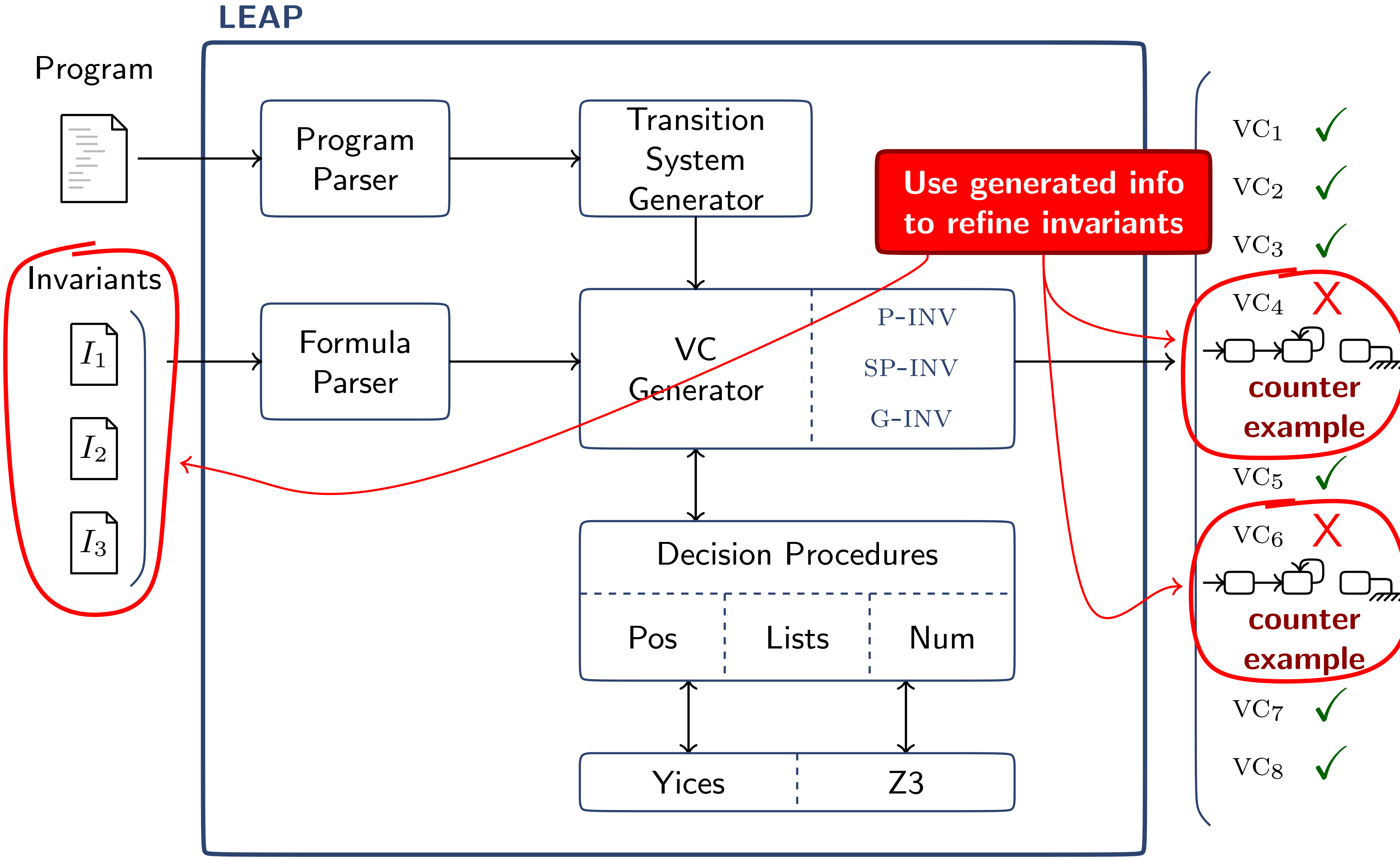


Future Ideas

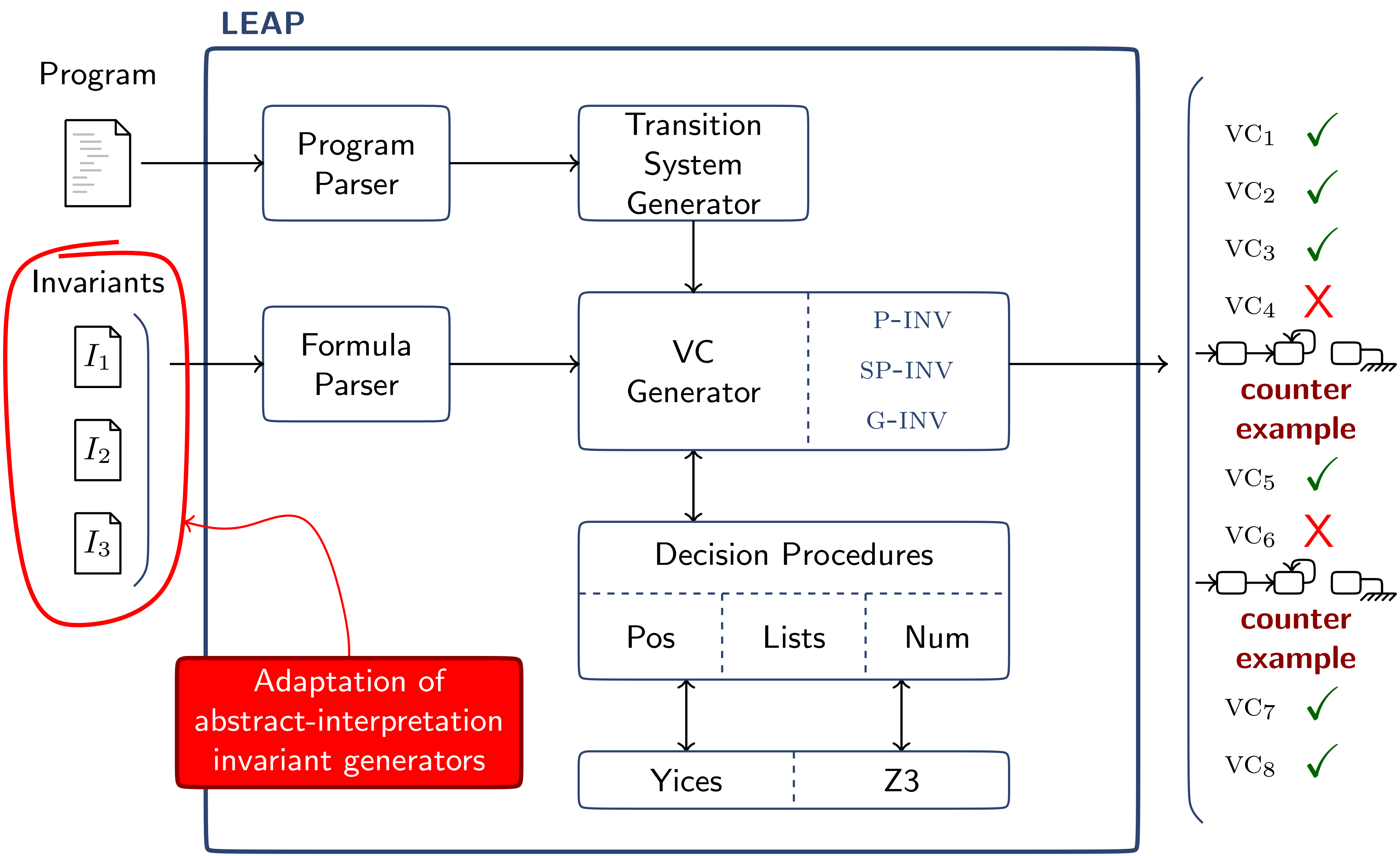


Future Ideas

refine supoported invariants



Future Ideas



Conclusions

- We presented a technique for verifying **parametrized invariance**
- Requires verification of a **bounded number of VCs**...
- ... **independently** of number of **threads**
- **Implemented** on LEAP
- **Easily extensible** for other **languages and theories**
- Uniform verification of **safety properties**...
- ... a step **towards liveness verification**