Verifying Concurrent Memory Reclamation Algorithms with Grace

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From Hoare Logic to Separation Logic

Noam Rinetzky Tel Aviv University, Israel

Technion, 1/July/2013

Verifying Concurrent Memory Reclamation Algorithms with Grace

European Symposium on Programming (ESOP) 2013

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research problem

- verify non-blocking data structures with explicit memory management

non-blocking concurrent data structures

- stack, queue, set
- highly concurrent
  - no global locks protecting the entire DS
  - fine grained concurrency

research problem

- verify non-blocking data structures with explicit memory management
  - memory safety
  - no memory leaks
  - no anomalies due to reallocation (ABA)
  - correctness (atomicity)

concurrency is challenging

- non-blocking concurrent data structures
  - stack, queue, set
- challenges
  - interference
  - pointers
  - dynamic allocation (malloc)

concurrency very is challenging

- non-blocking concurrent data structures
  - stack, queue, set
- challenges
  - interference
  - pointers
  - dynamic allocation (malloc)
  - **deallocation (free)**
    - **reallocation (free ; malloc)**

concurrent counter (with GC)

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int *C;
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int inc() {
 int v, *s, *n;
 n = new int;
 do{
  s = C;
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 return v;
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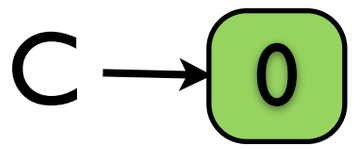


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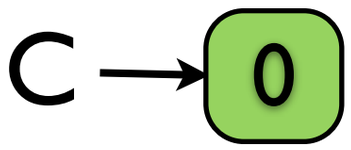


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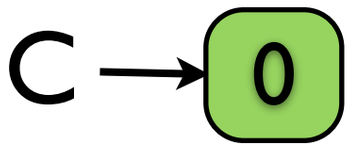

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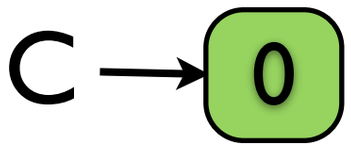

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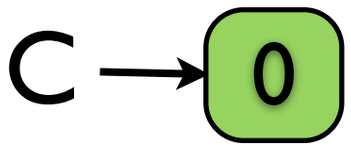

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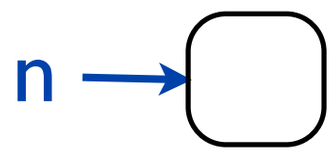


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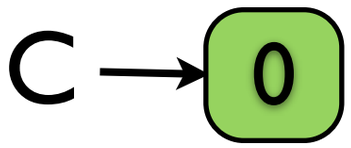

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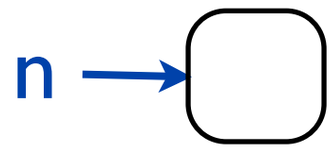


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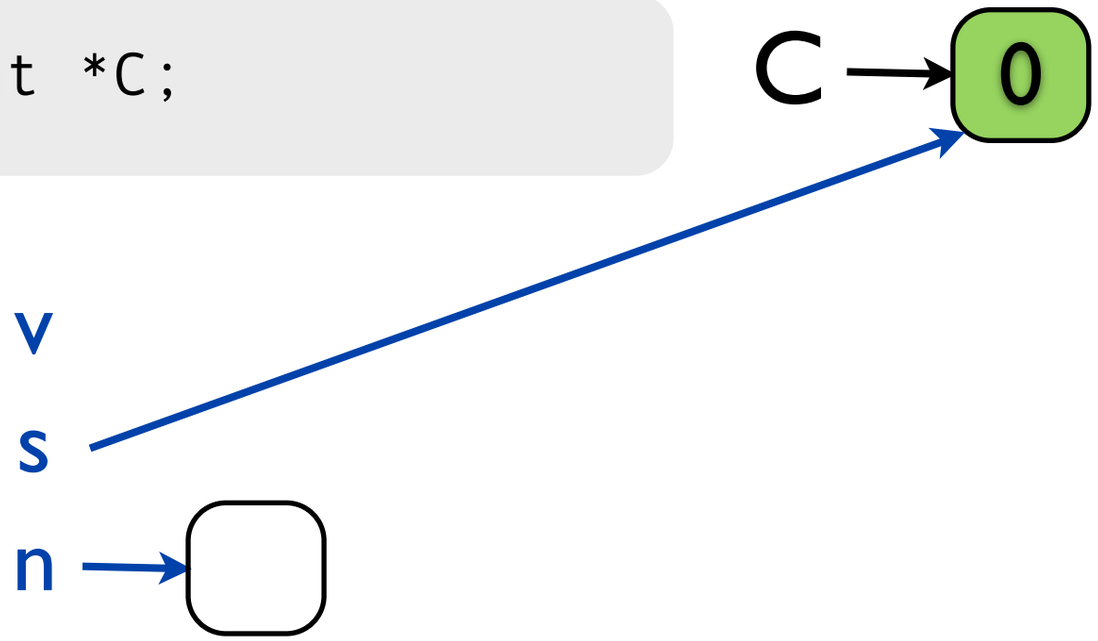


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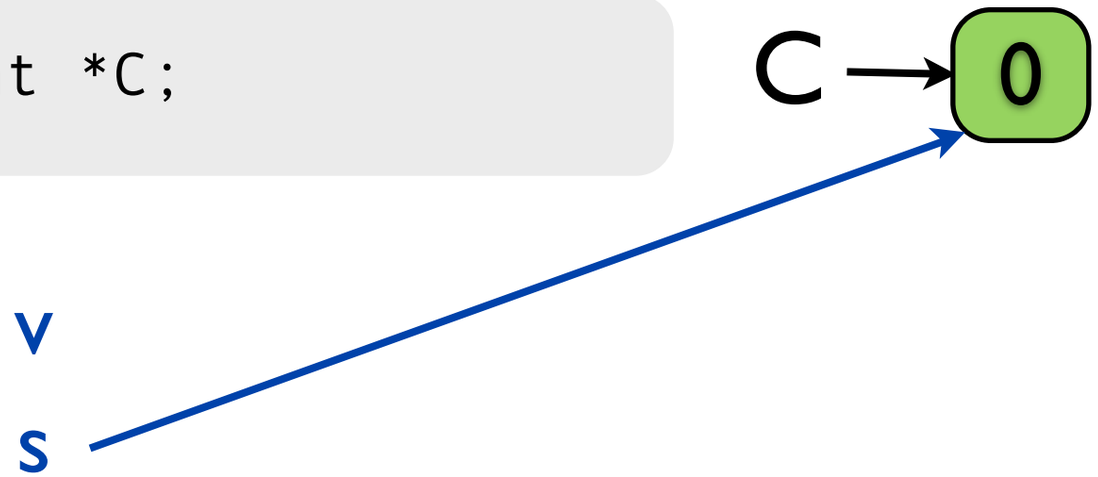


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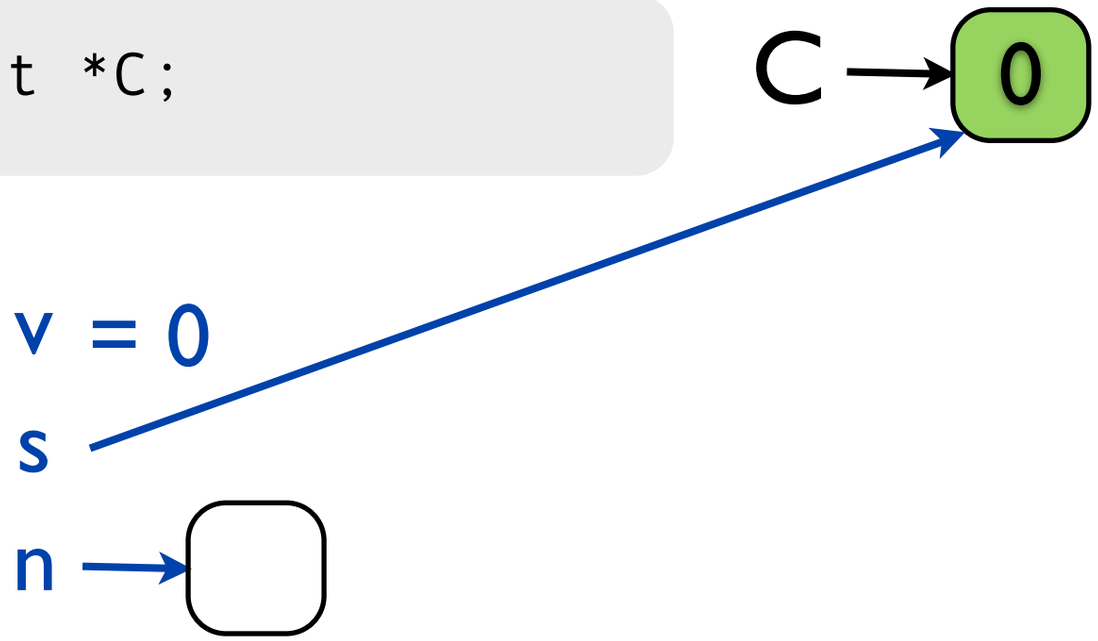


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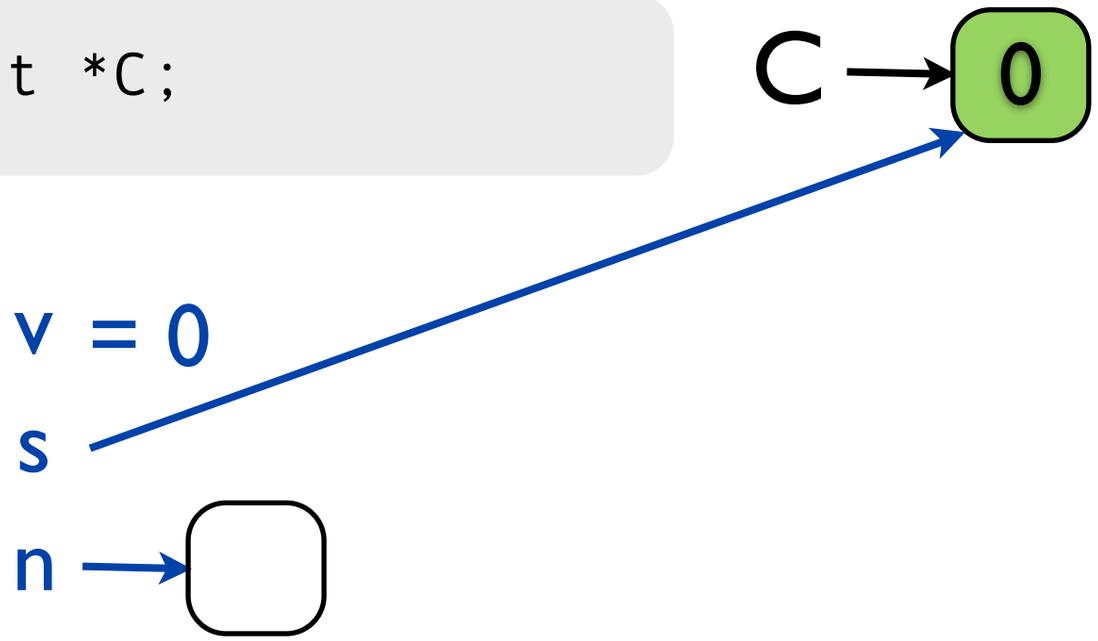


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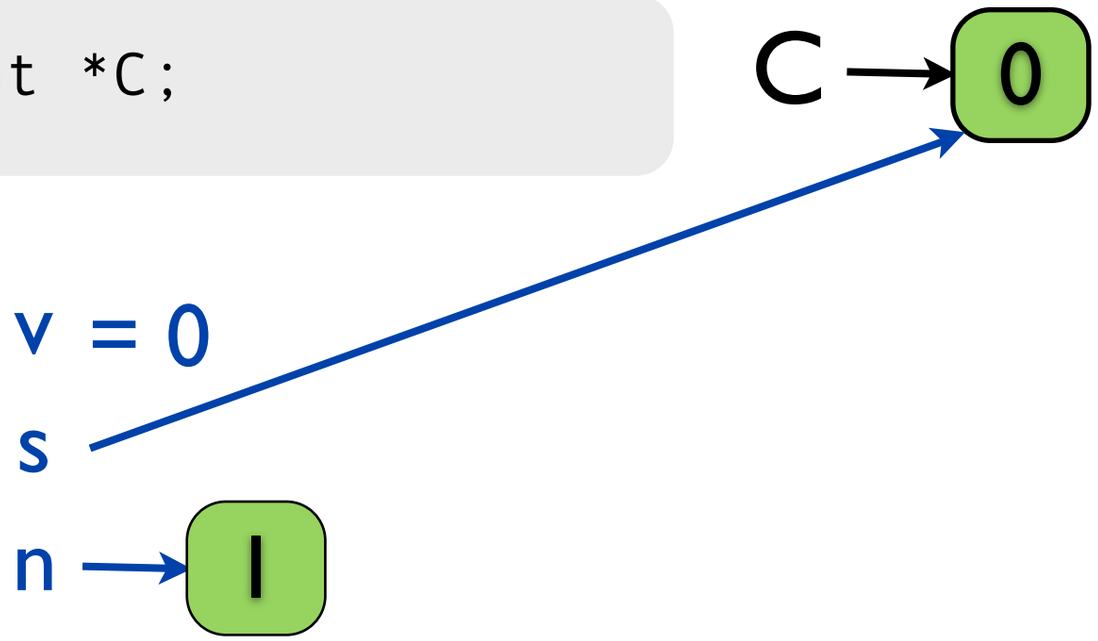


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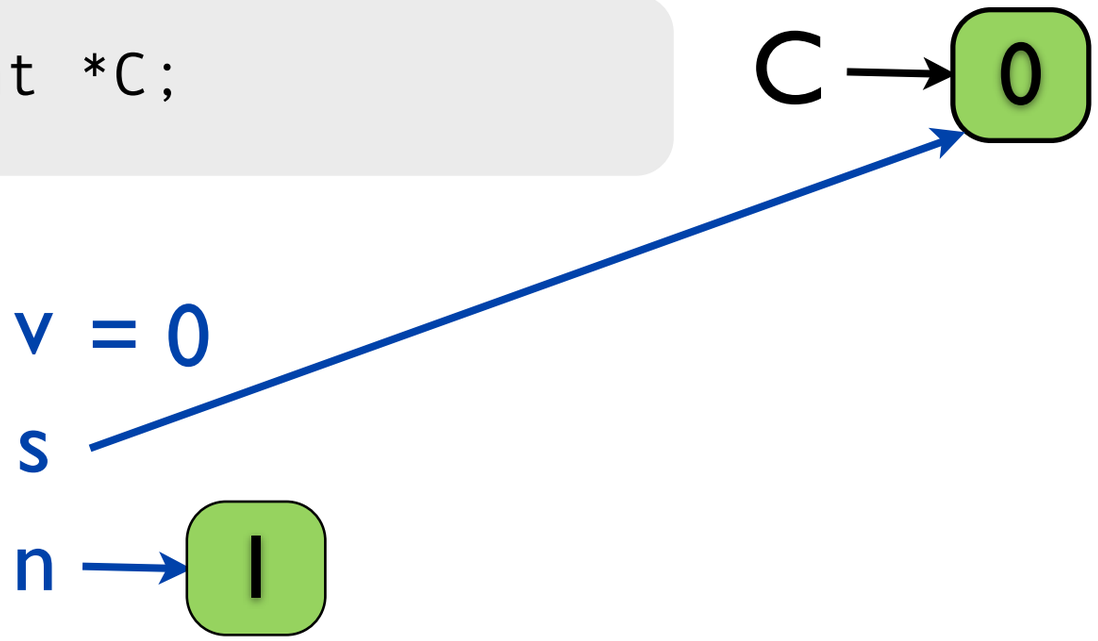


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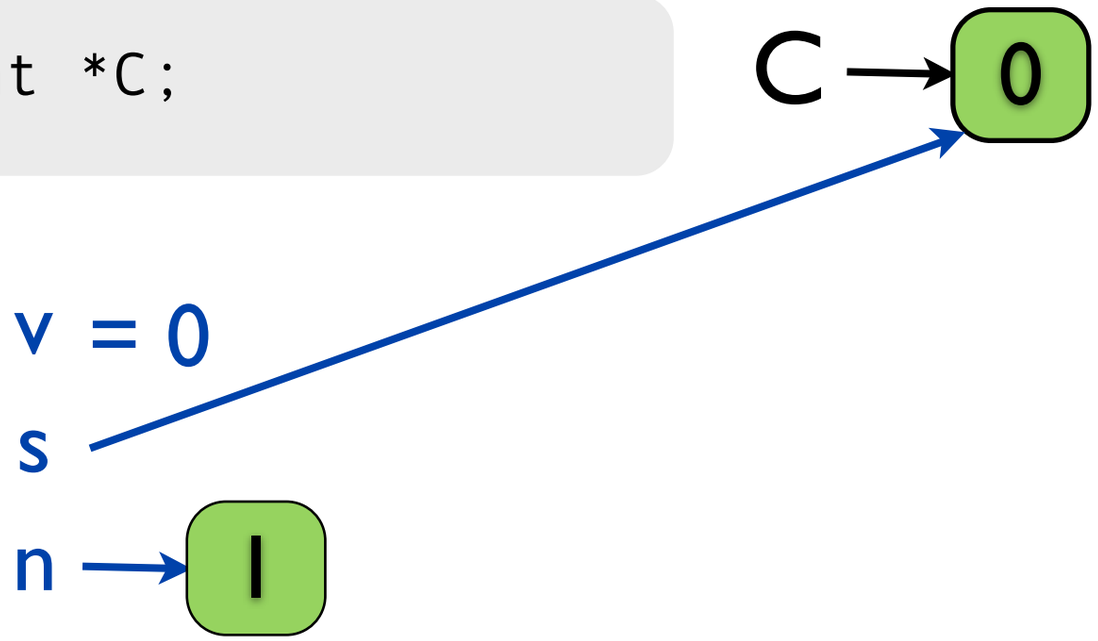


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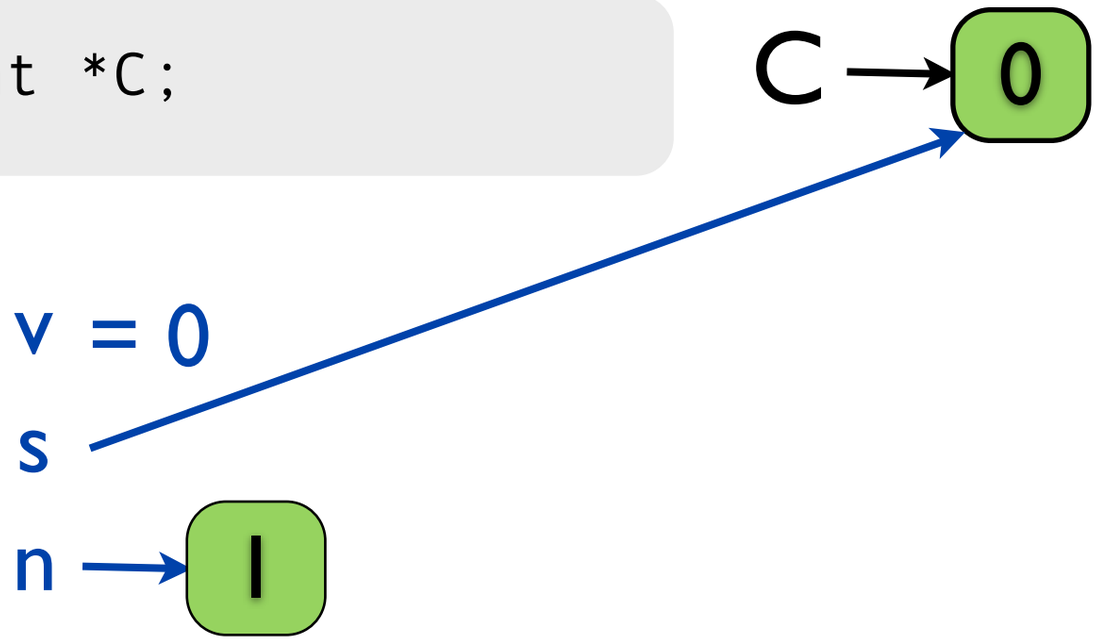


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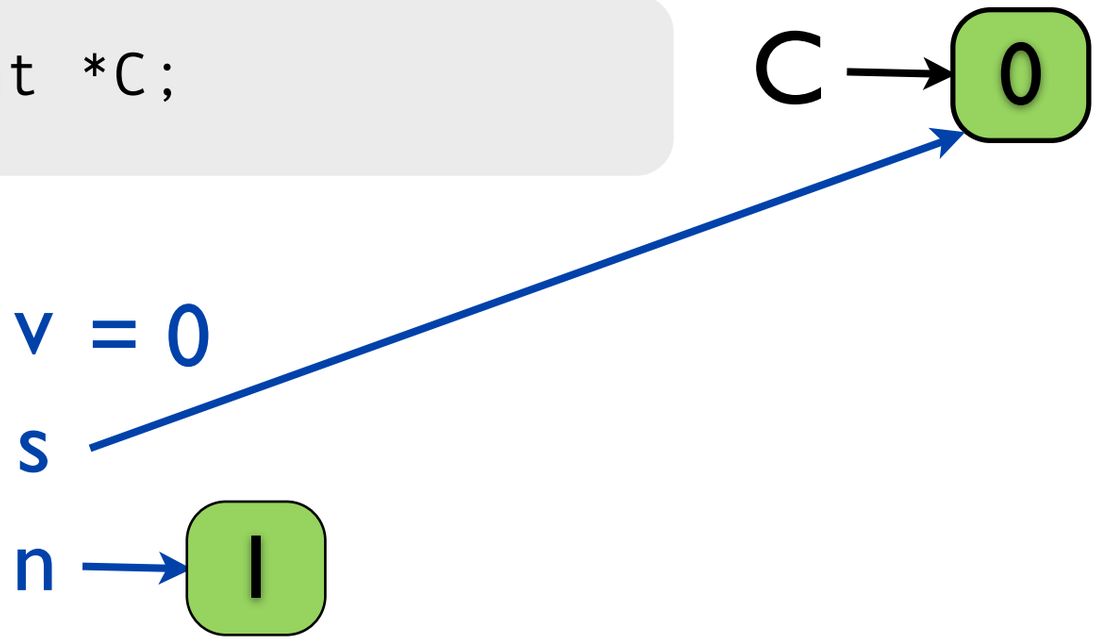


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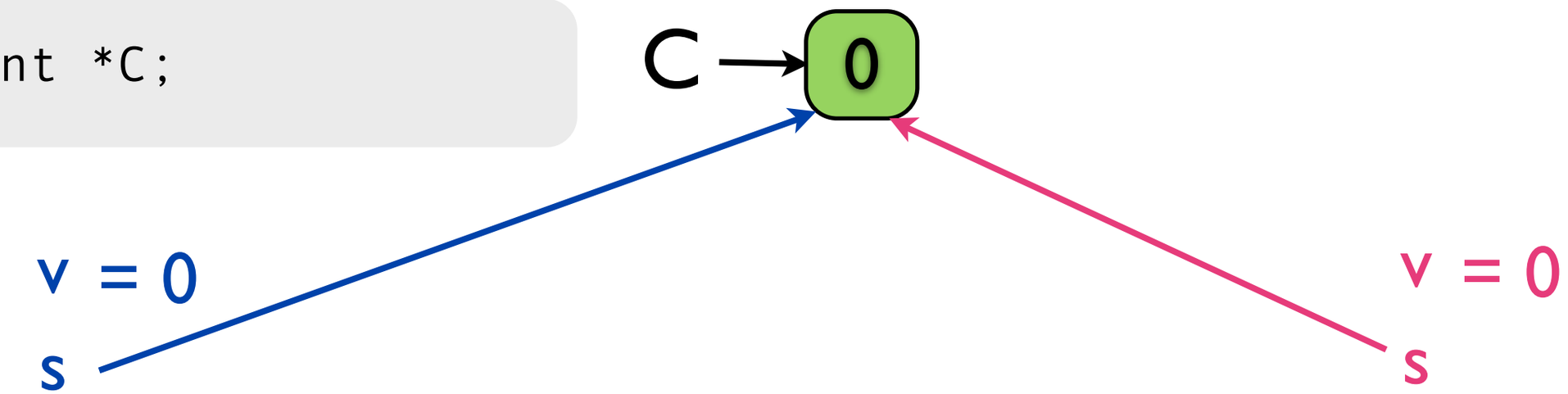

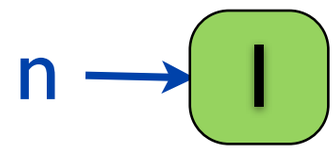


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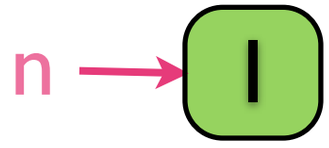


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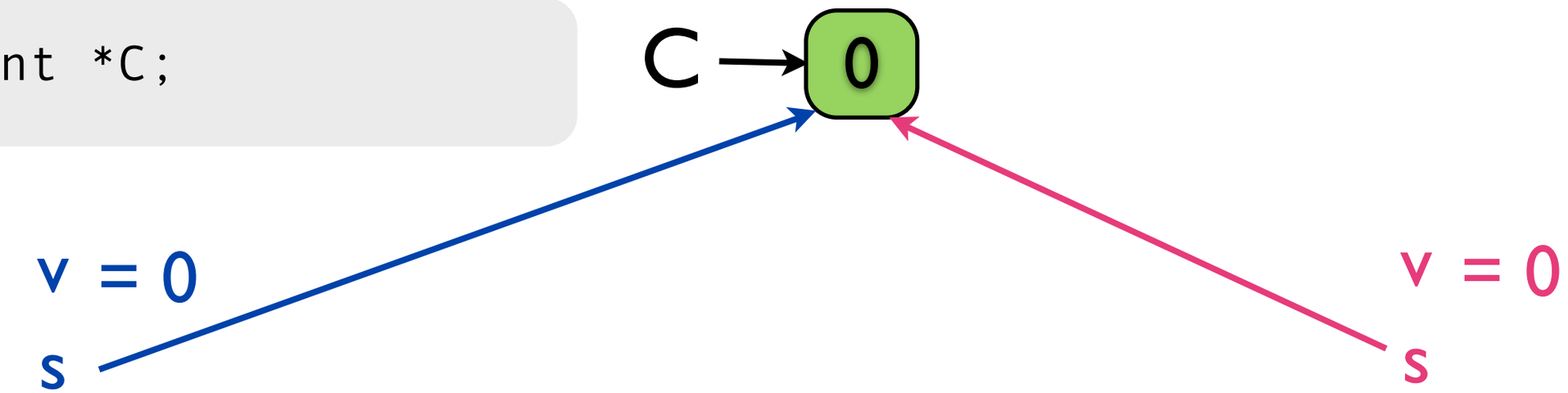

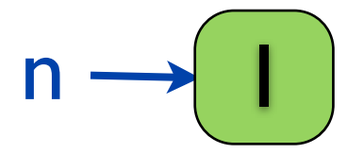


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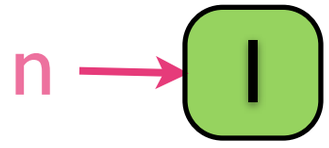


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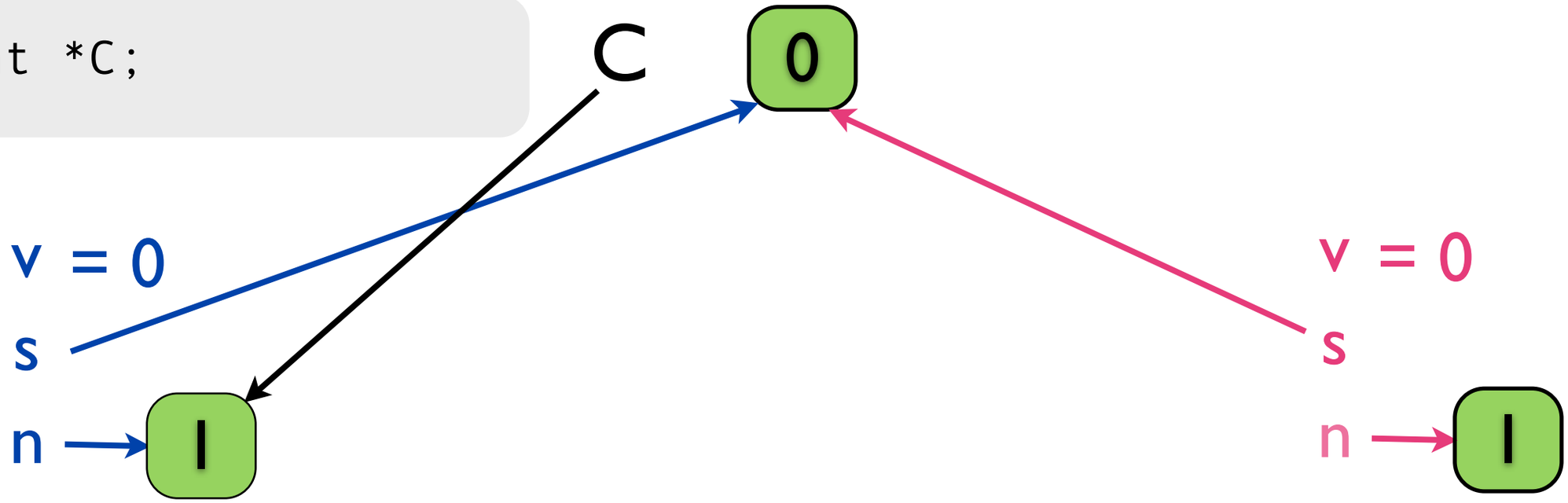


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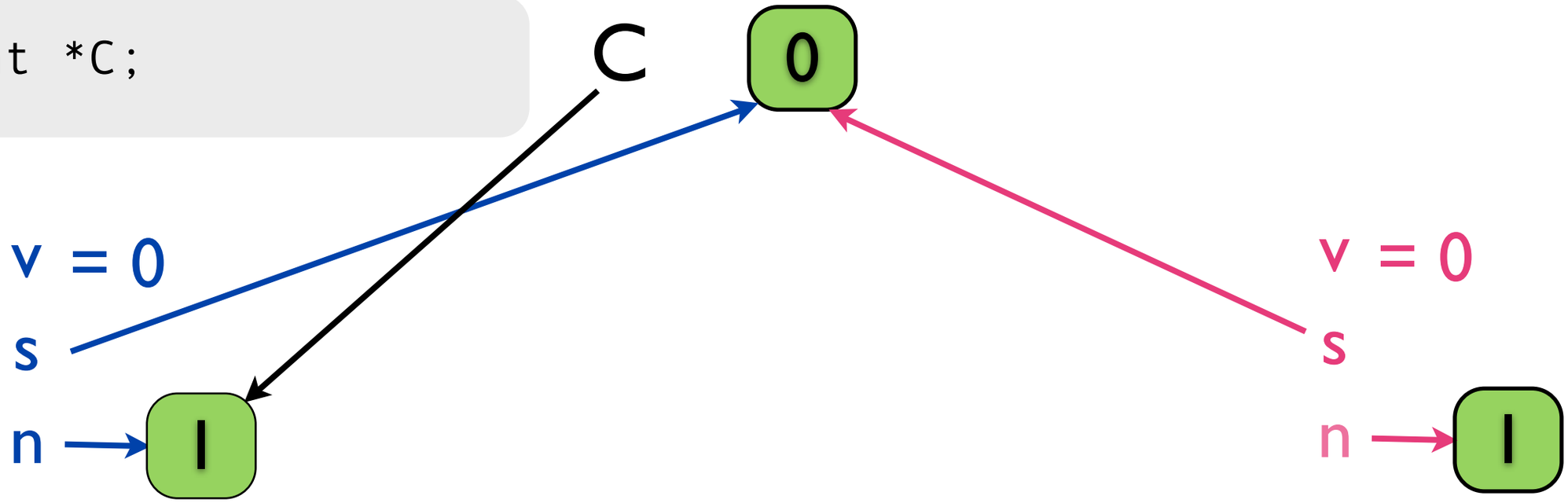


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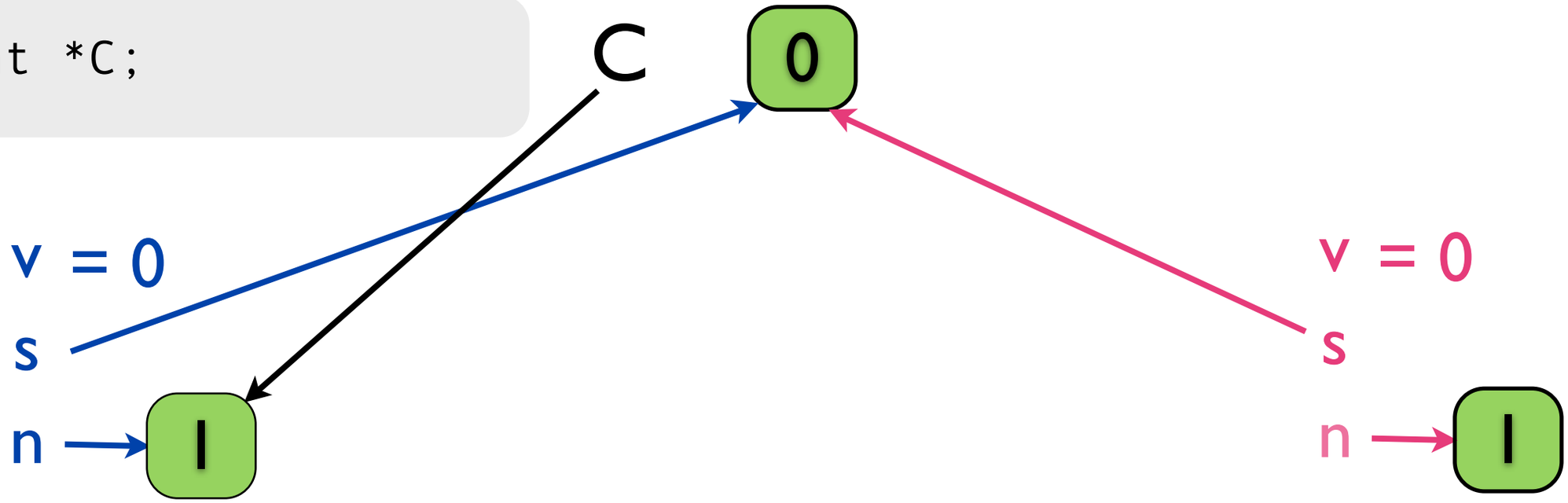


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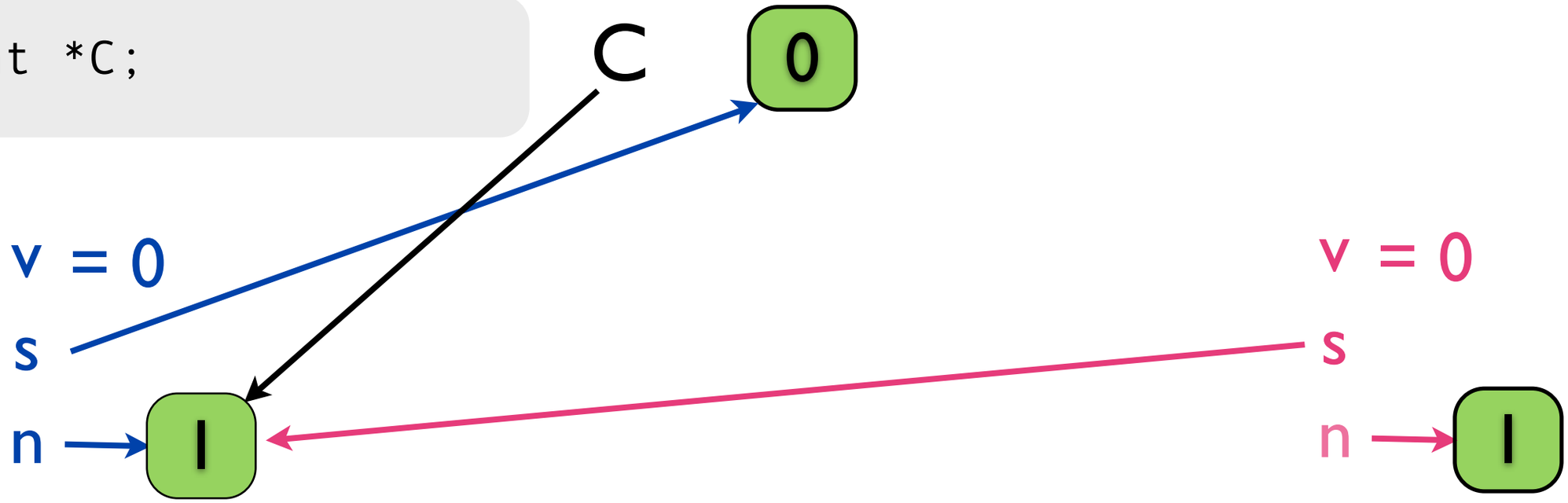


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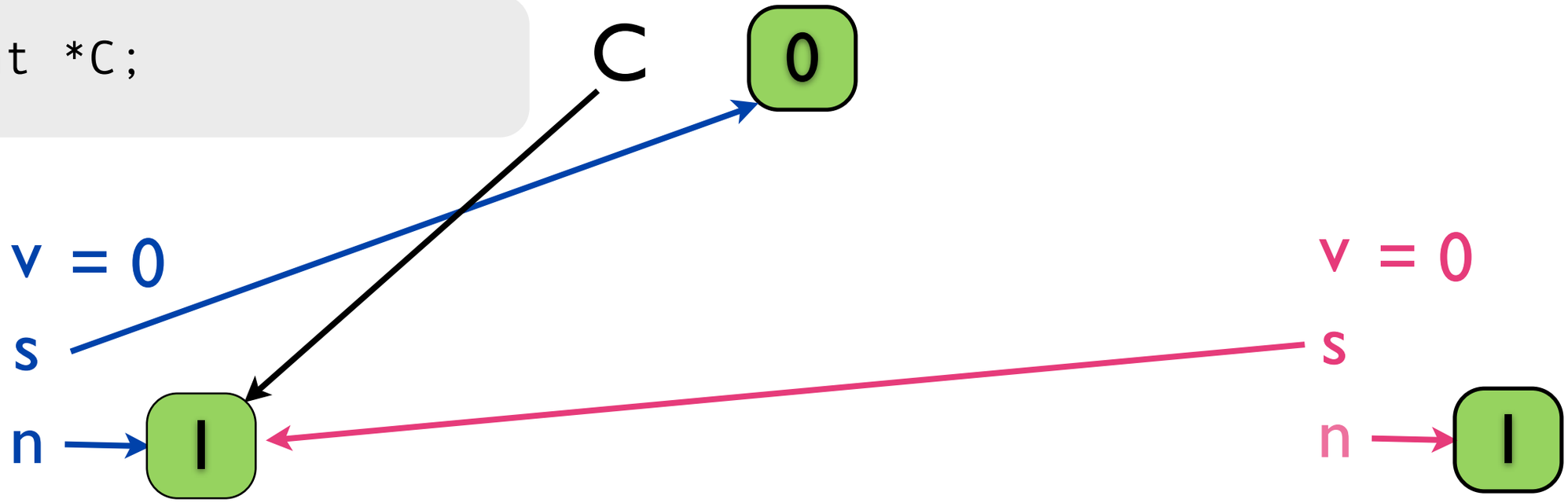


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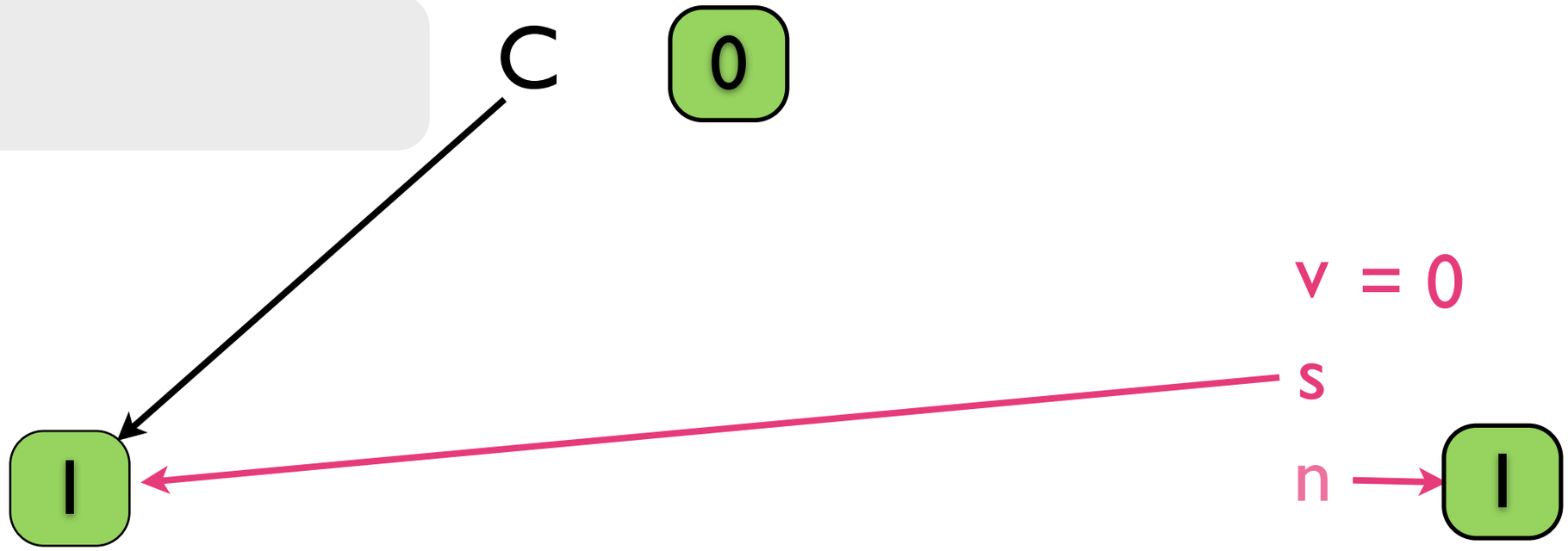


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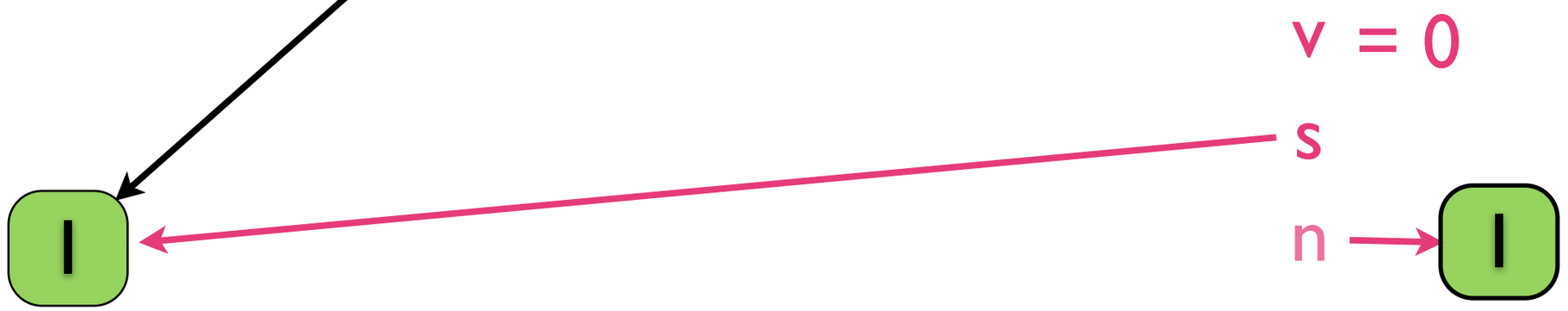


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  s = C;
  v = *s;
  *n = v + 1;
 }while(!CAS(&C,s,n));

 return v;
}
```

no GC ⇒ memory leaks

```
int *C;
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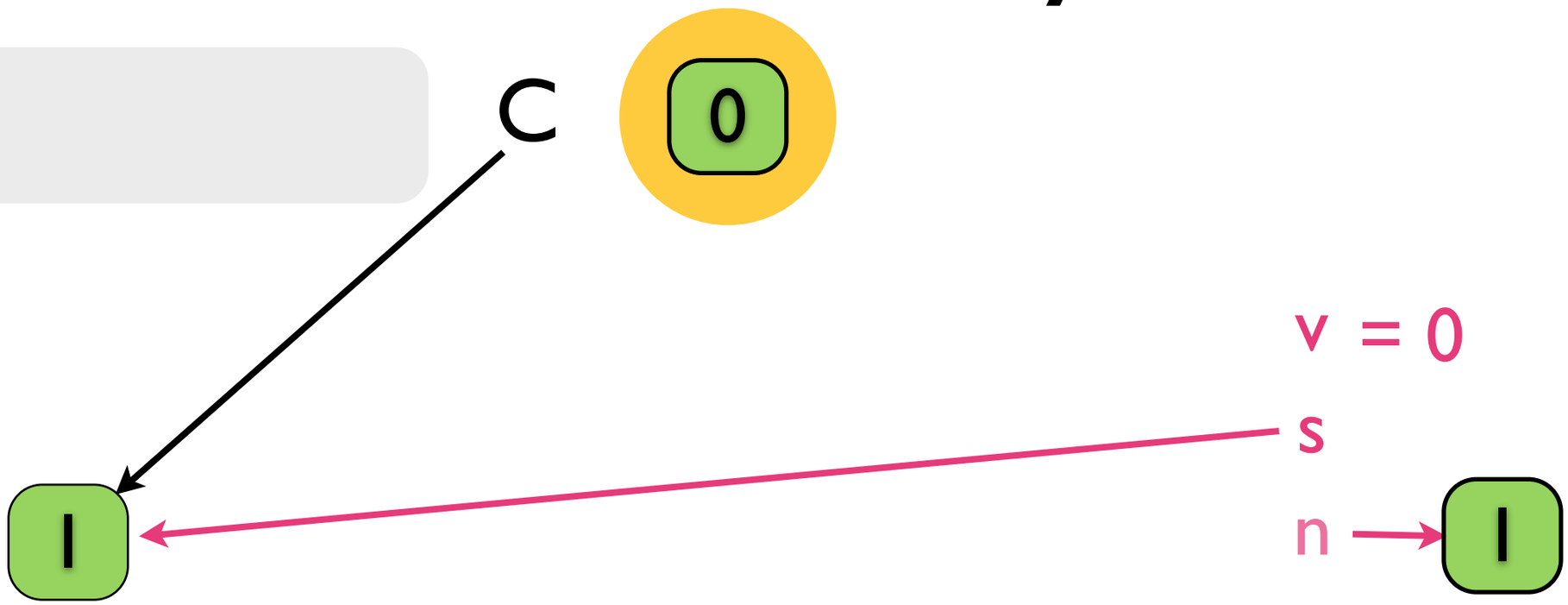


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CAS(&C,s,n)≔
 <if(C==s)
    C=n;
    return true
  else
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naive fix

```
int *C;
```

C


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}
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problem: memory safety

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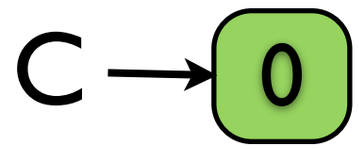


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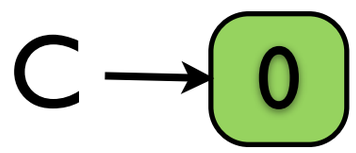


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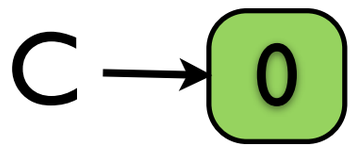

v

s

n

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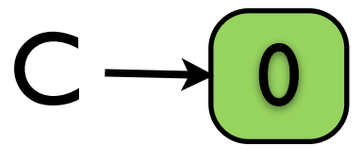

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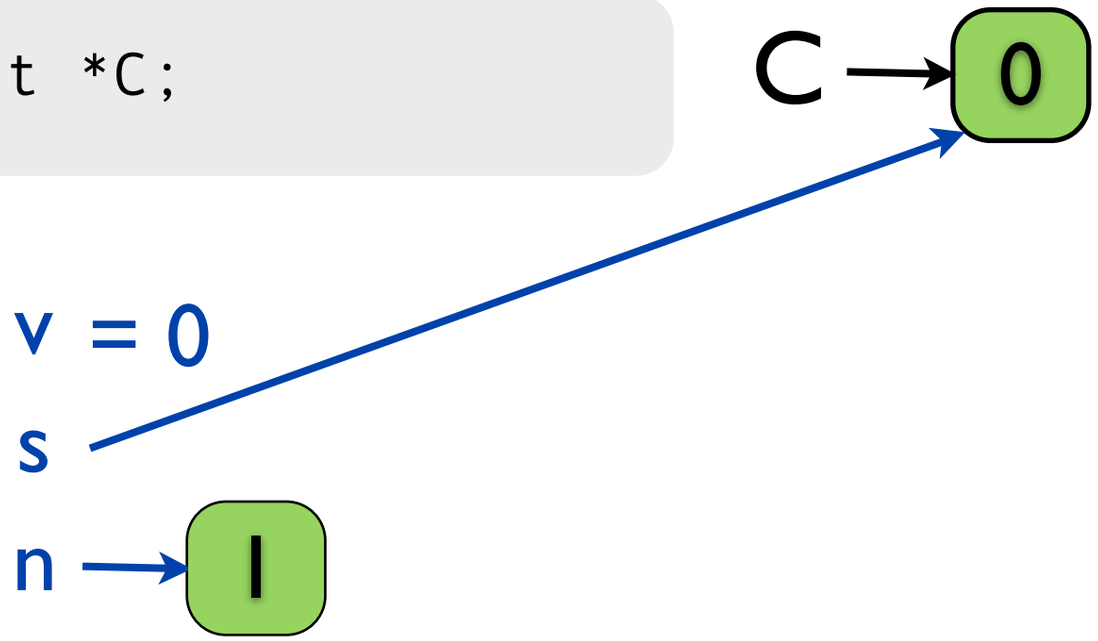


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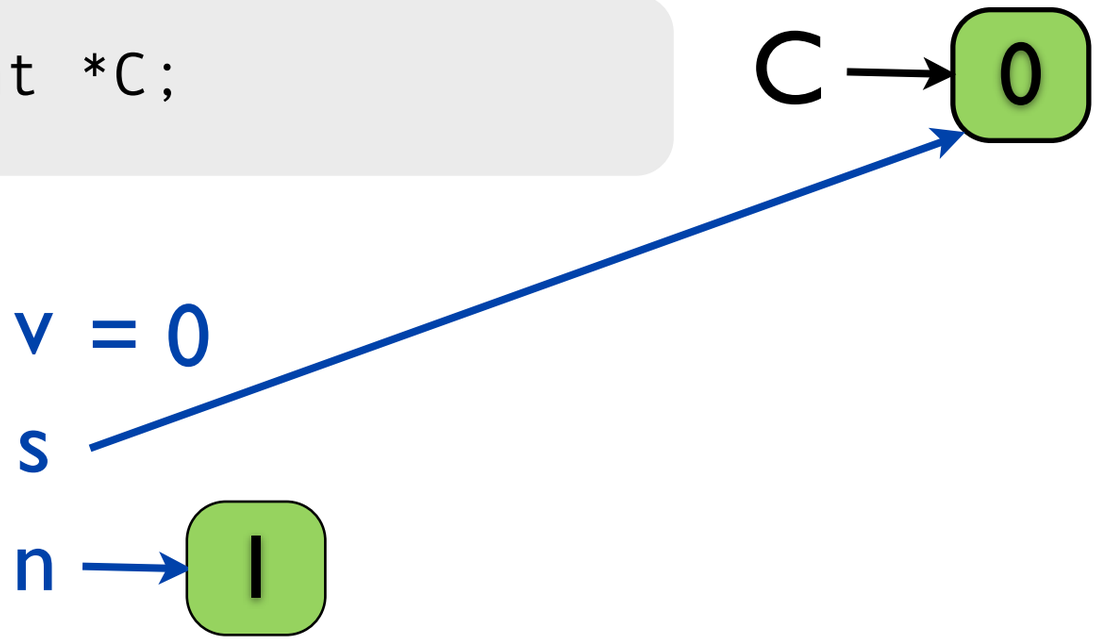


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```

problem: memory safety

```
int *C;
```

v = 0

s

n

1

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int inc() {
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v

s

n

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problem: memory safety

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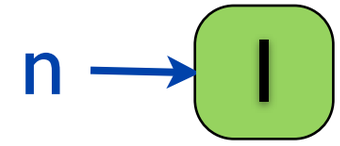
v = 0

s

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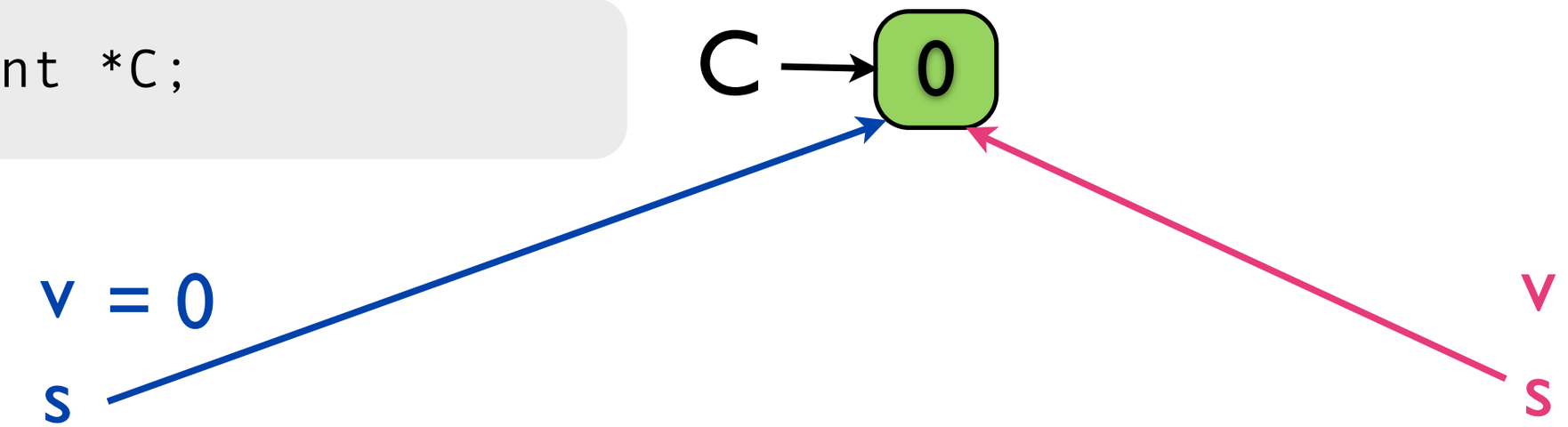

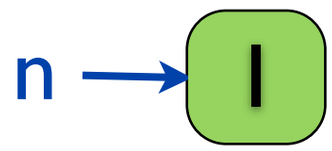


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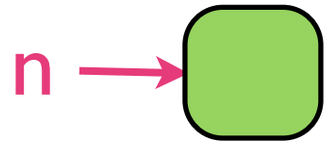


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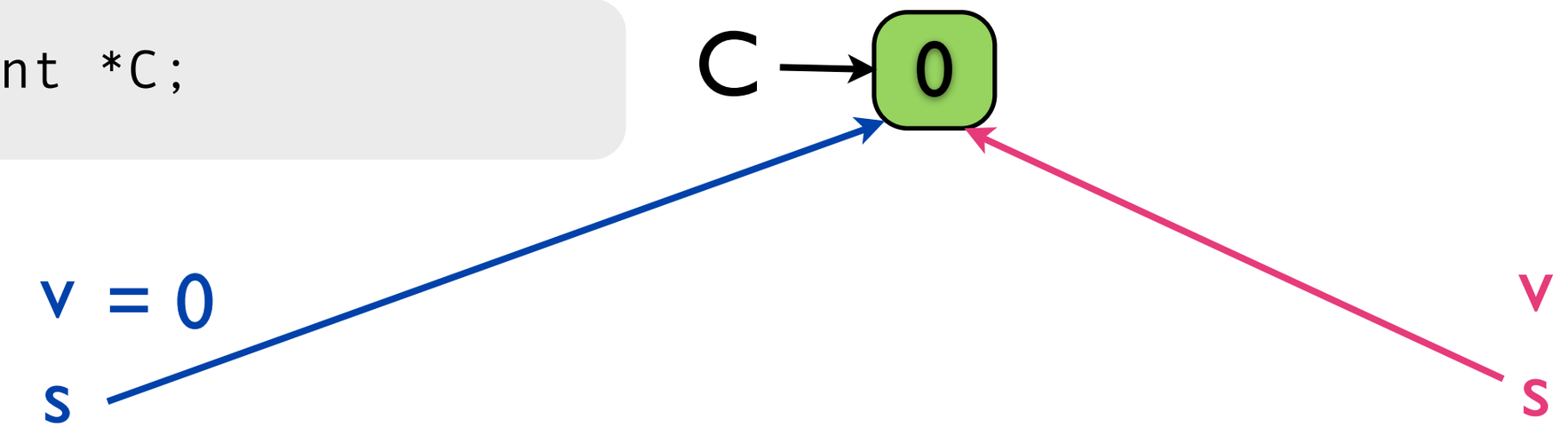

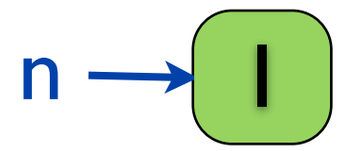


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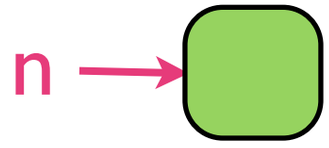


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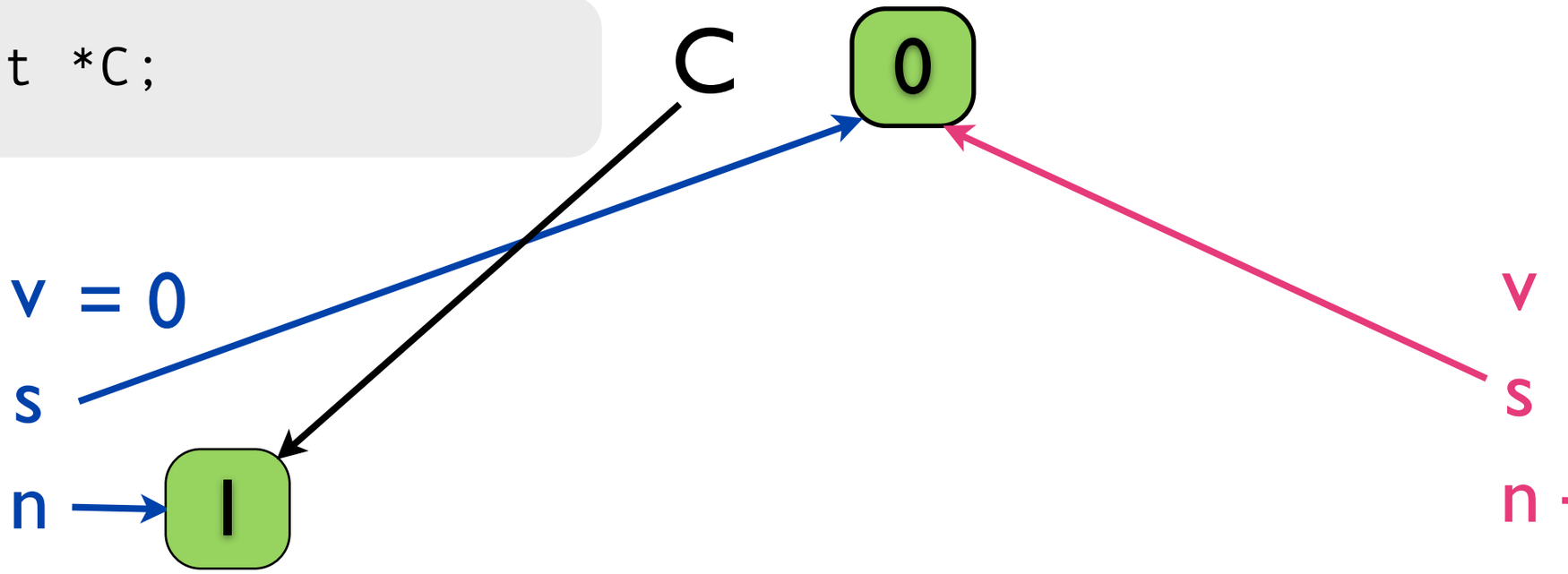


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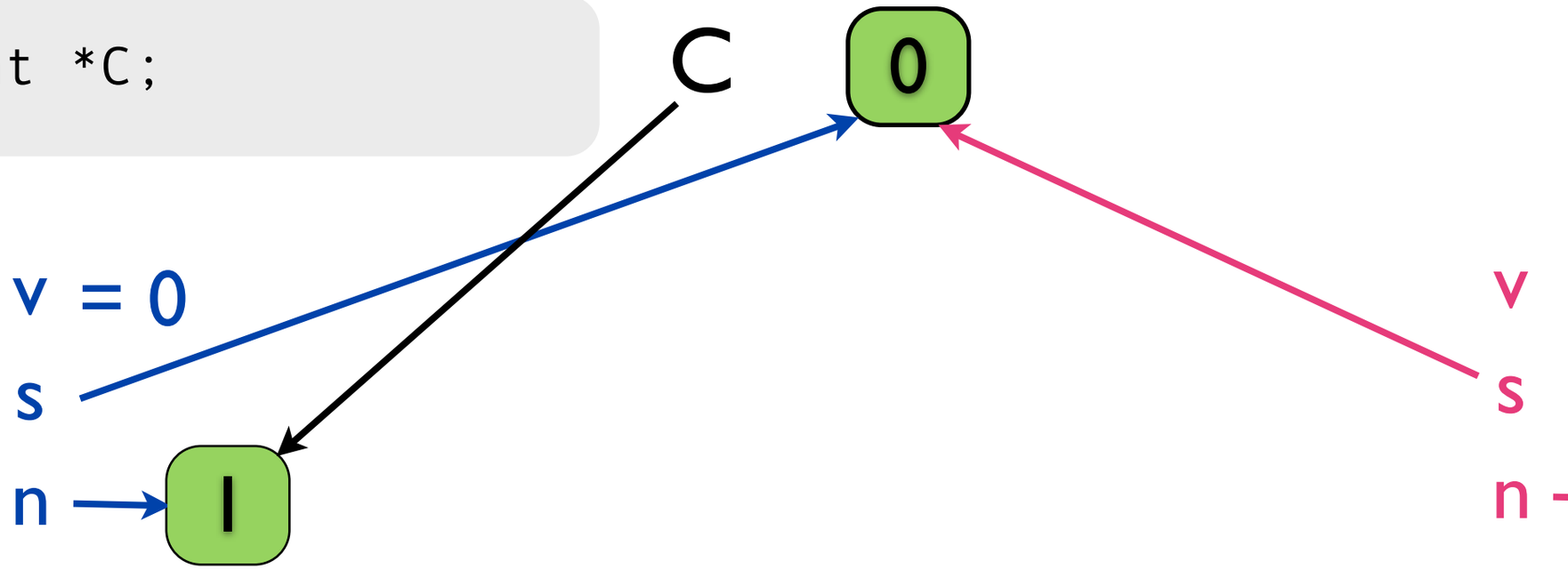


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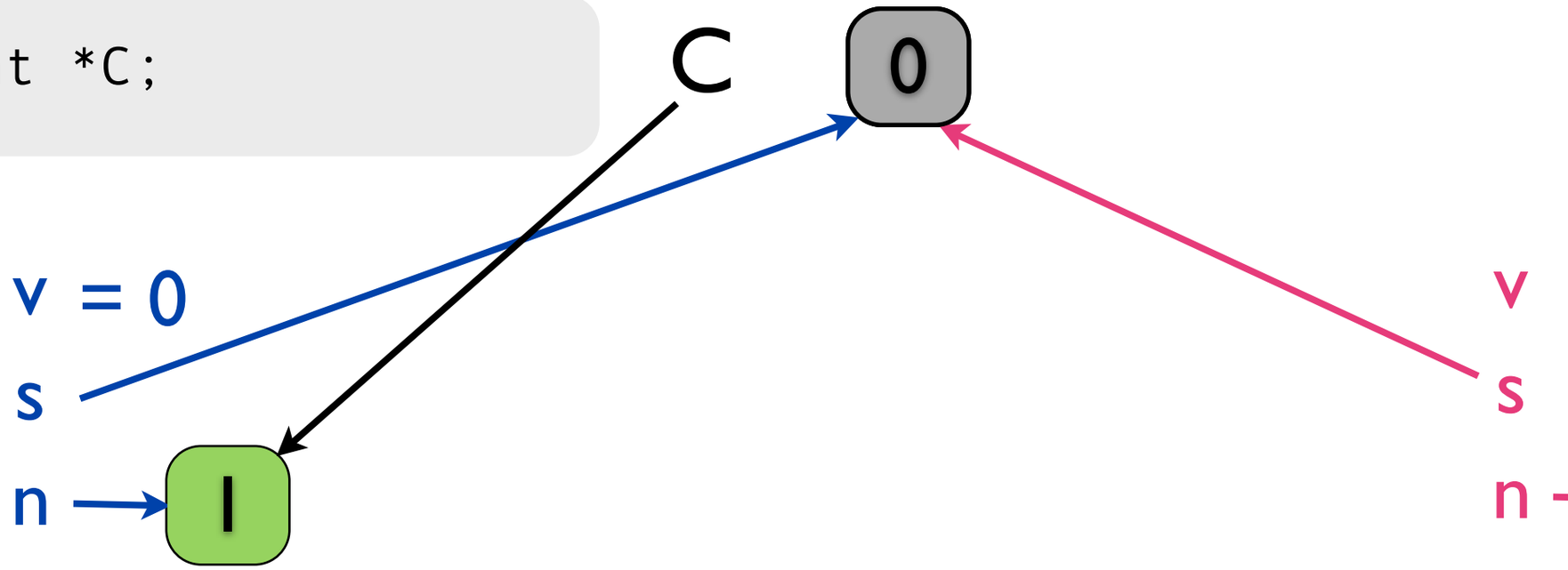


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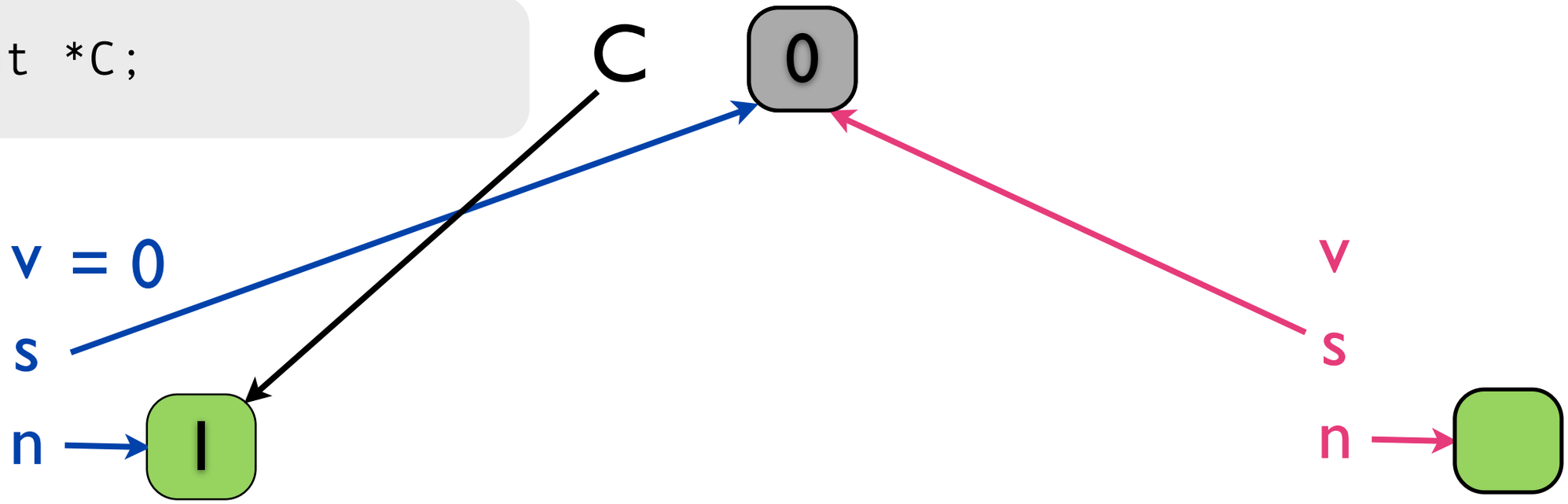


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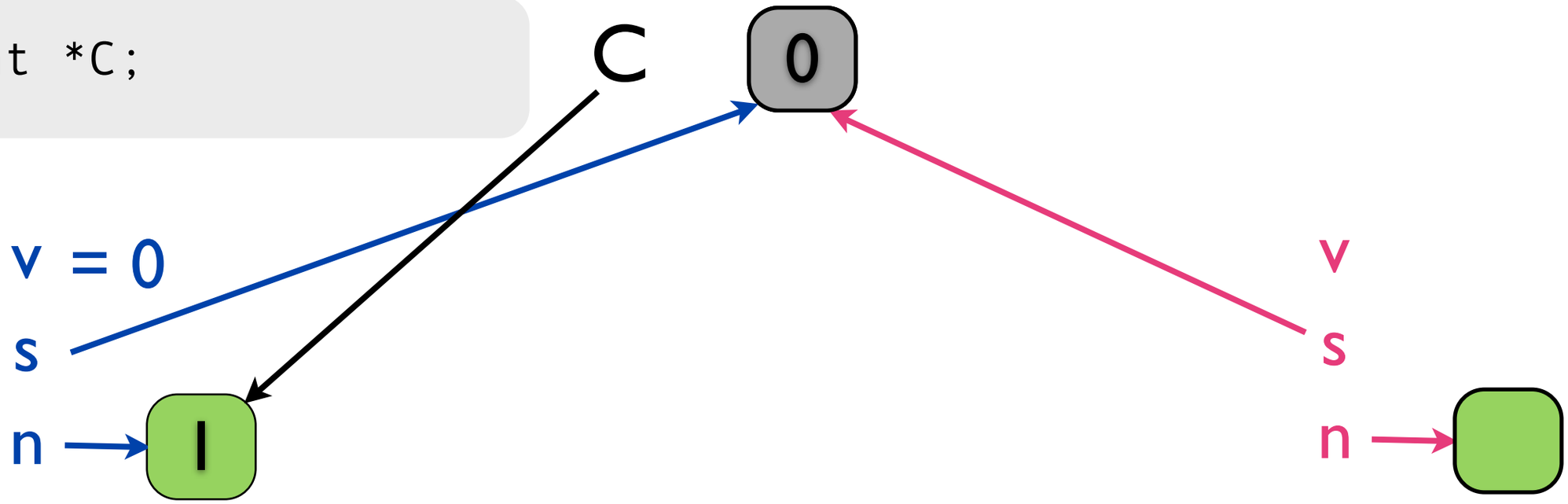


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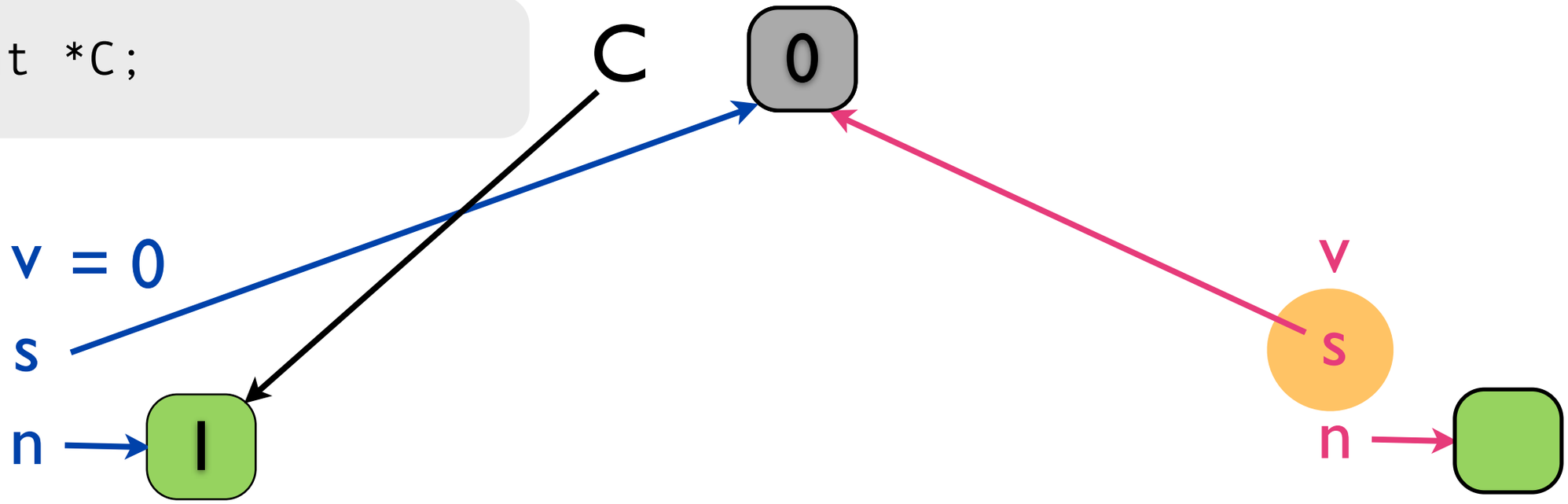


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problem: correctness

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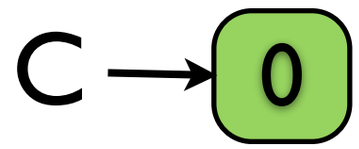


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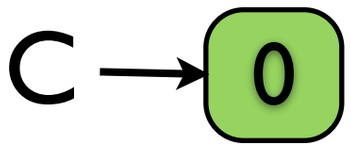


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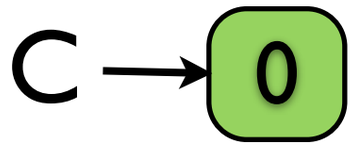

v

s

n

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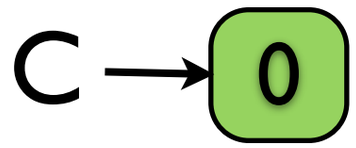

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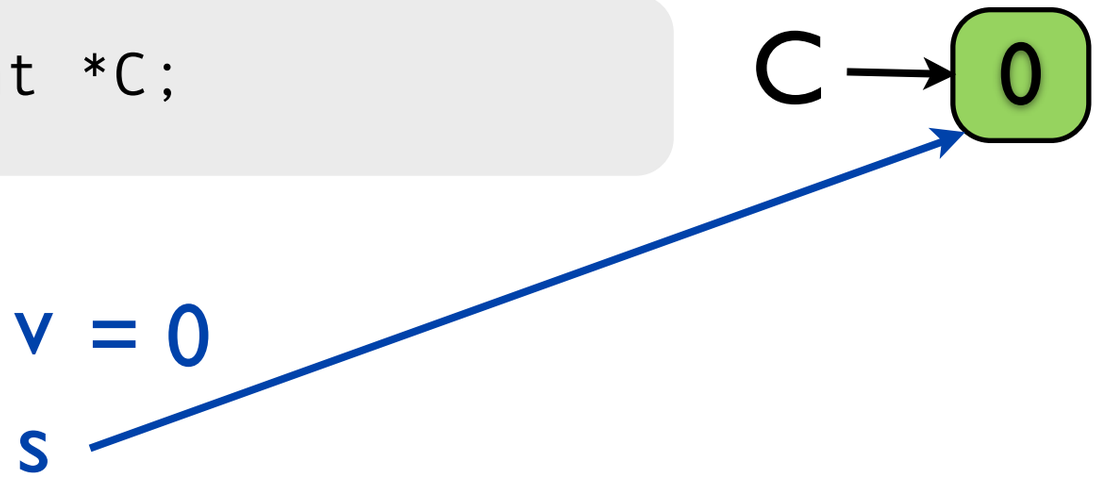


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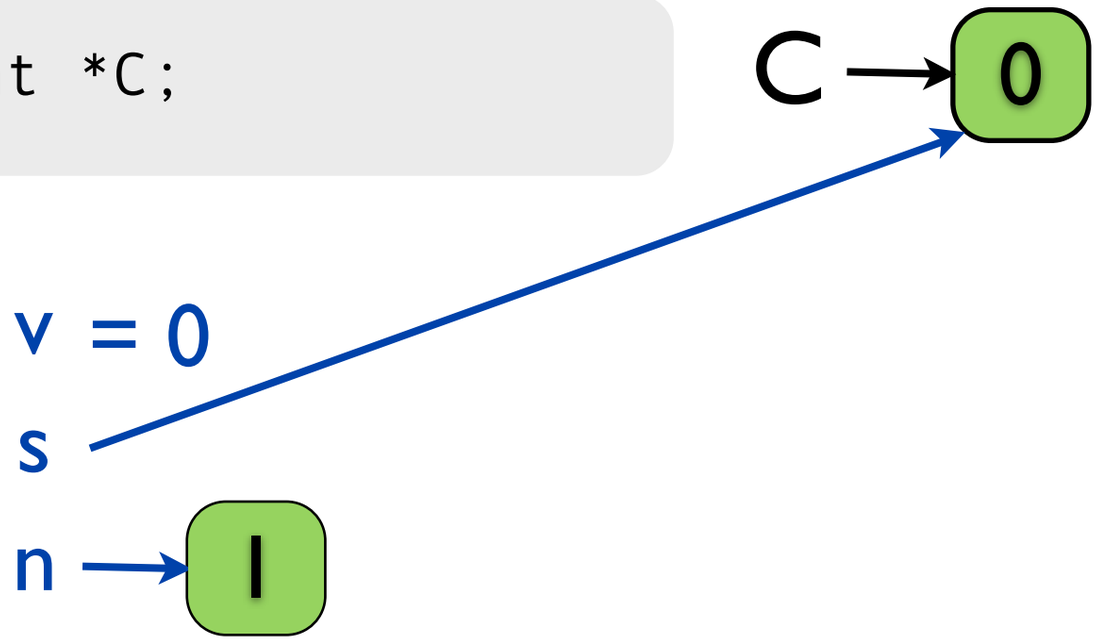


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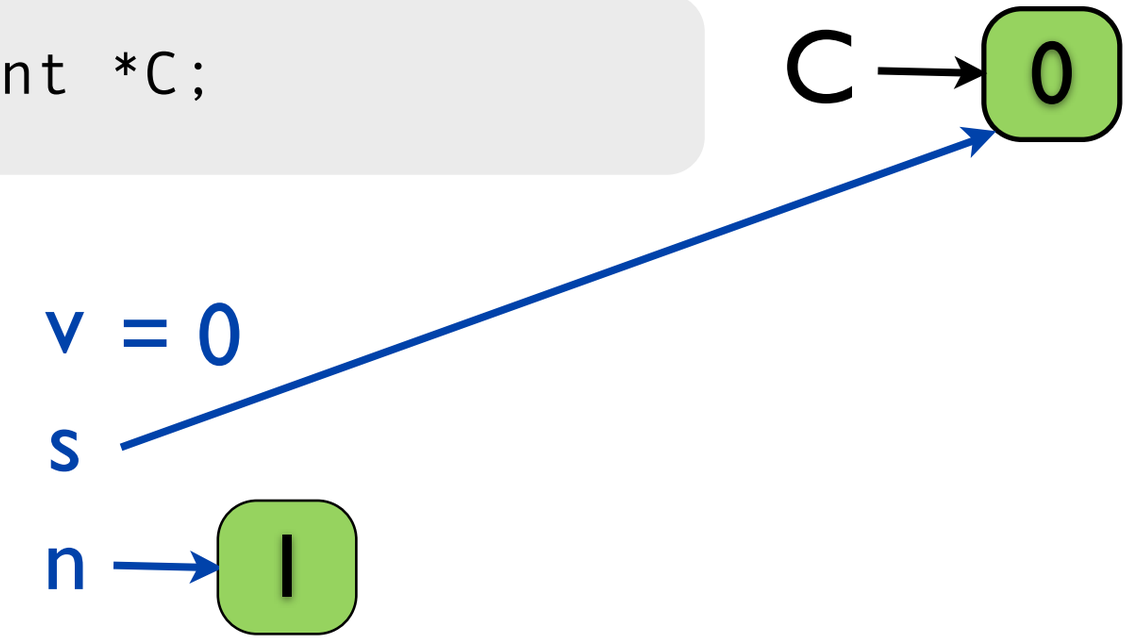


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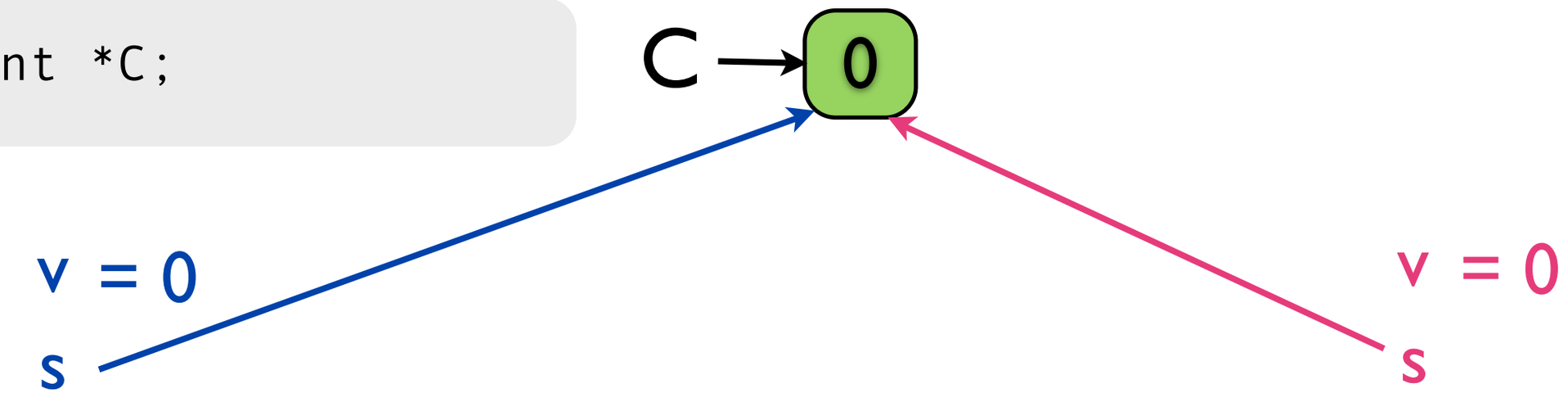

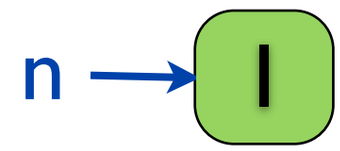


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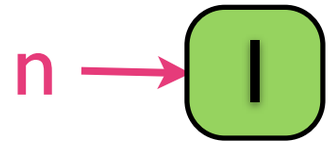


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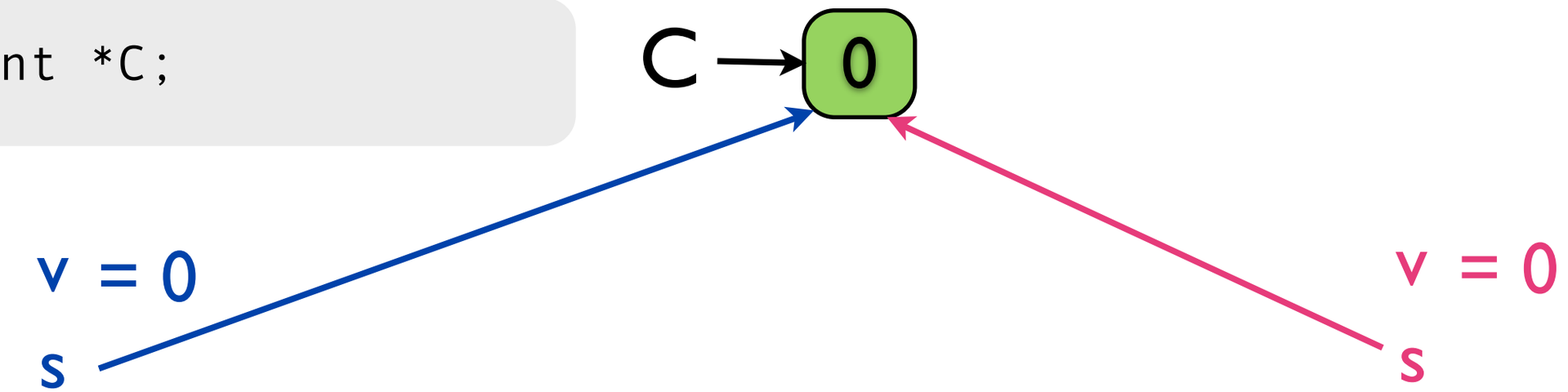

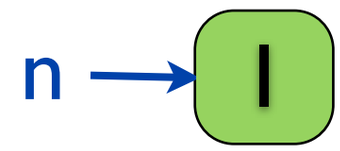


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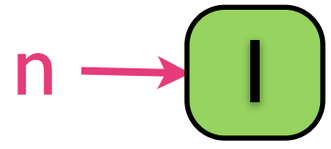


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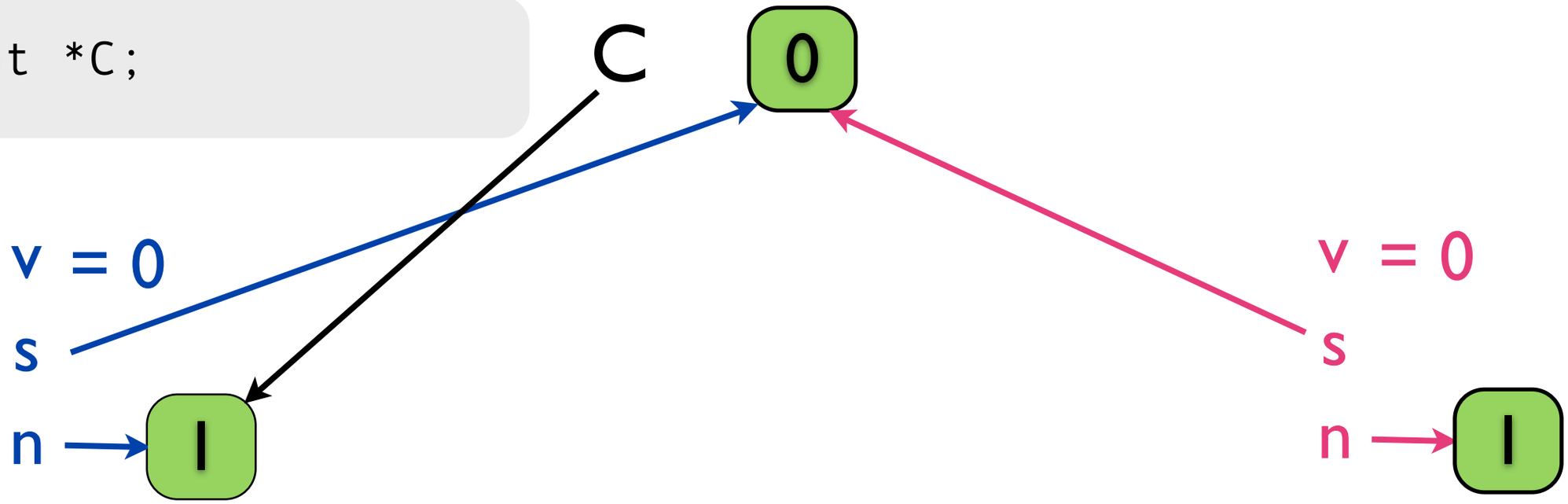


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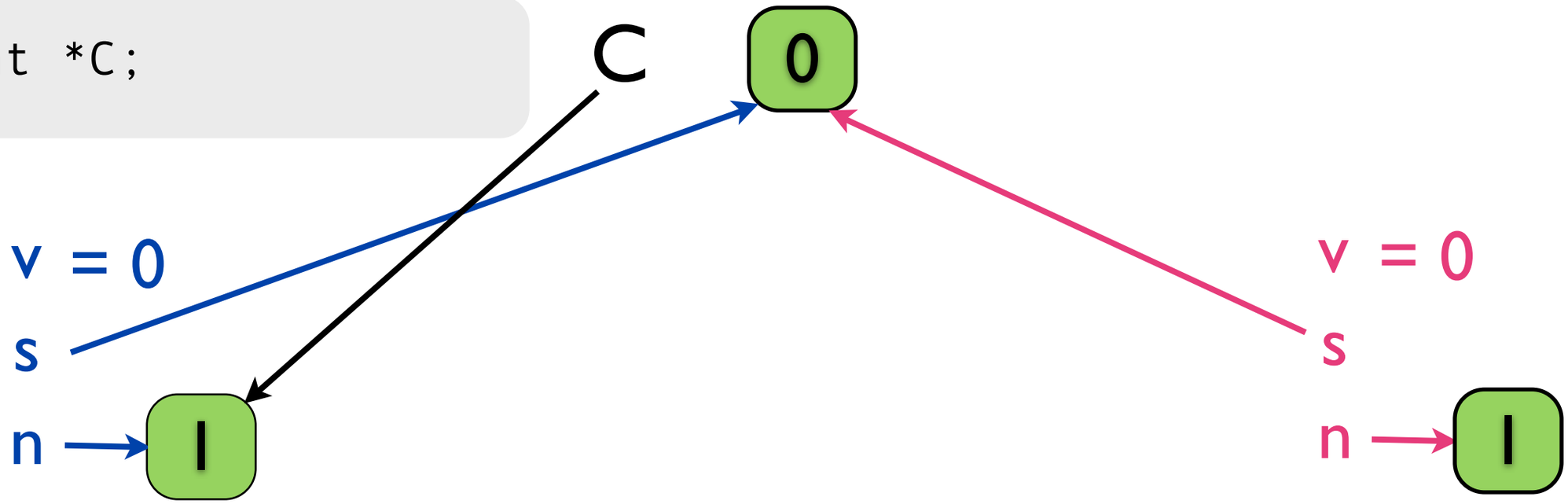


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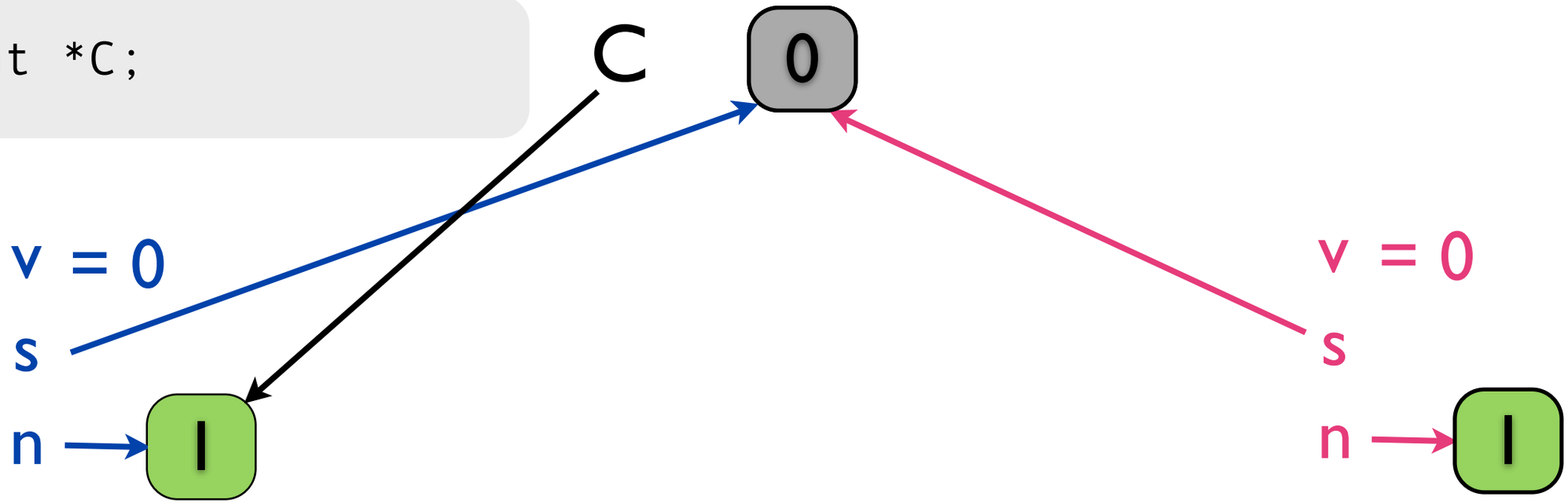


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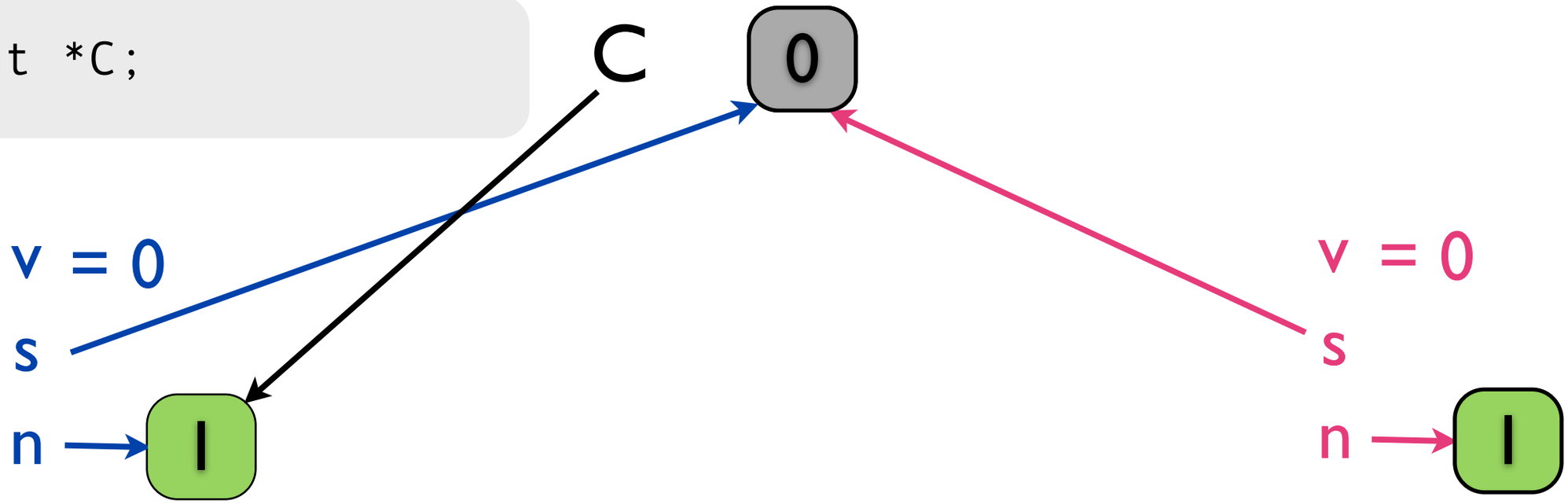


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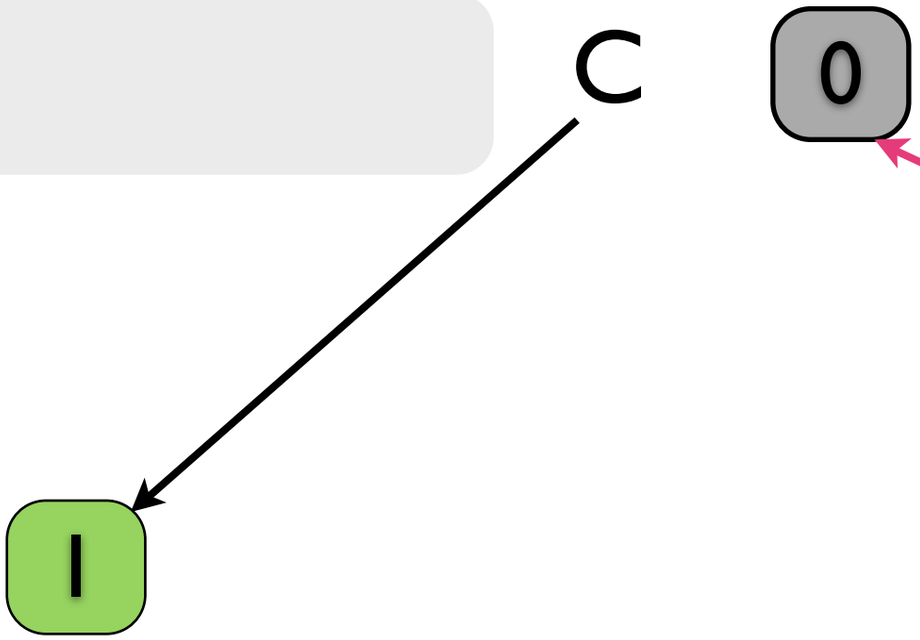


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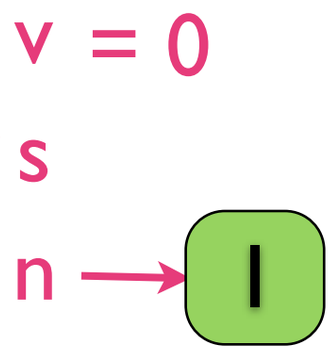


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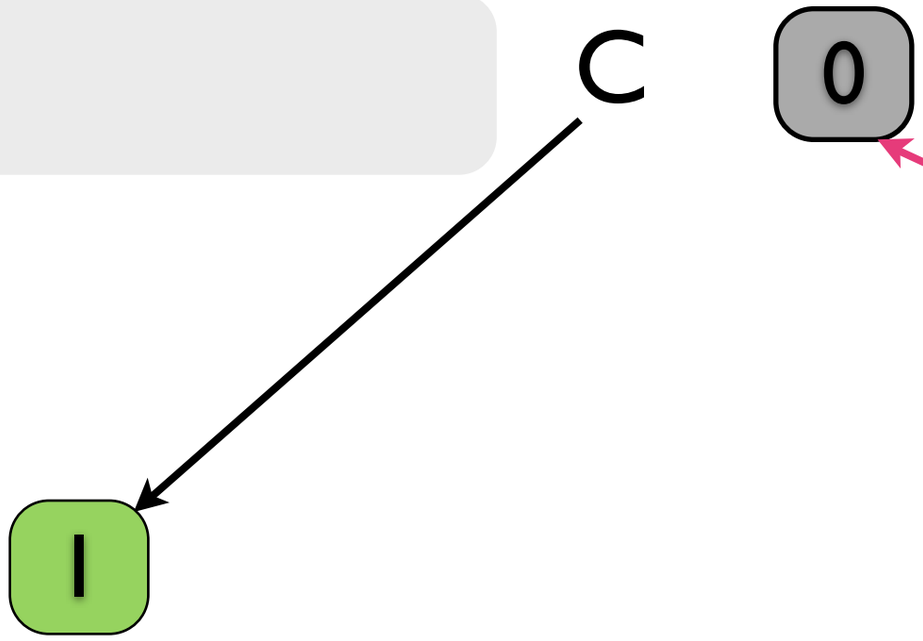


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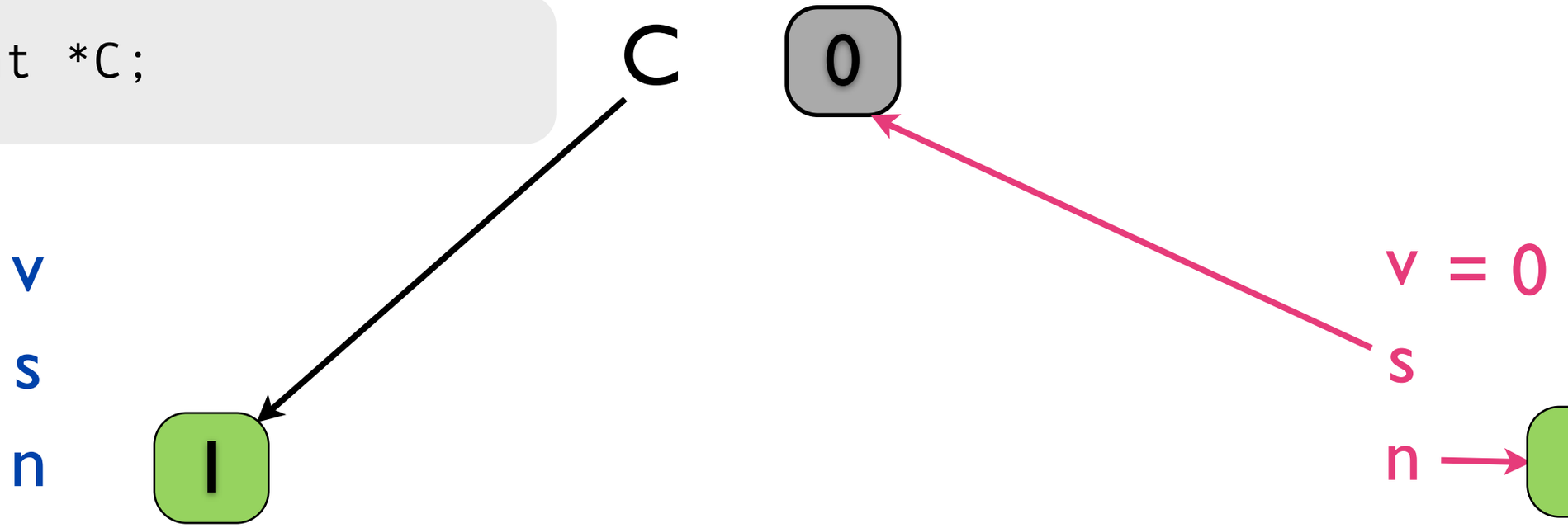


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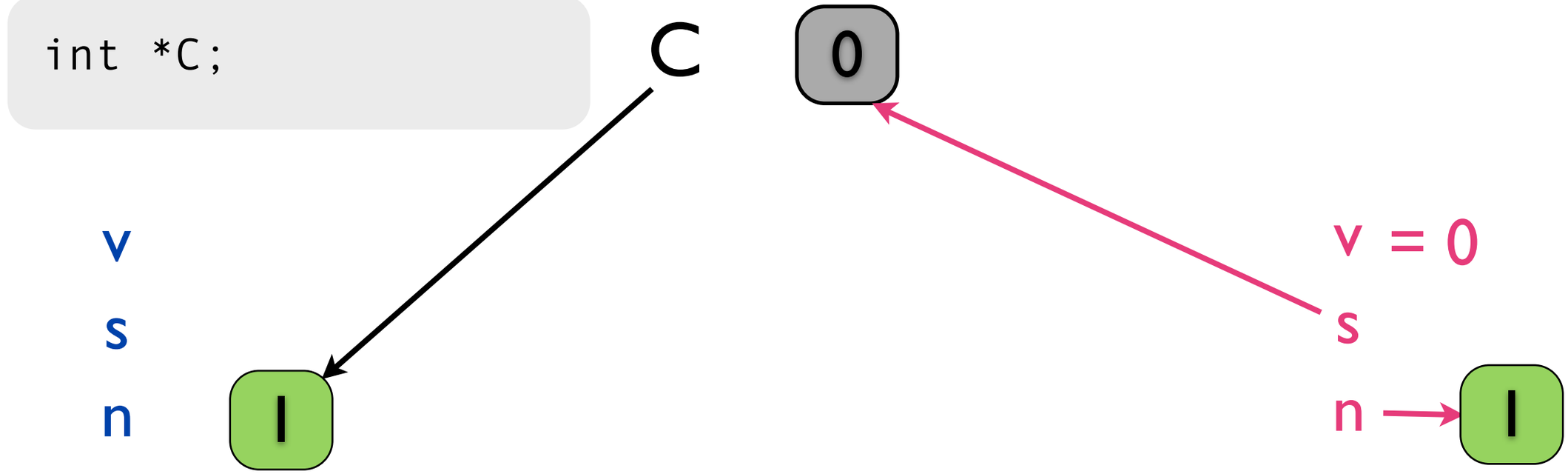


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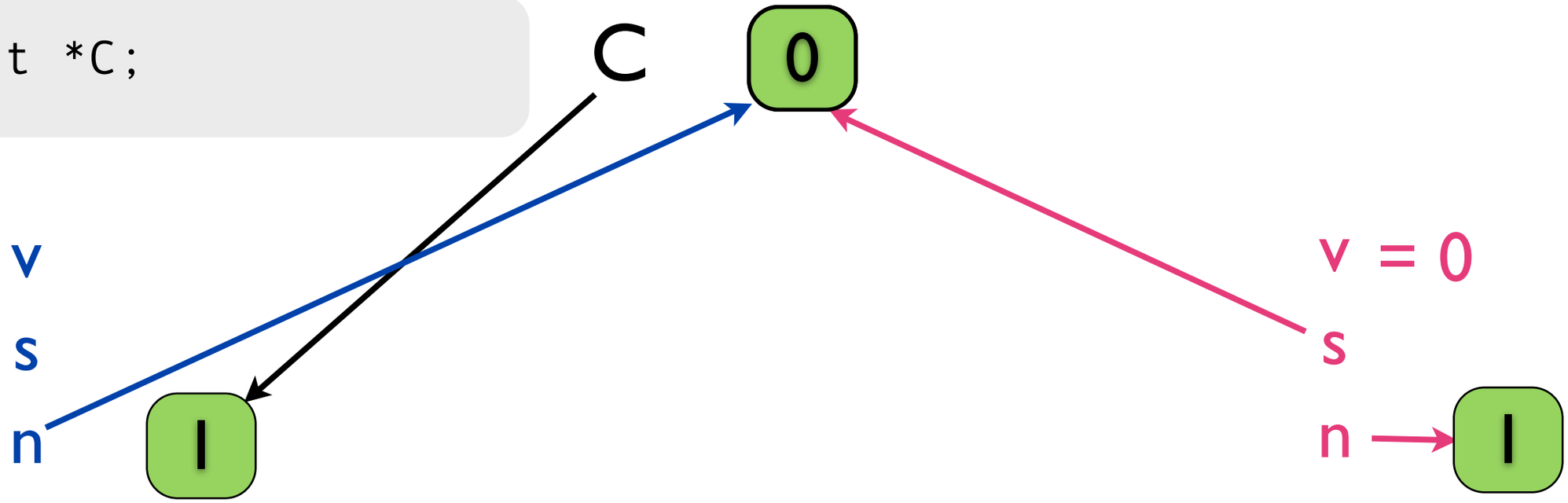


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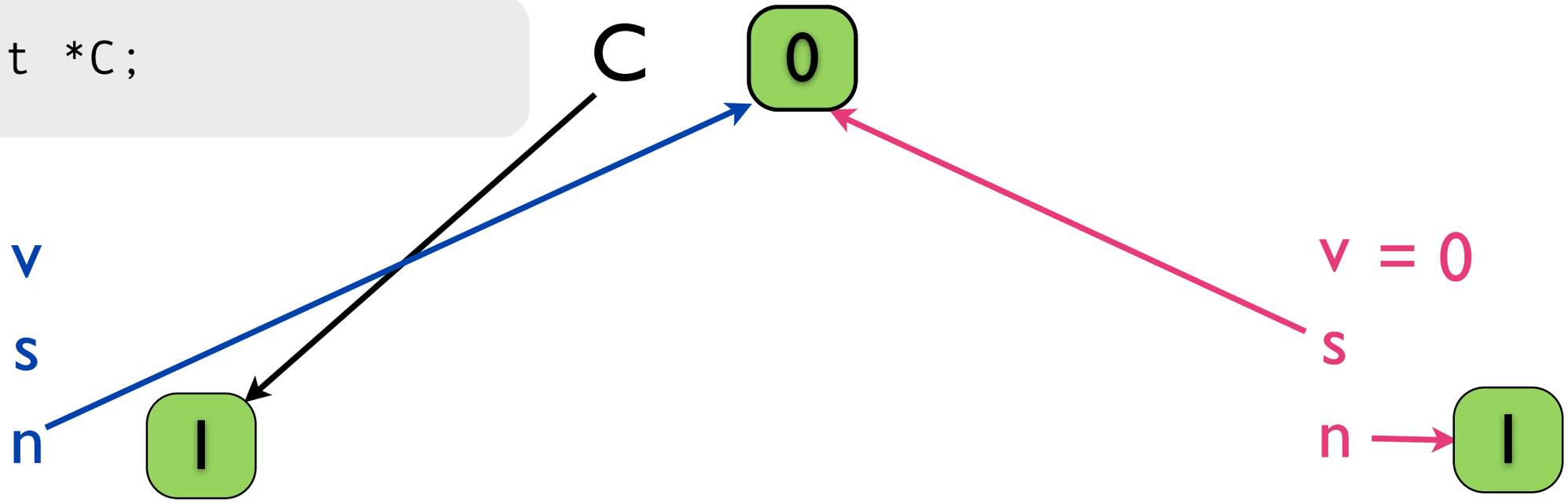


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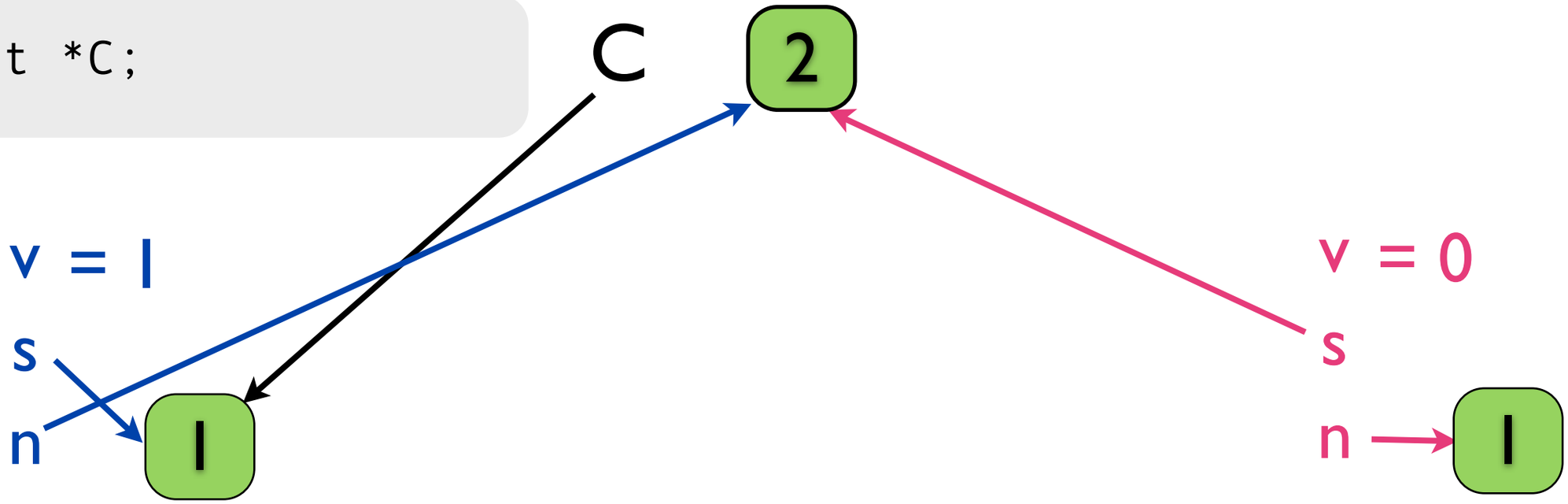


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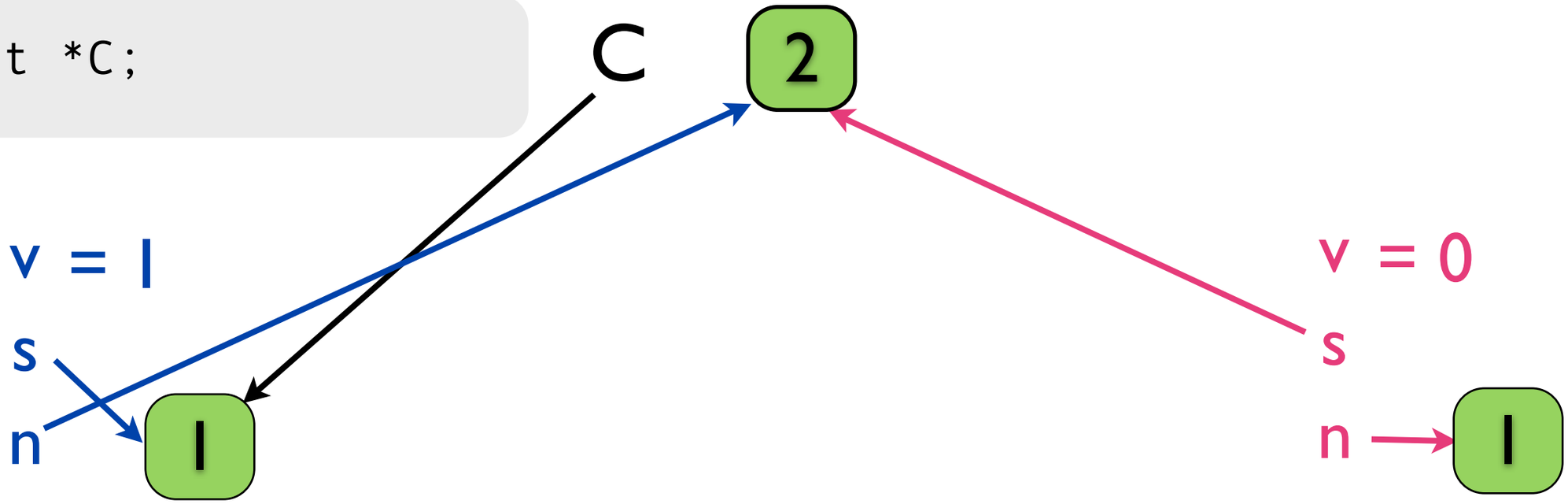


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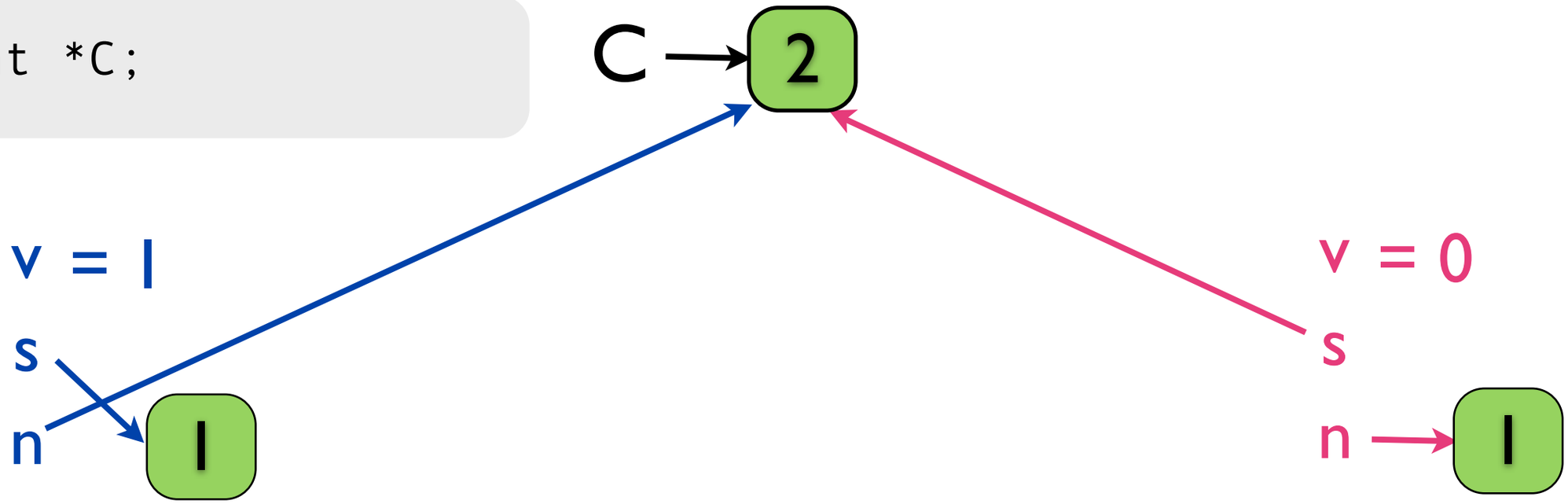


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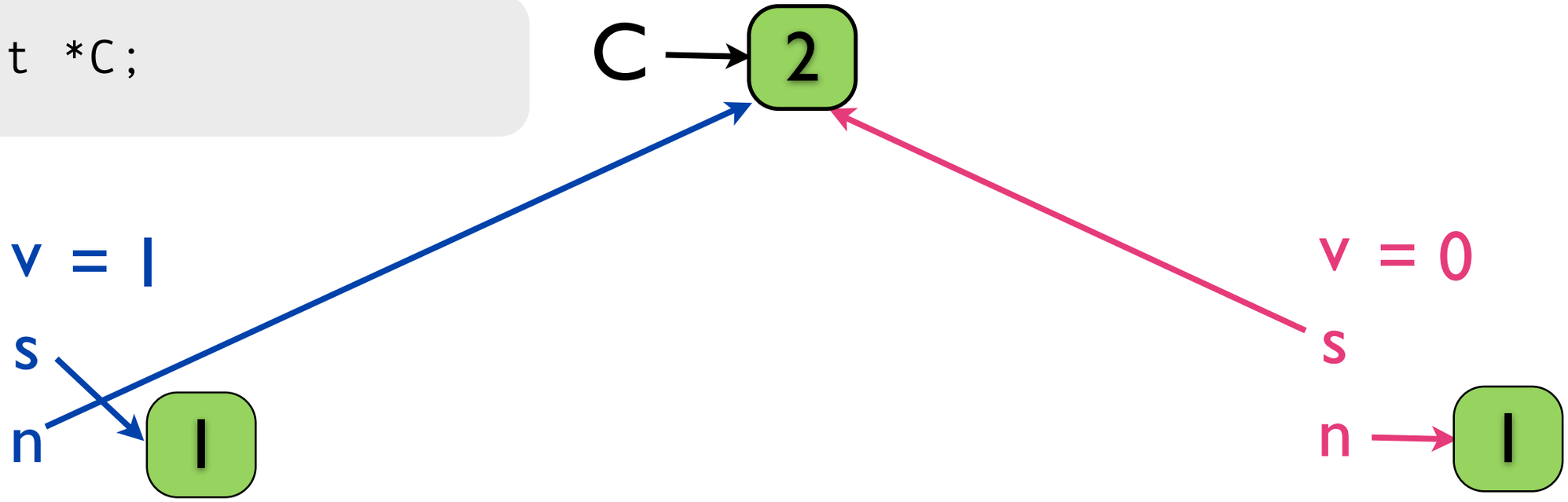


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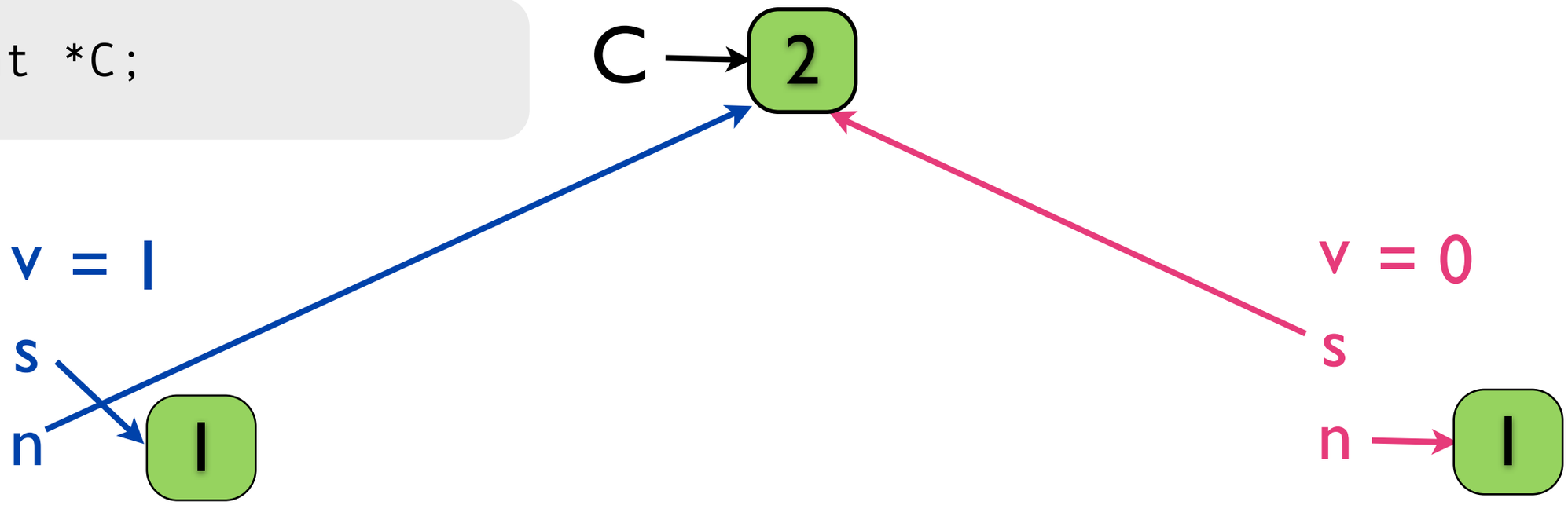


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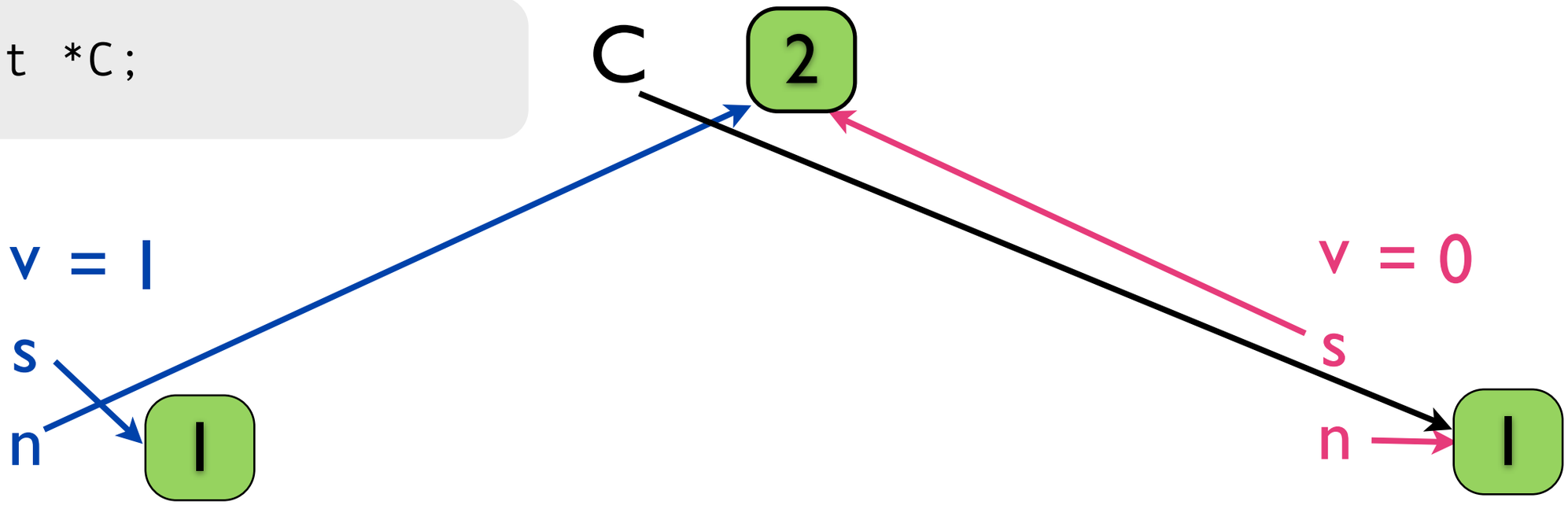


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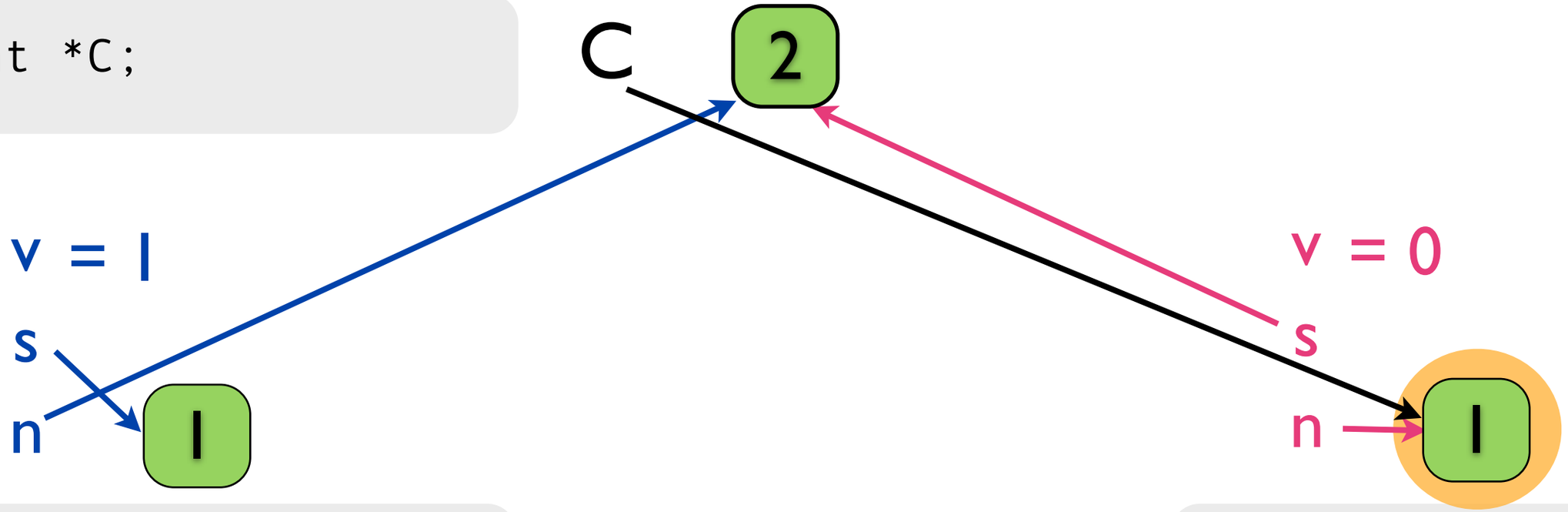


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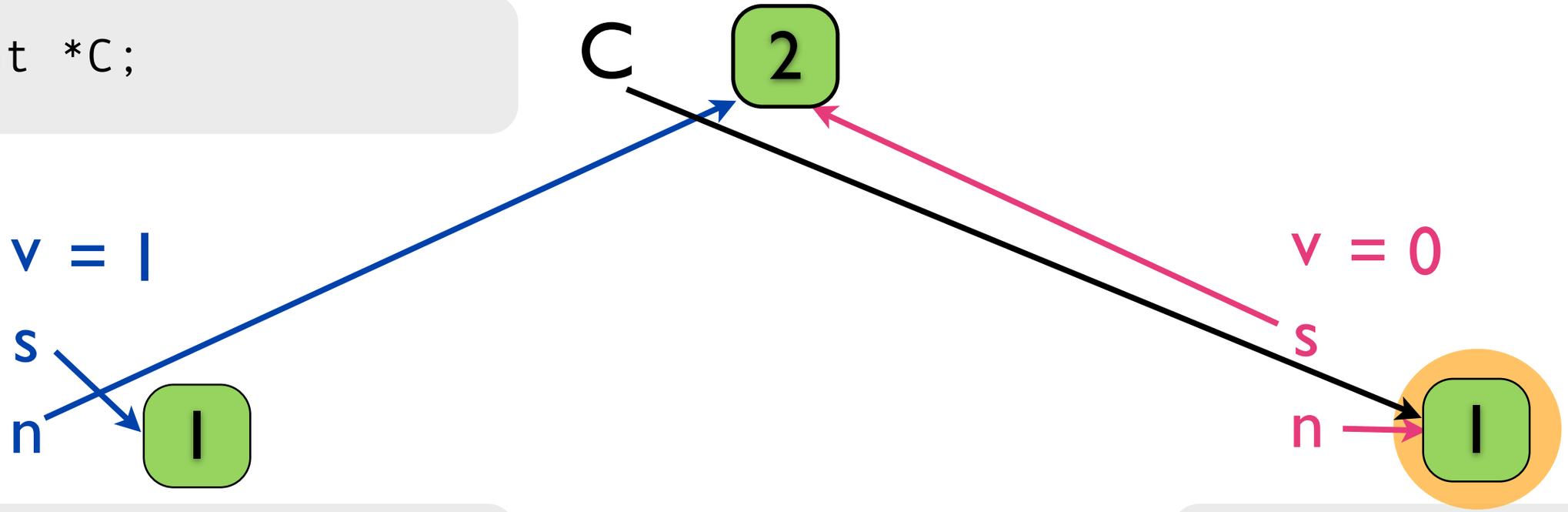


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ABA anomaly

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reclamation algorithms (paradigms)

- **Hazard Pointers** [Michael'02]
- **RCU** [McKenney+'98]
- **Epoch** [Fraser+'03]

main insight

- **Hazard Pointers** [Michael'02]
- **RCU** [McKenney+'98]
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- pattern of temporal synchronization invariant
  - grace periods
- same logical idiom for invariant preservation

main contributions

- exposing algorithmic core
  - pattern for spatial-temporal synchronization
  - unified view based on grace periods

- pattern-based modular verification
  - past temporal separation logic (*since*)
  - uniform proofs of realistic algorithms

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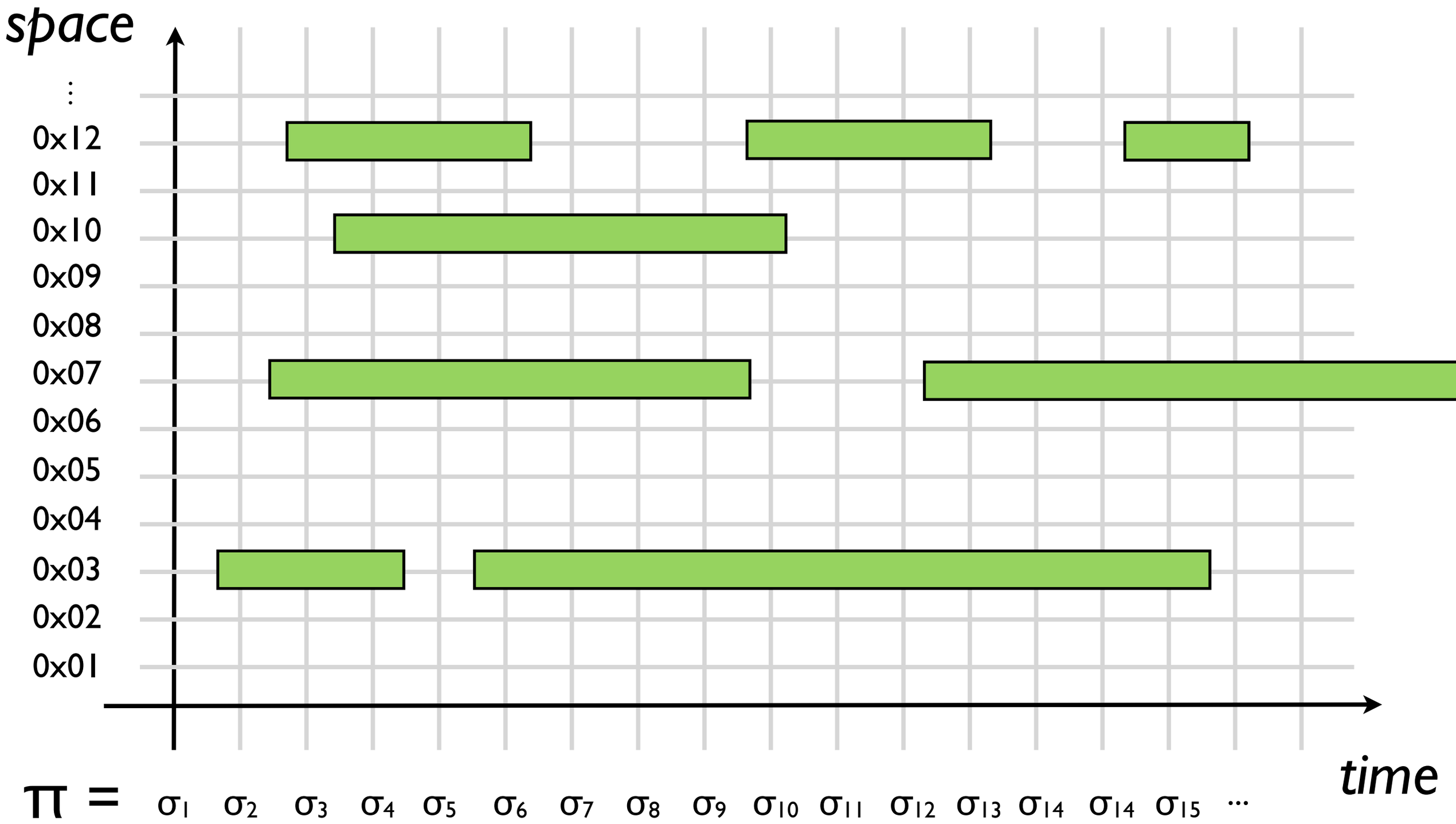
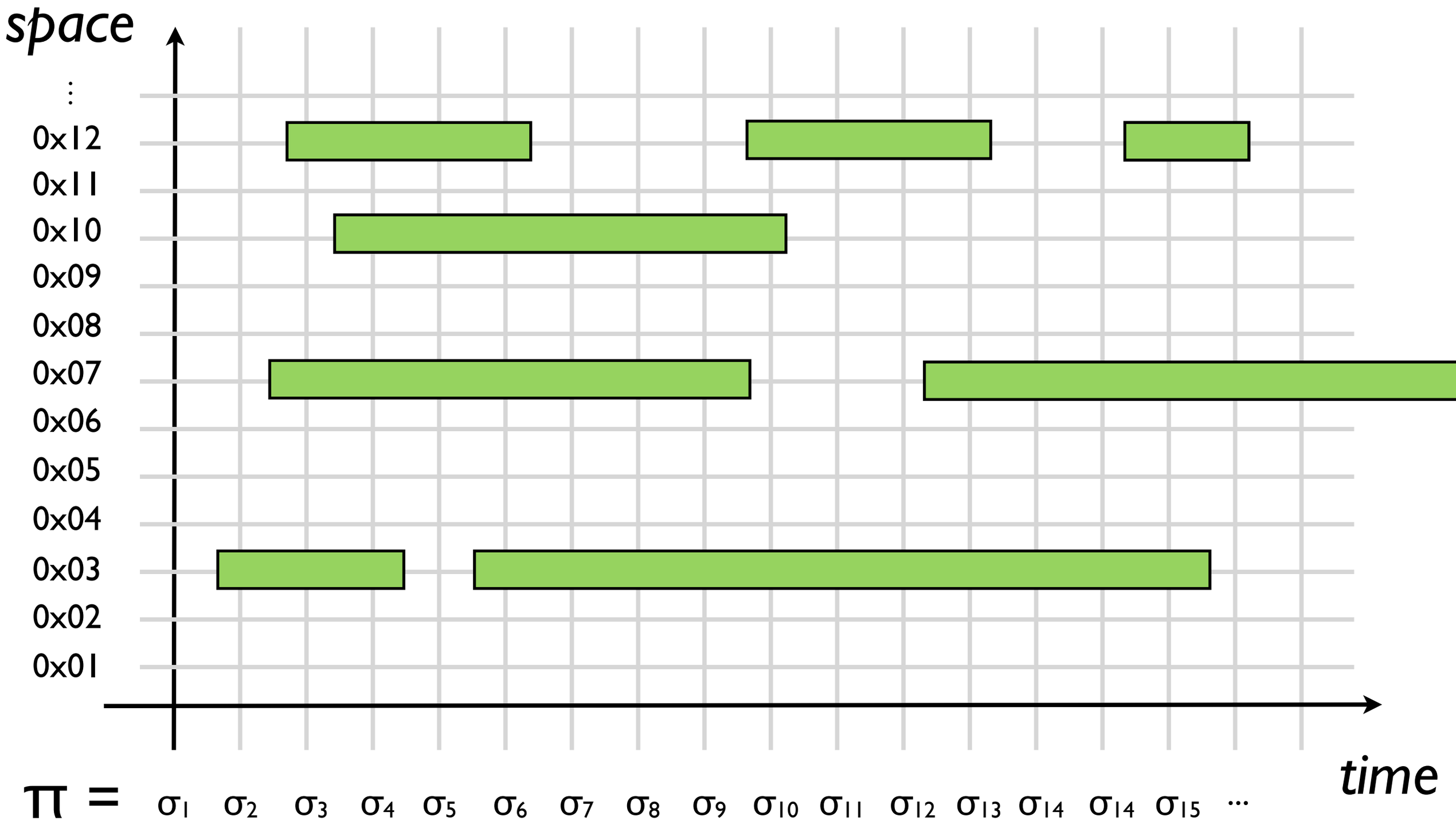
dual views: space / time



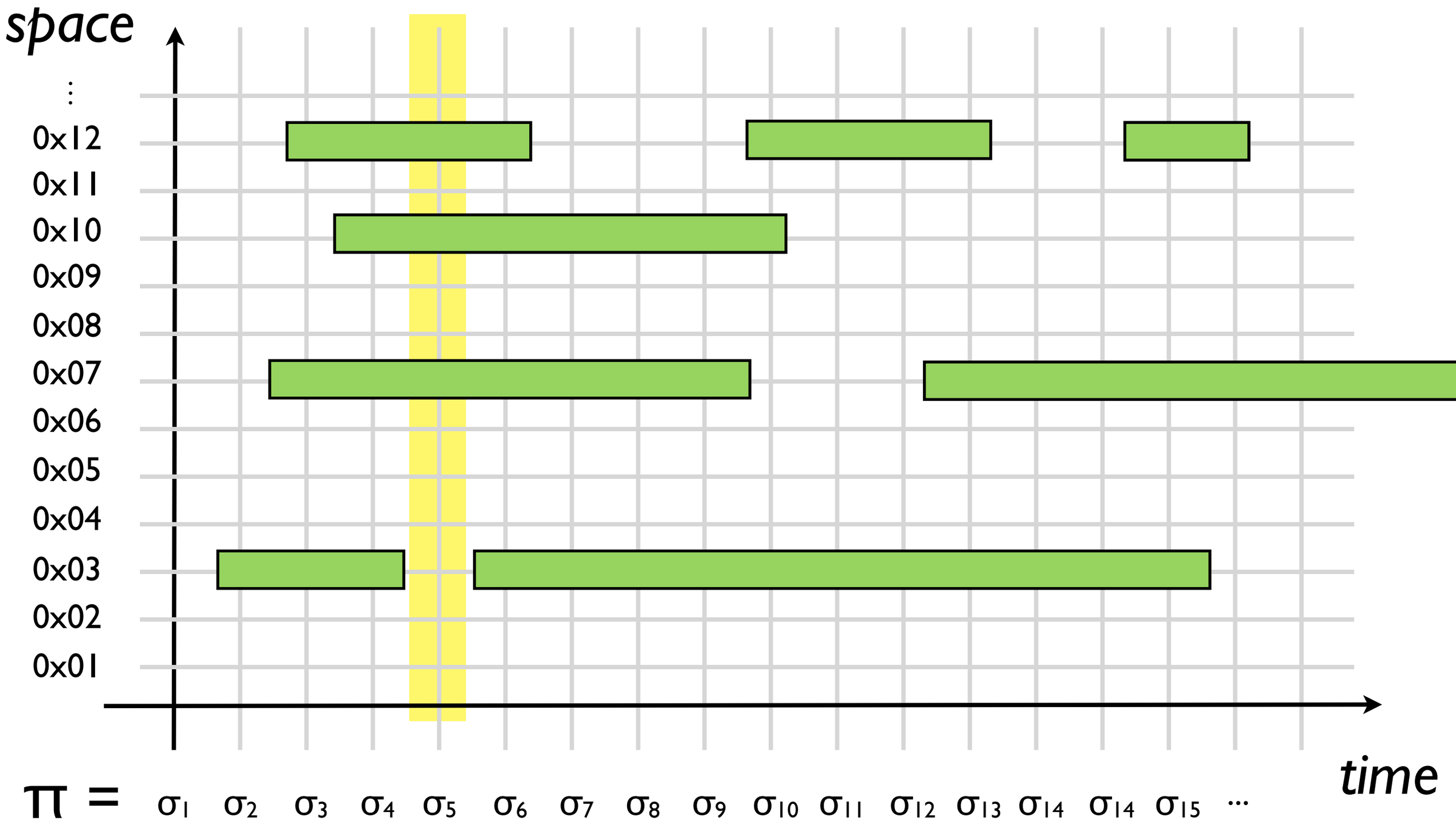
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

dual views: space / time



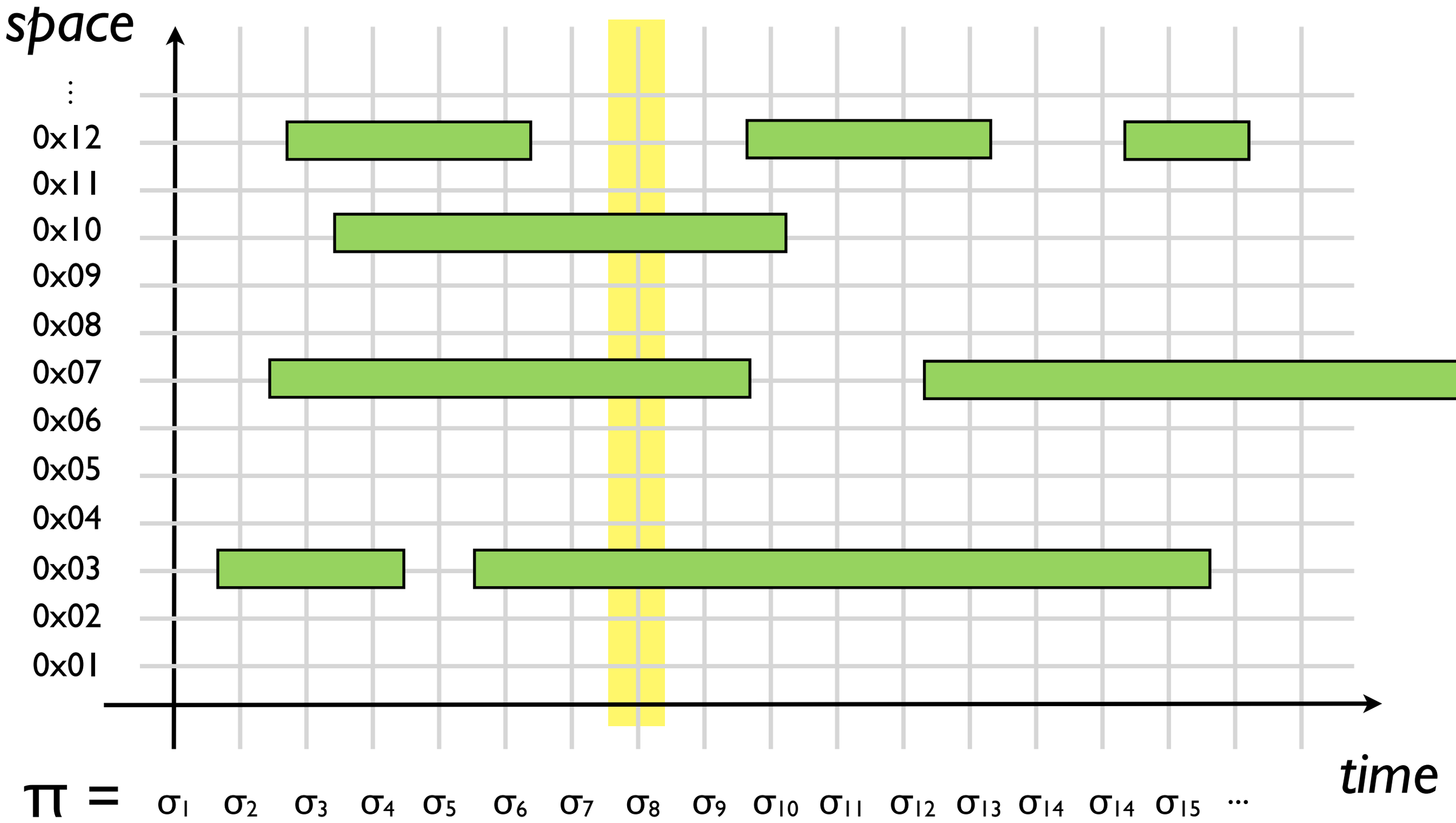
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

dual views: space / time



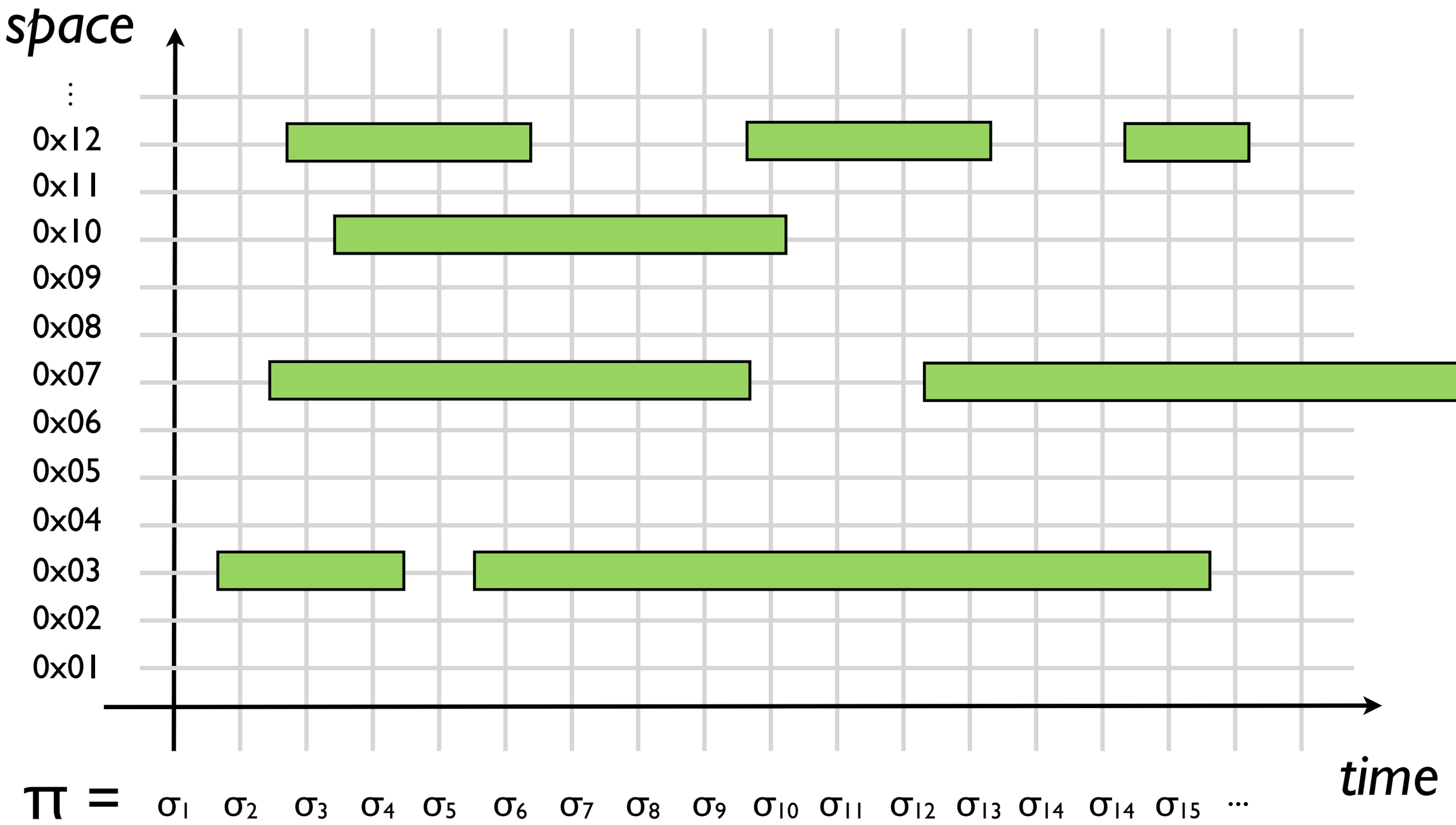
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

dual views: space / time



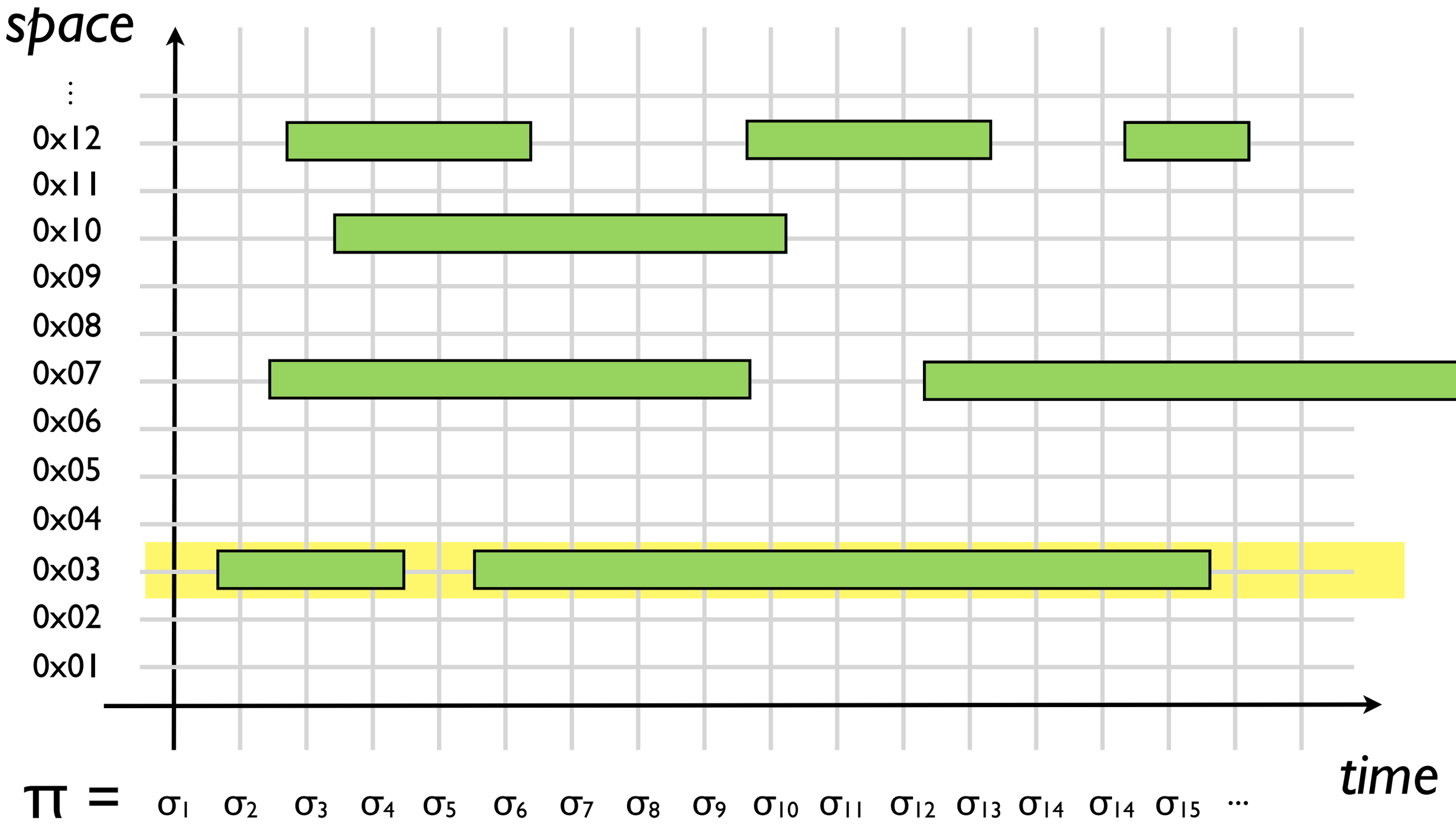
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

dual views: space / time



$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

dual views: space / time



$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

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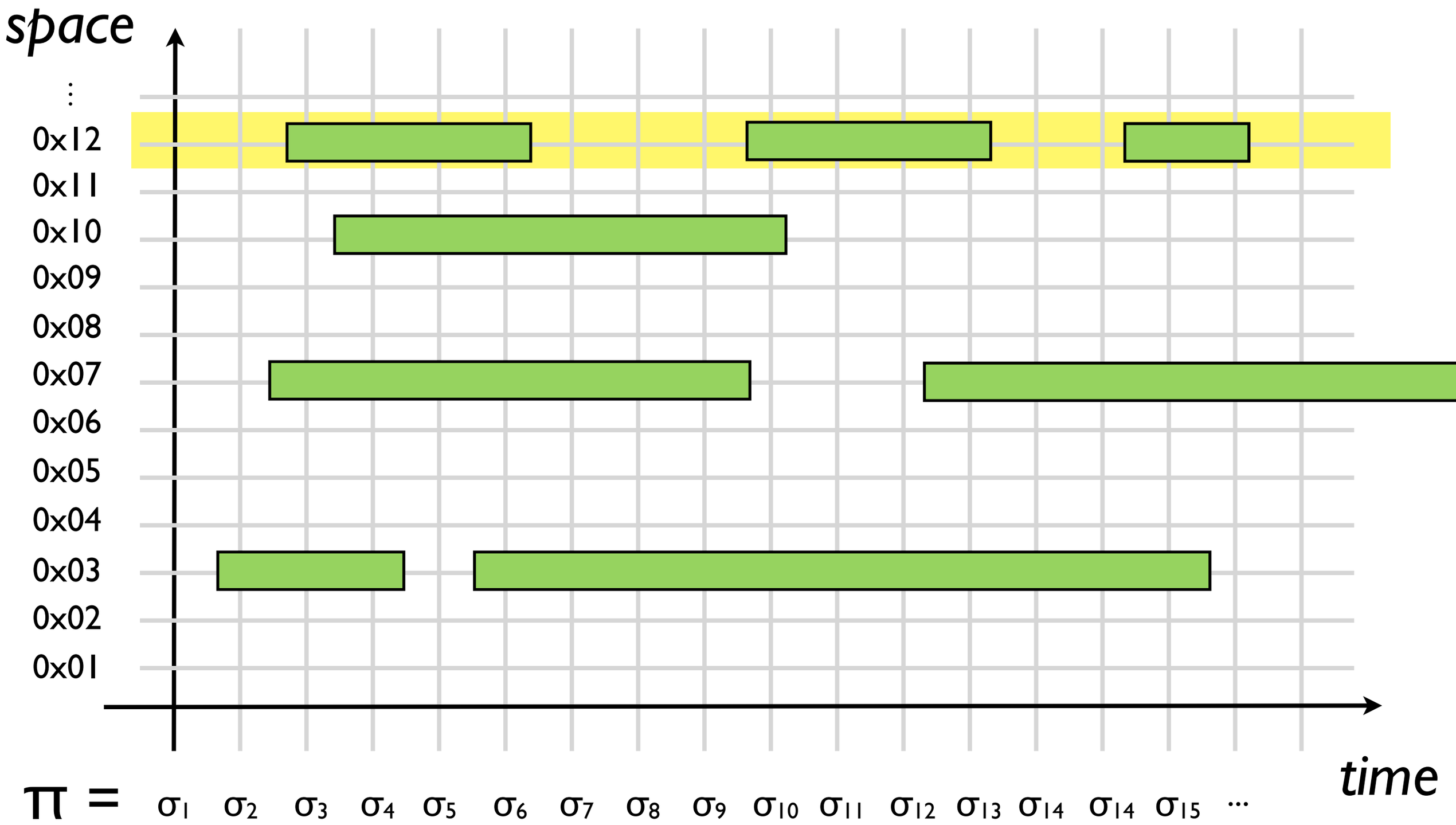
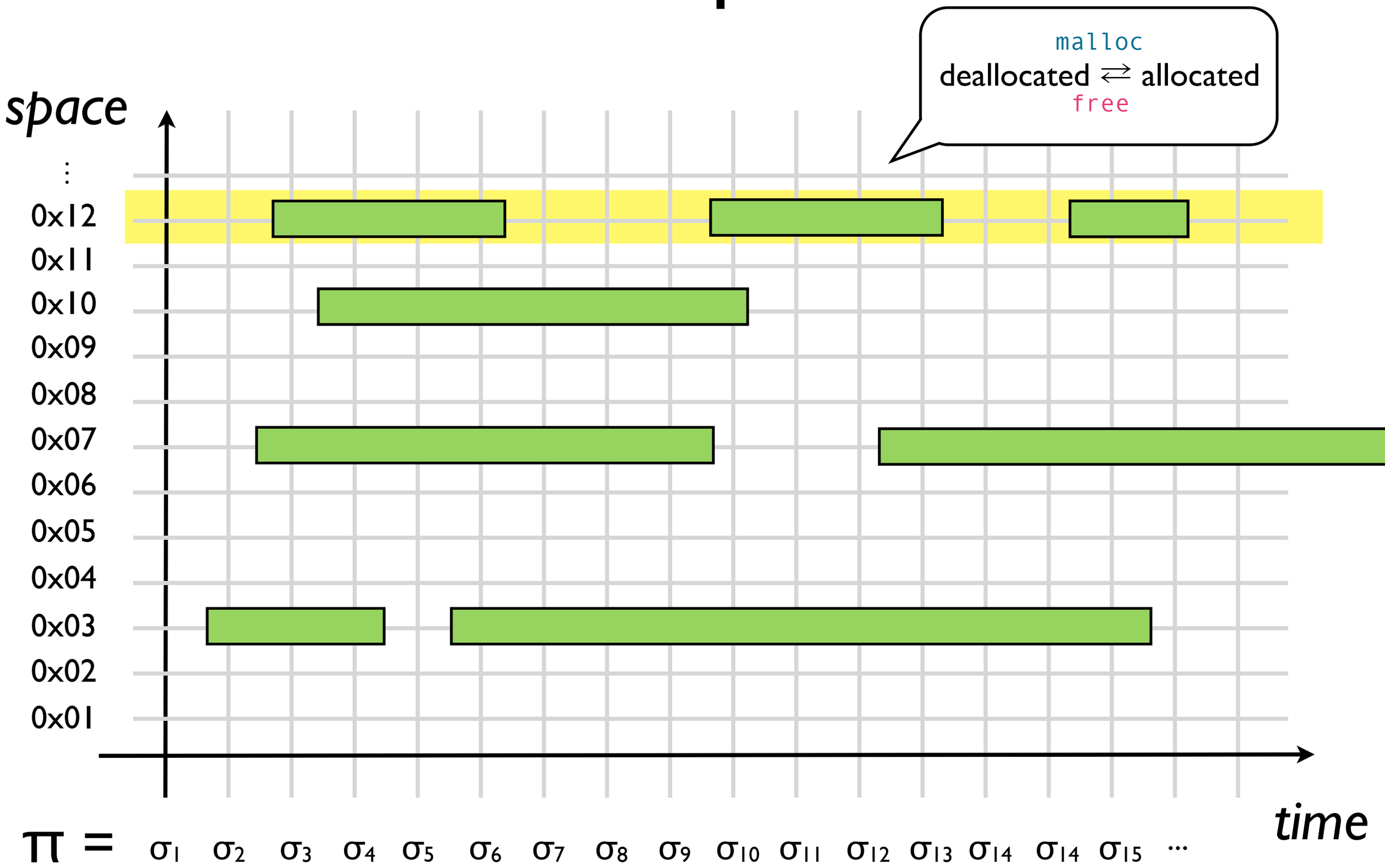


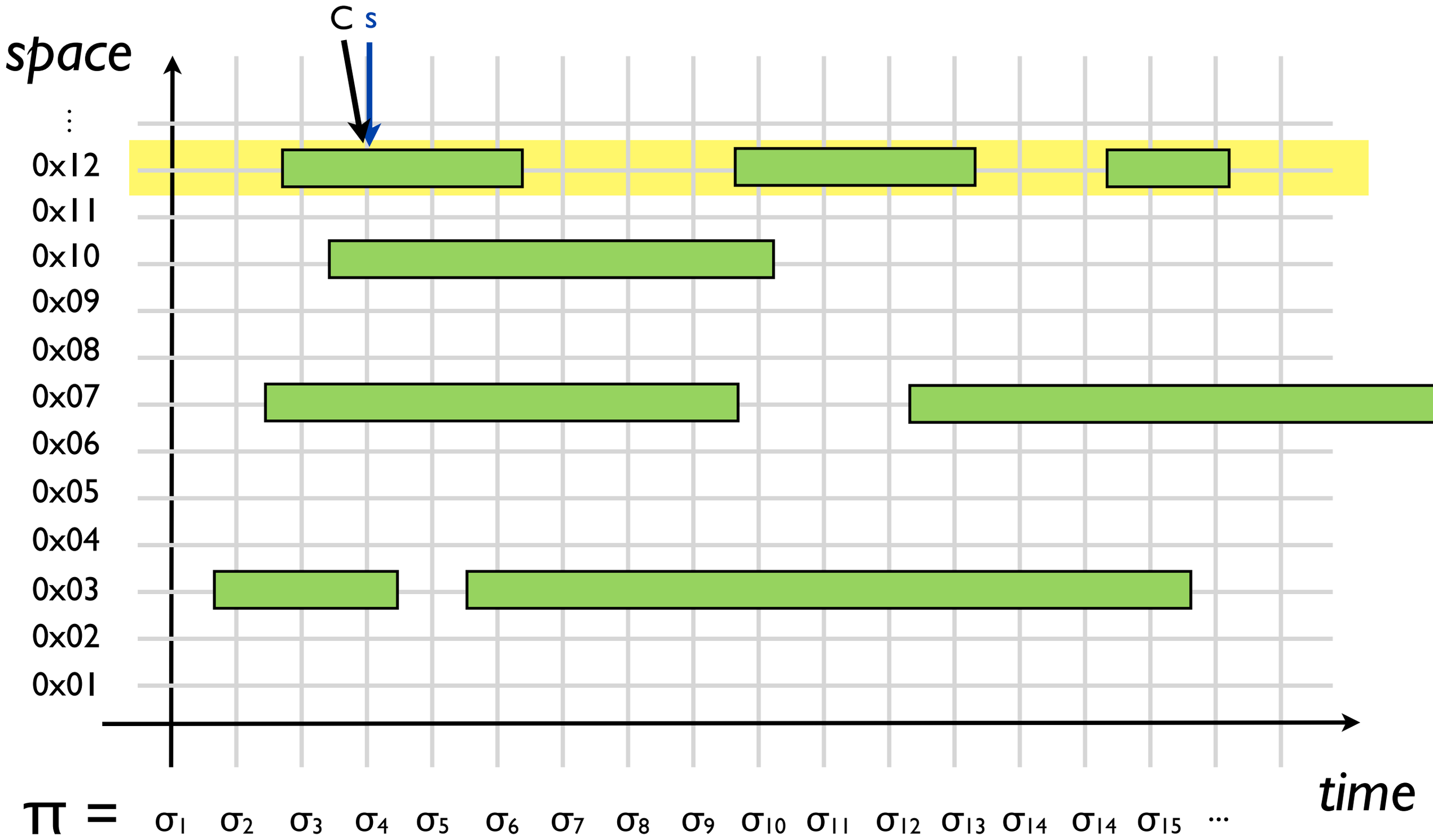
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

dual views: space / time



$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \cdots$

memory safety (temporal view)



$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

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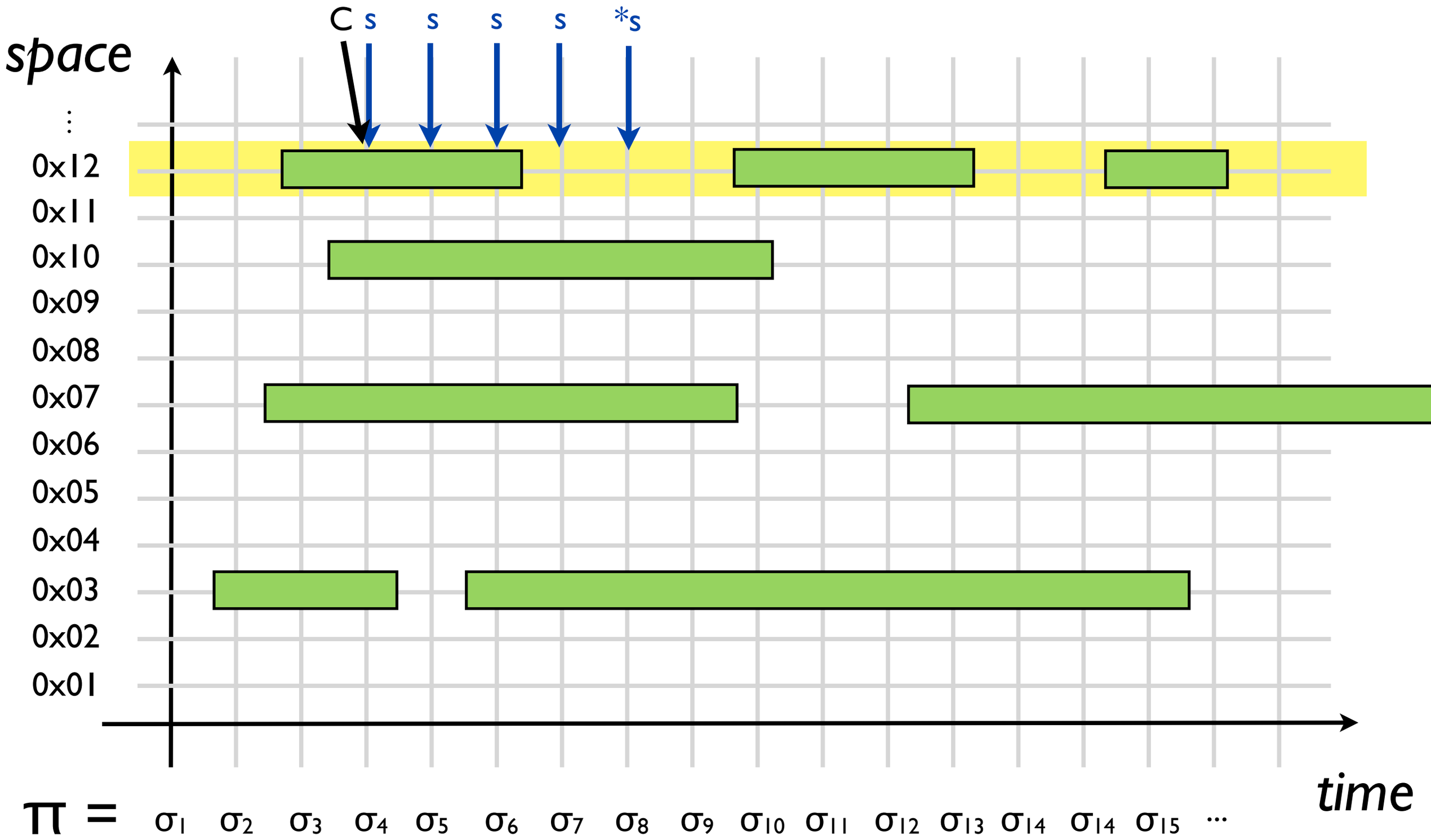
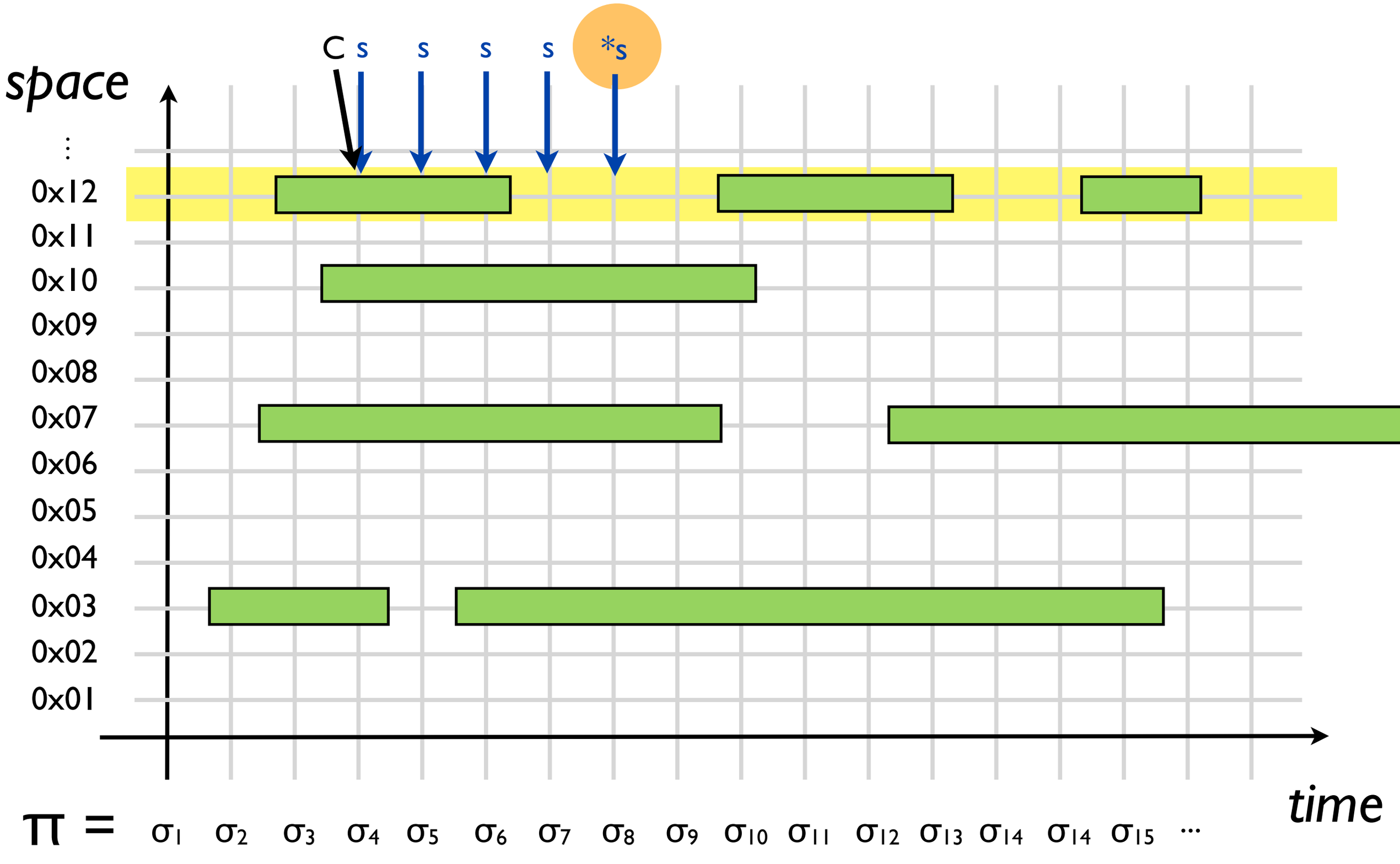


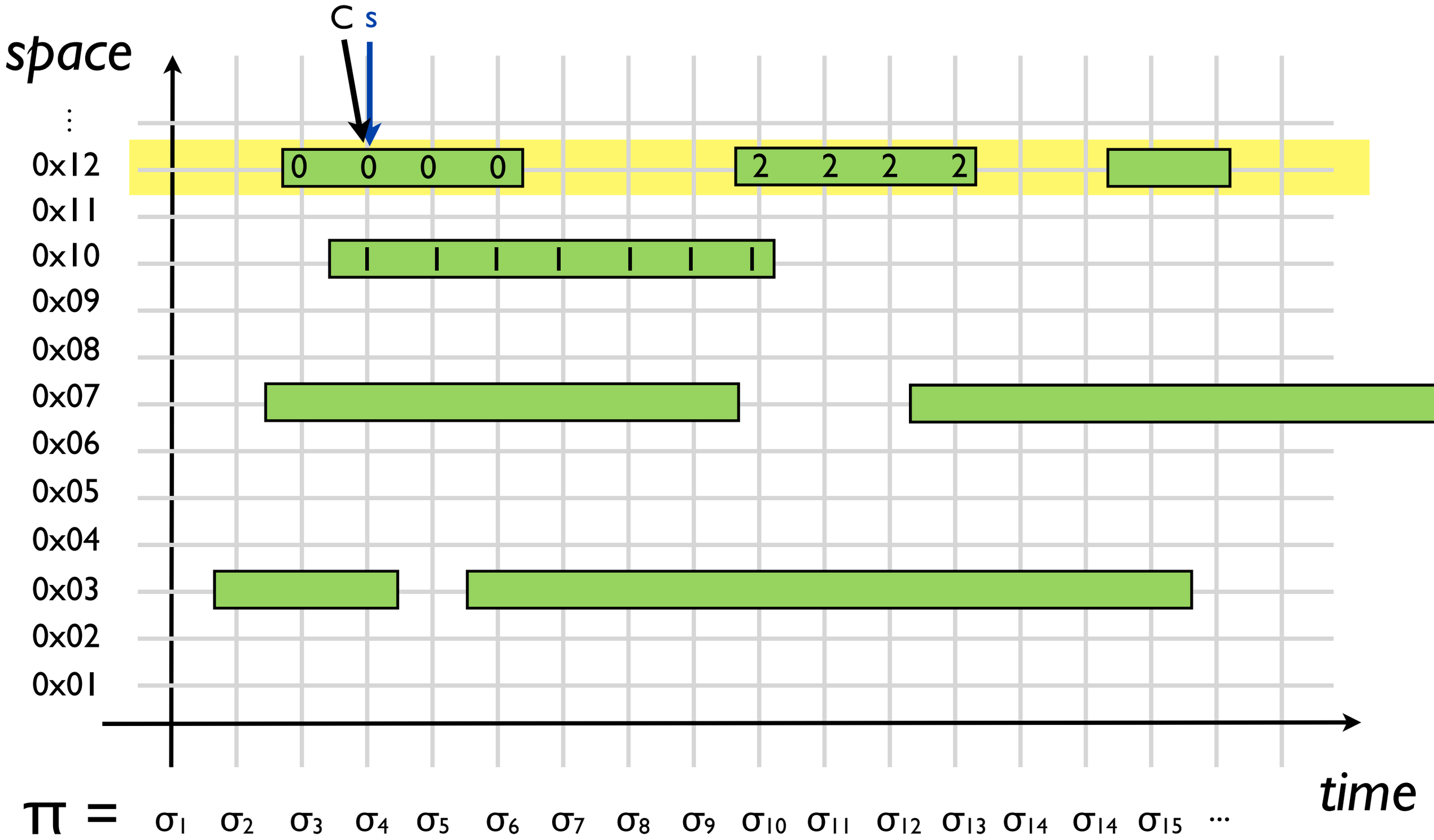
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

memory safety (temporal view)



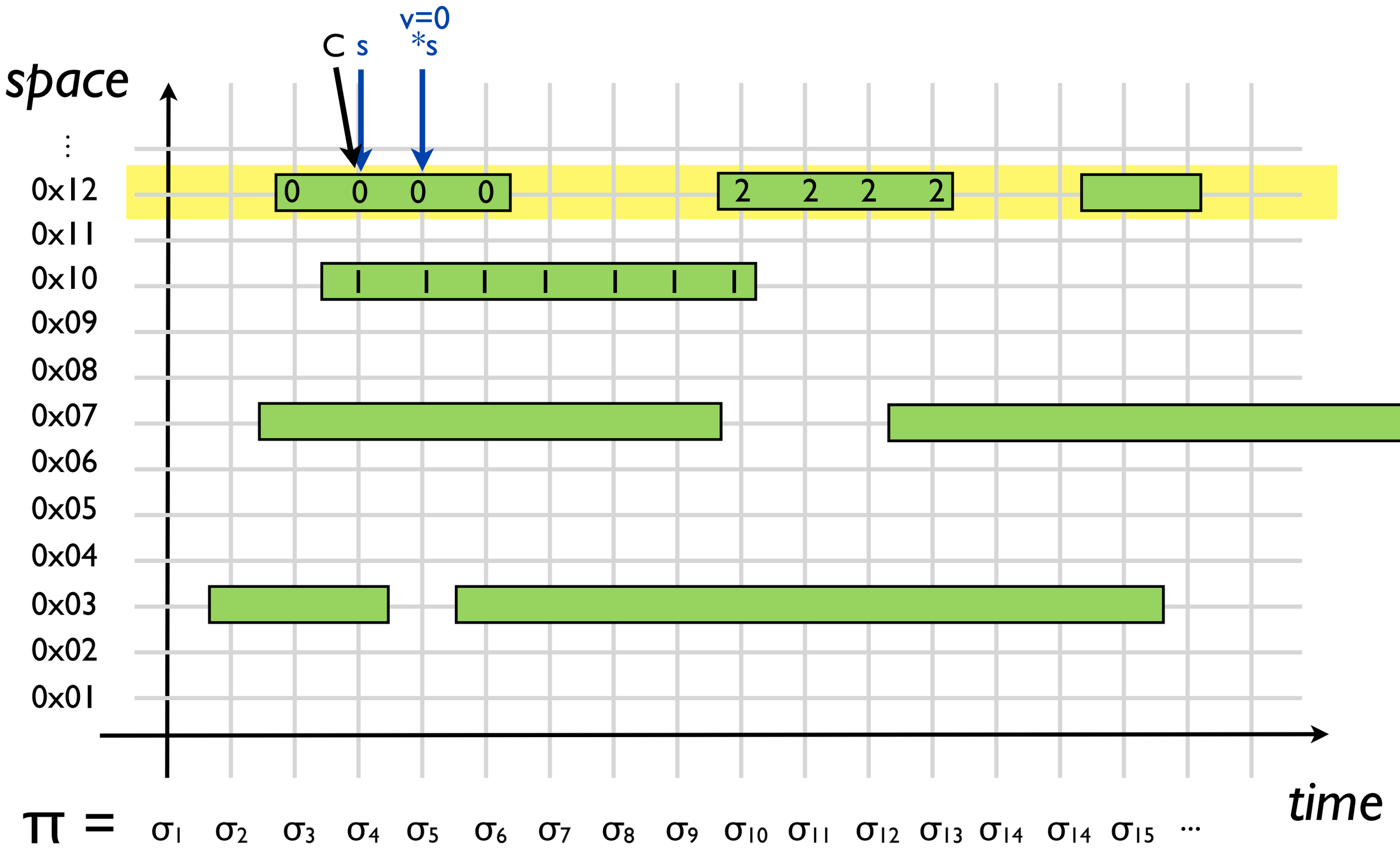
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \cdots$

ABA (temporal view)



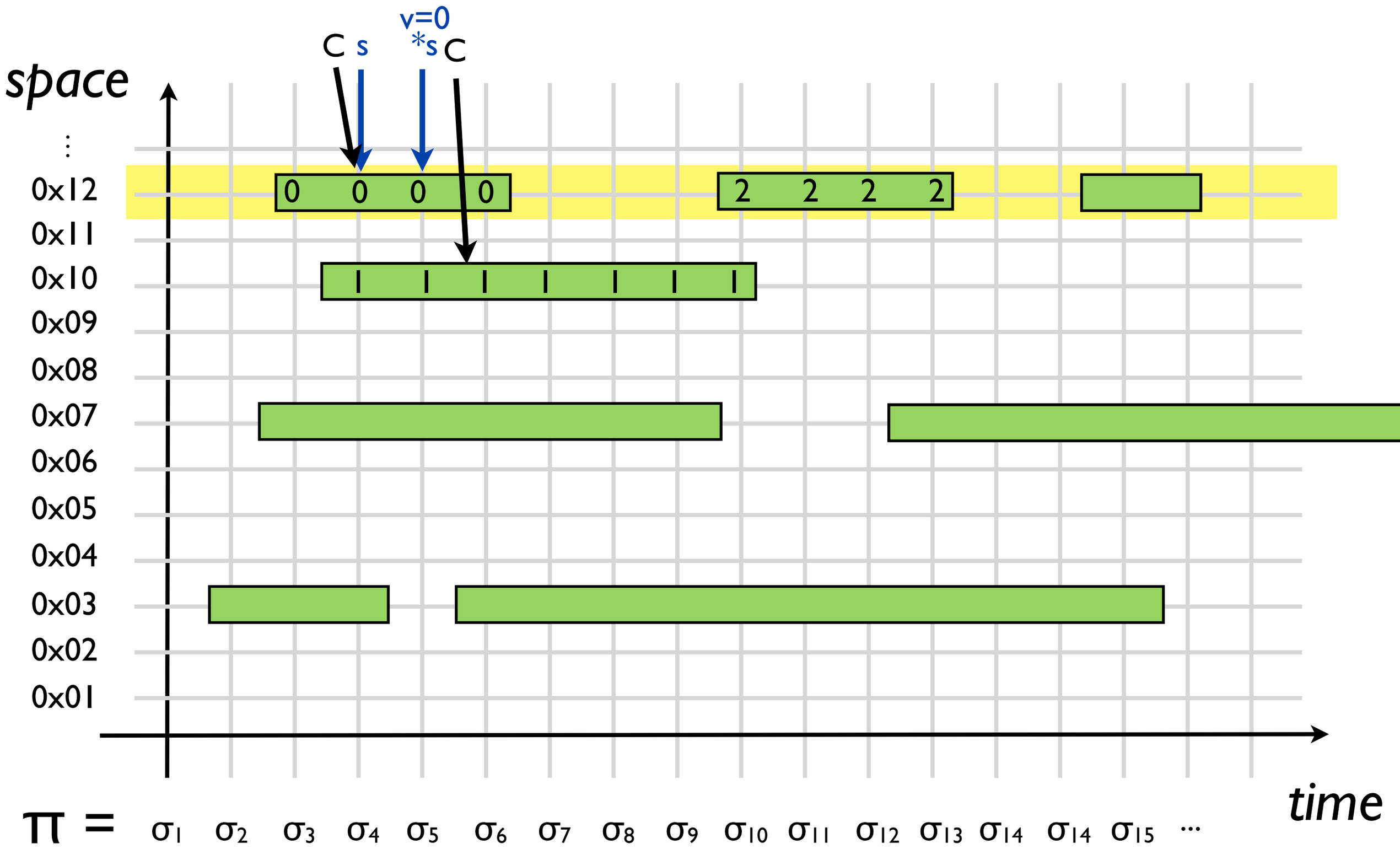
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

ABA (temporal view)



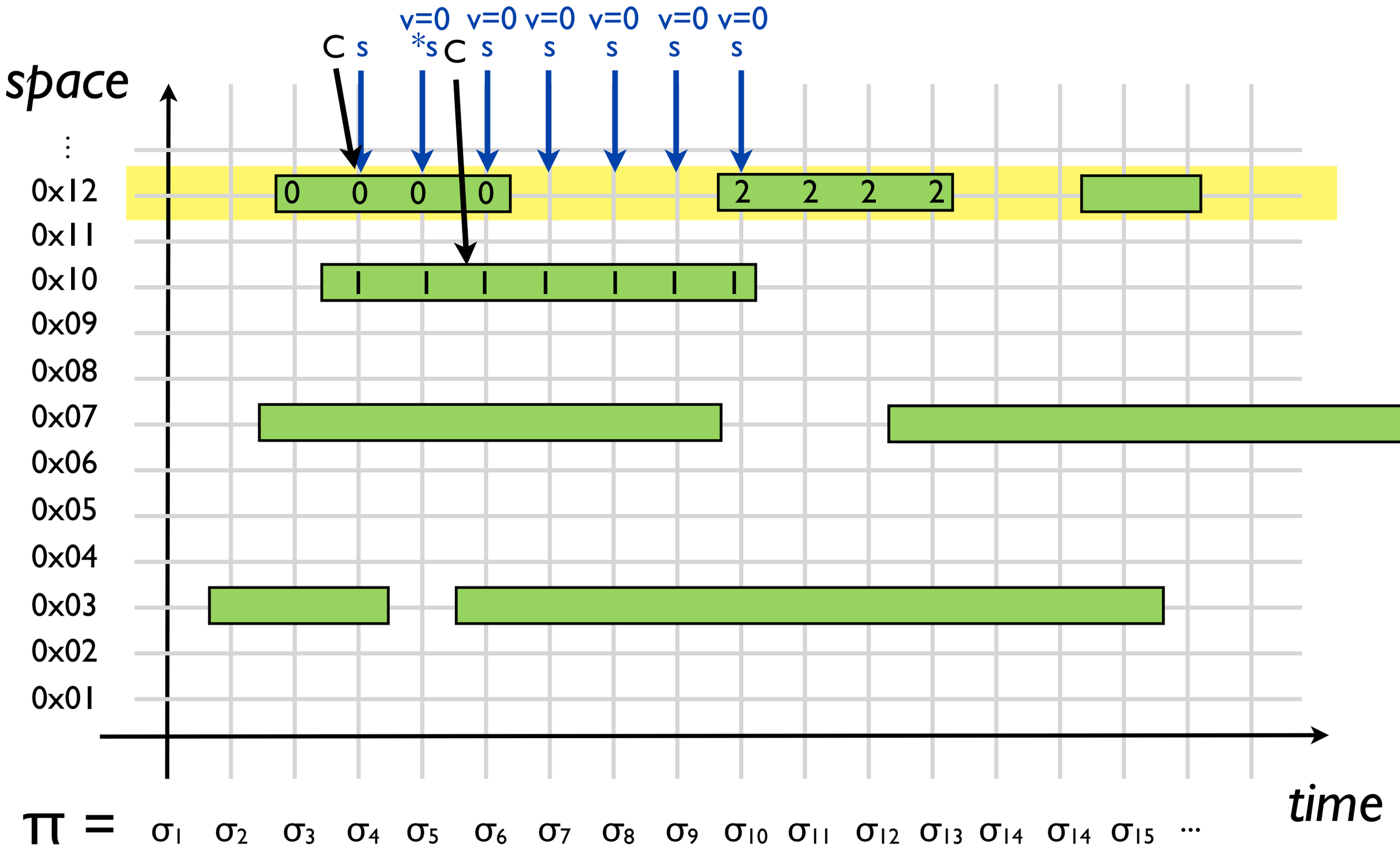
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \cdots$

ABA (temporal view)



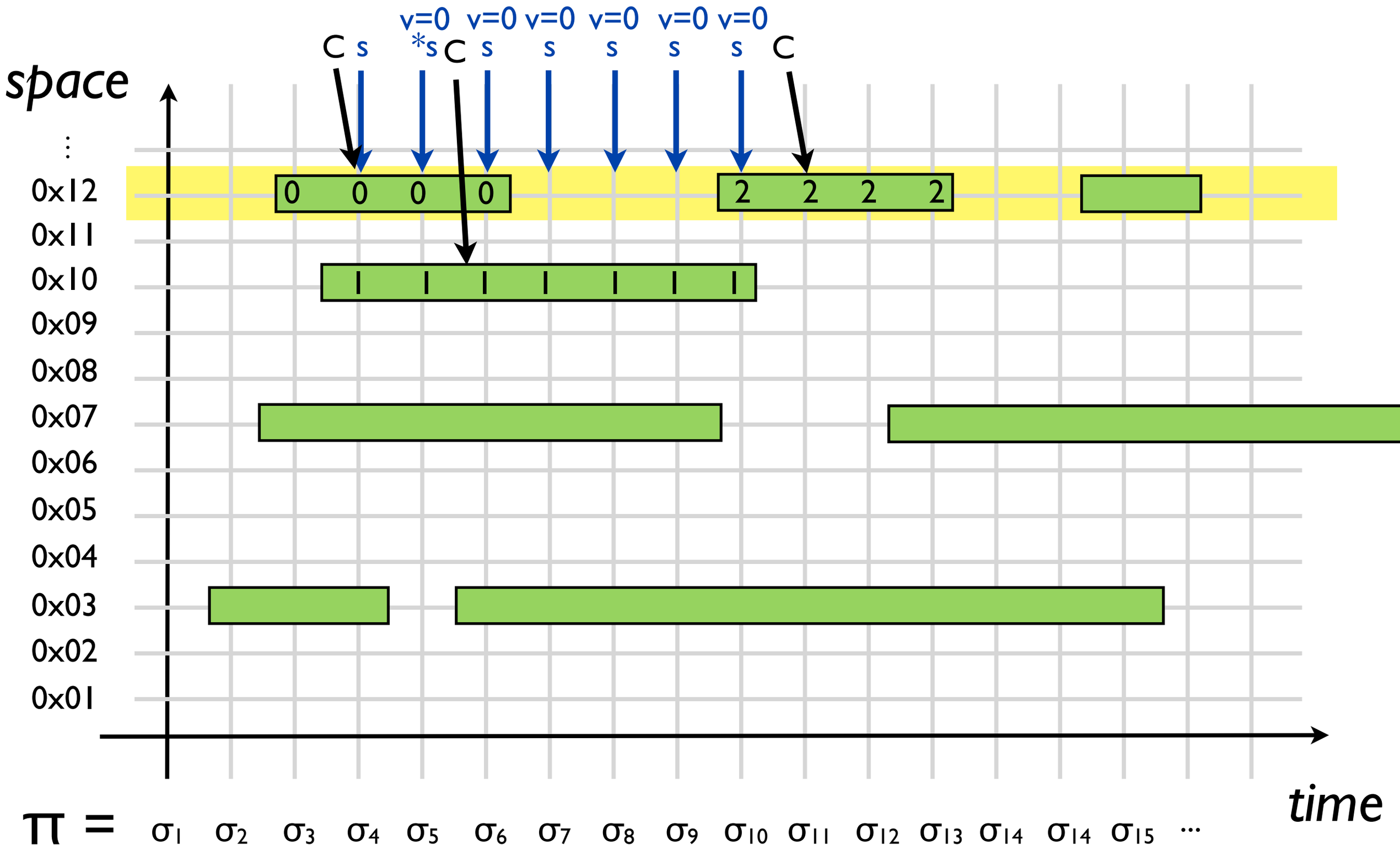
$\pi =$ σ_1 σ_2 σ_3 σ_4 σ_5 σ_6 σ_7 σ_8 σ_9 σ_{10} σ_{11} σ_{12} σ_{13} σ_{14} σ_{14} σ_{15} …

ABA (temporal view)



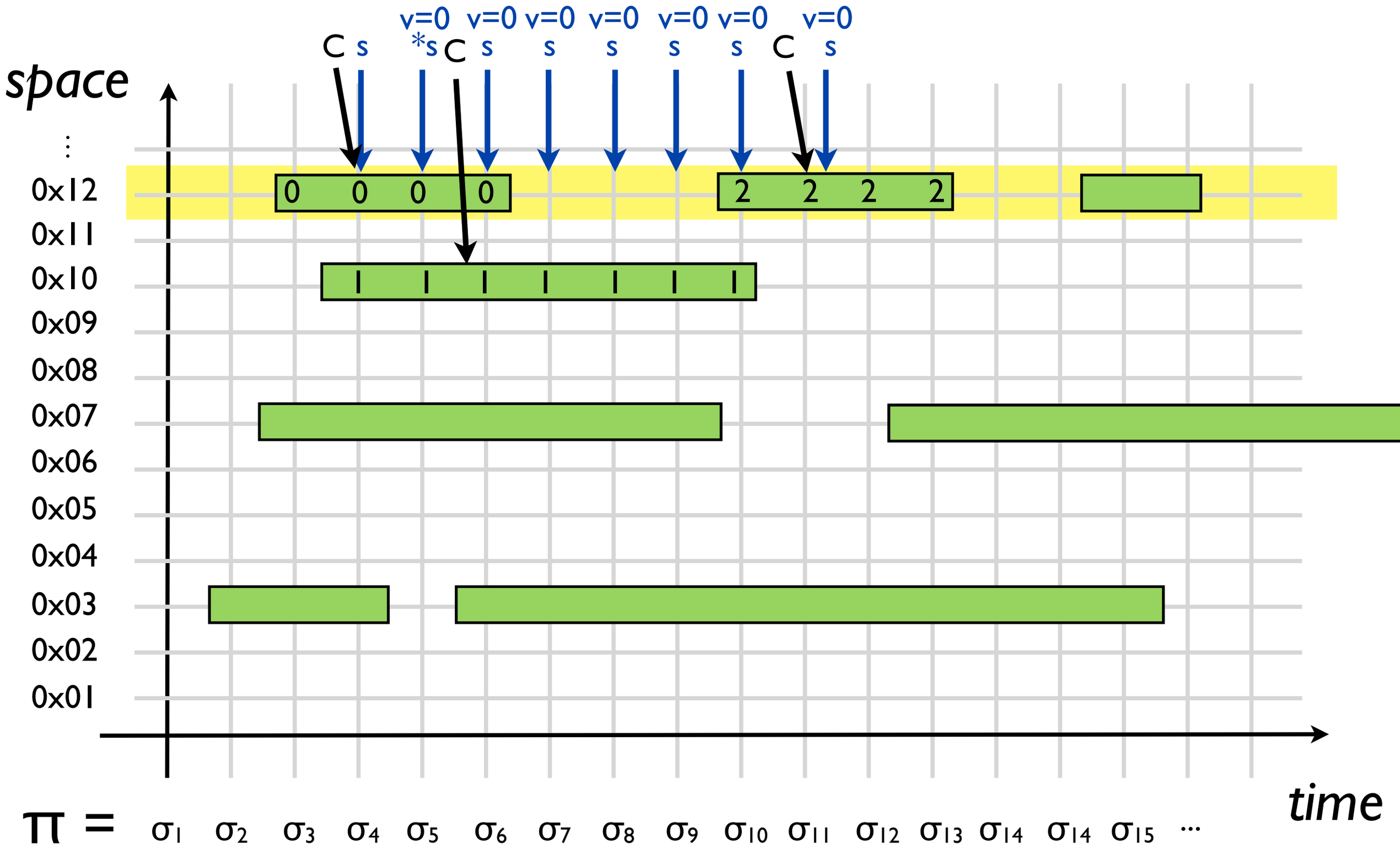
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

ABA (temporal view)



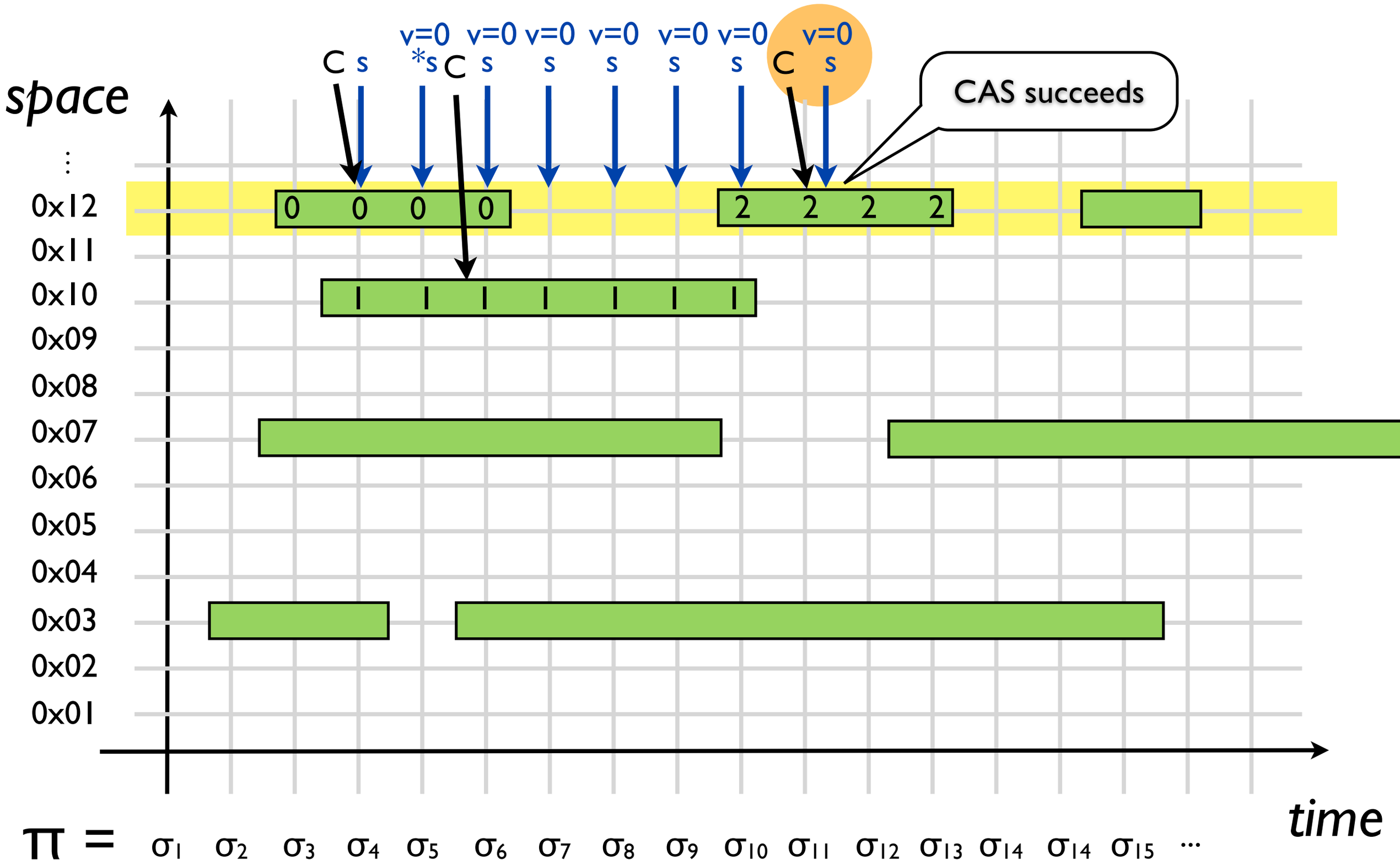
$\pi = \sigma_1\ \sigma_2\ \sigma_3\ \sigma_4\ \sigma_5\ \sigma_6\ \sigma_7\ \sigma_8\ \sigma_9\ \sigma_{10}\ \sigma_{11}\ \sigma_{12}\ \sigma_{13}\ \sigma_{14}\ \sigma_{14}\ \sigma_{15}\ \ldots$

ABA (temporal view)



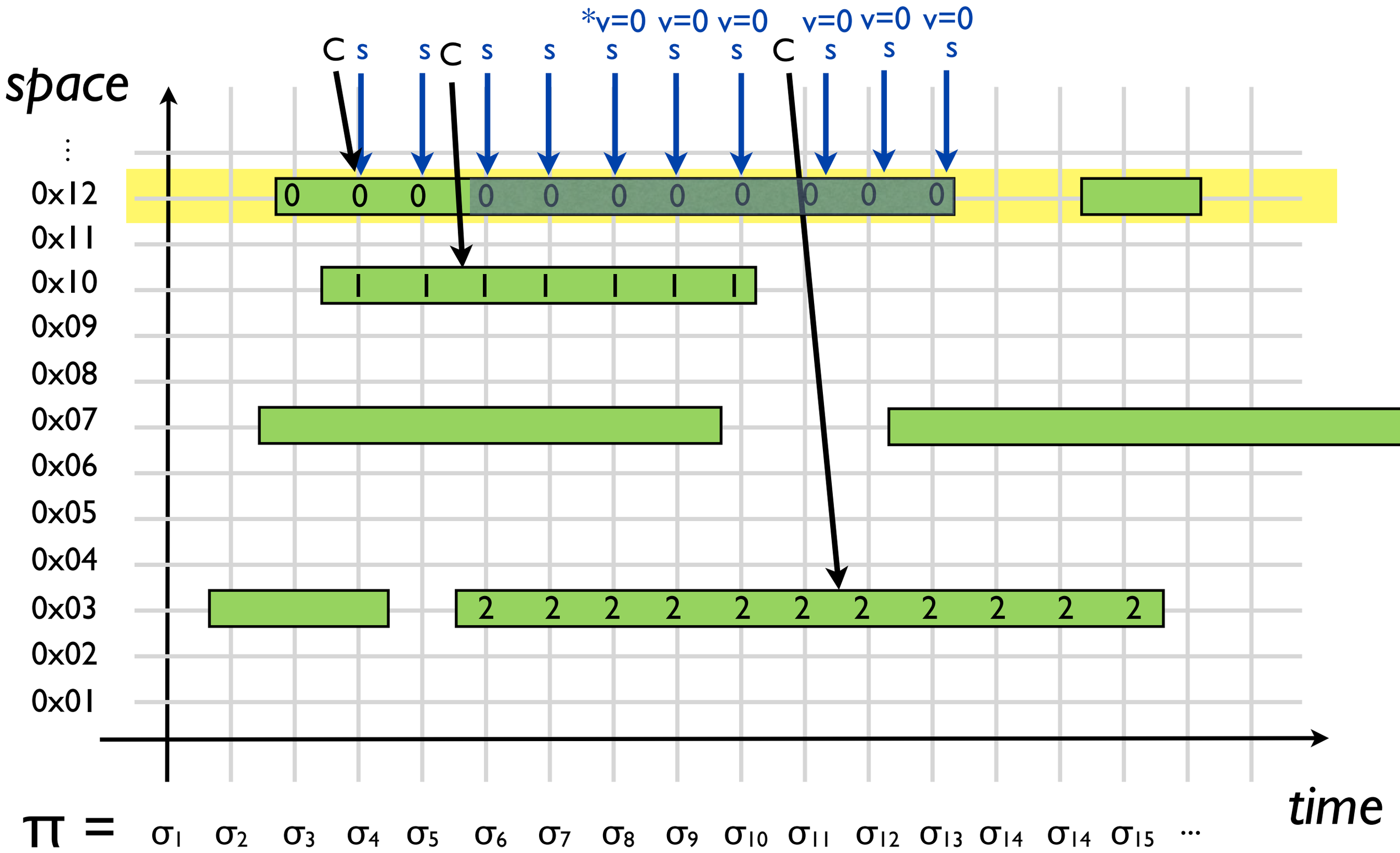
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

ABA (temporal view)



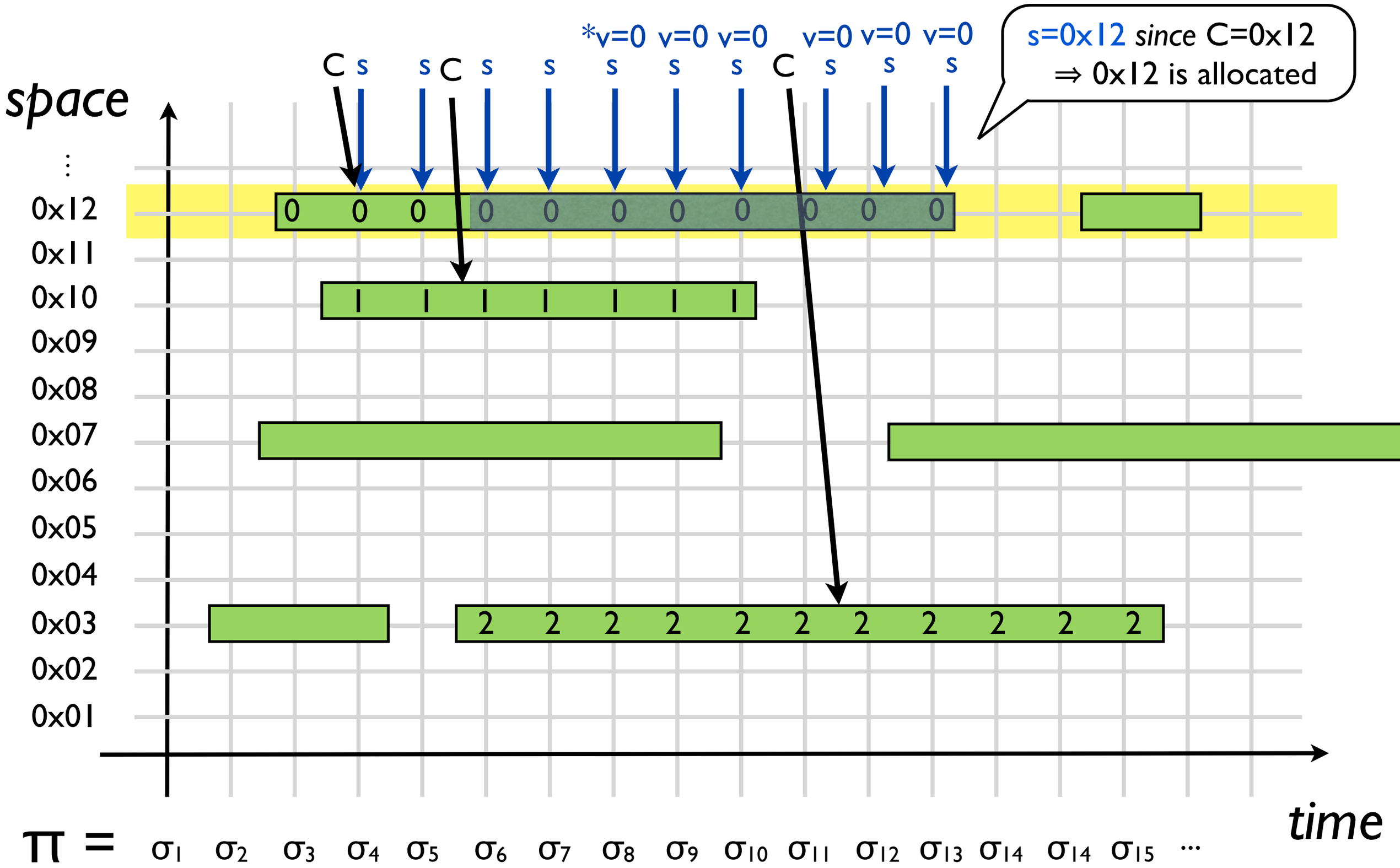
$\pi = \sigma_1\ \sigma_2\ \sigma_3\ \sigma_4\ \sigma_5\ \sigma_6\ \sigma_7\ \sigma_8\ \sigma_9\ \sigma_{10}\ \sigma_{11}\ \sigma_{12}\ \sigma_{13}\ \sigma_{14}\ \sigma_{14}\ \sigma_{15}\ \ldots$

what we want, ideally



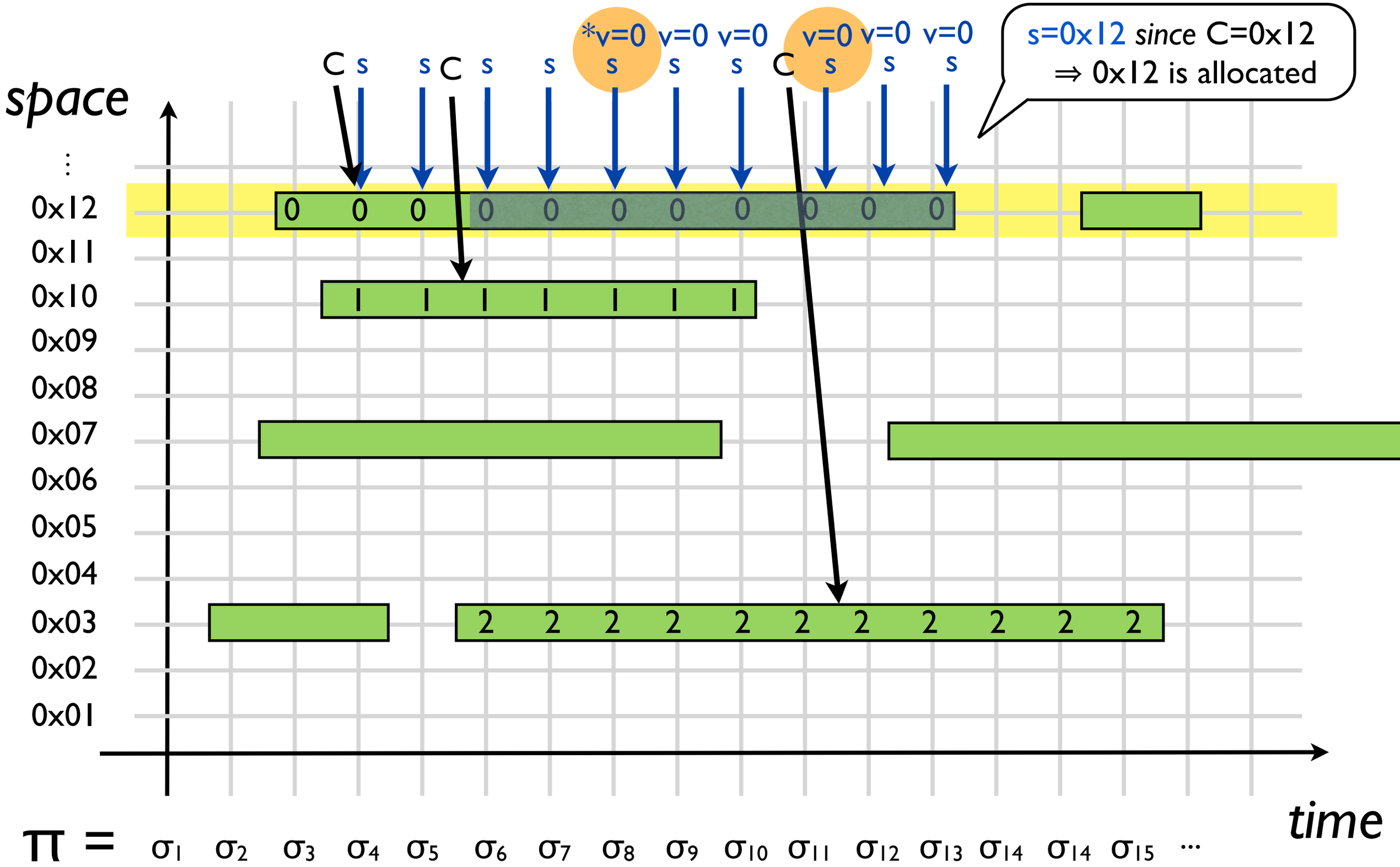
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \cdots$

what we want, ideally



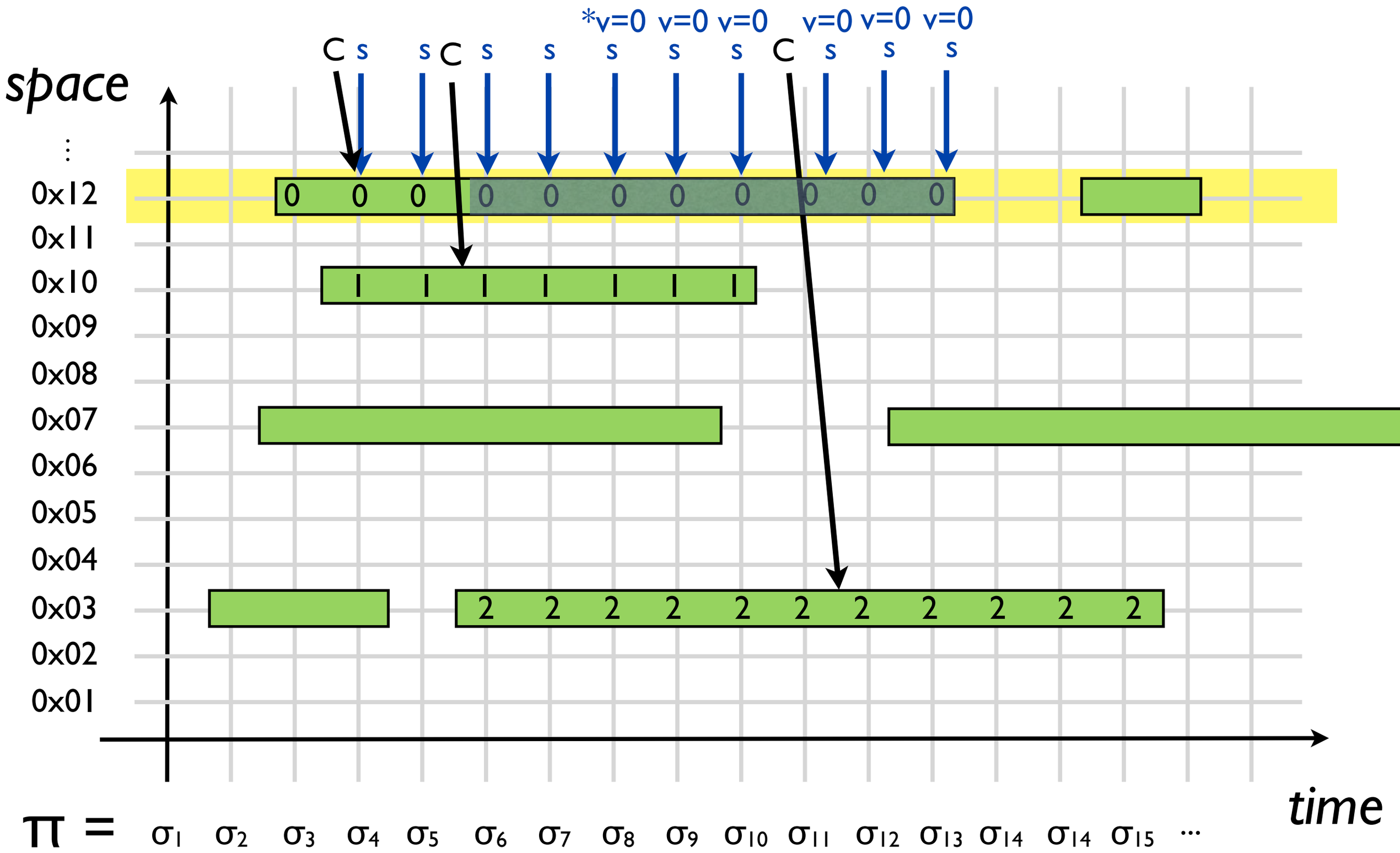
$\pi = \sigma_1\ \sigma_2\ \sigma_3\ \sigma_4\ \sigma_5\ \sigma_6\ \sigma_7\ \sigma_8\ \sigma_9\ \sigma_{10}\ \sigma_{11}\ \sigma_{12}\ \sigma_{13}\ \sigma_{14}\ \sigma_{14}\ \sigma_{15}\ \cdots$

what we want, ideally



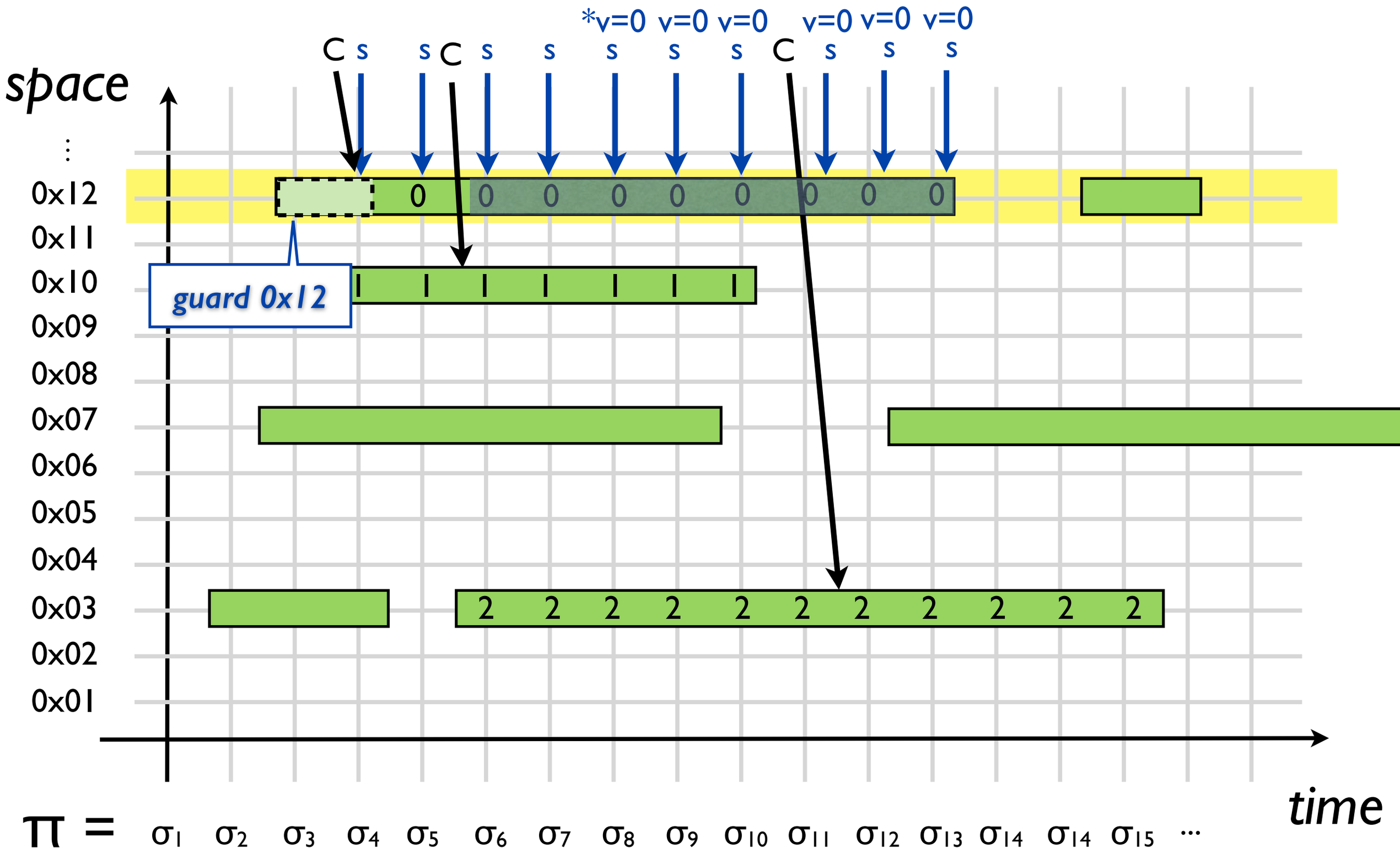
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \cdots$

grace-based synchronization



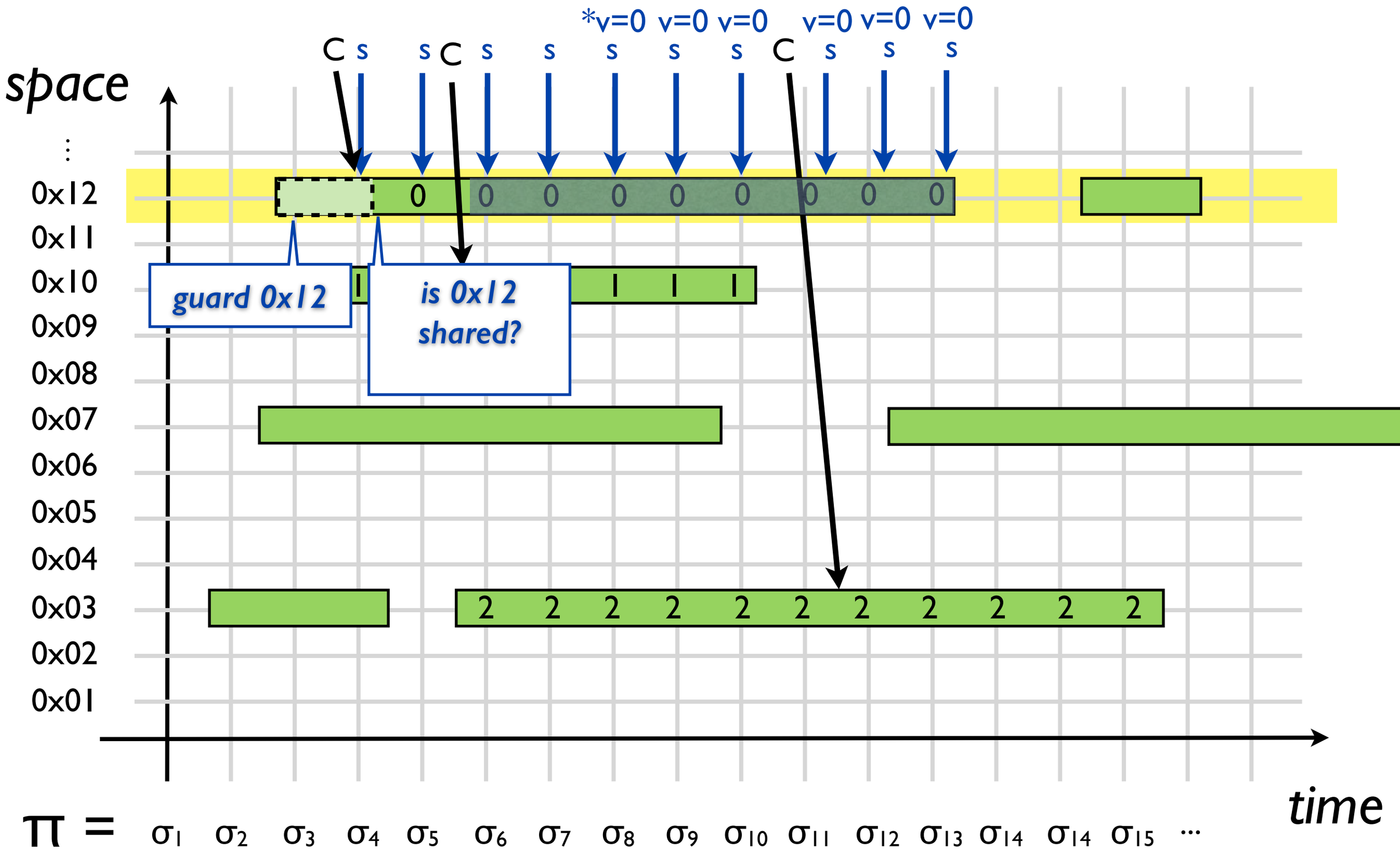
$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \cdots$

grace-based synchronization



$\pi = \sigma_1\ \sigma_2\ \sigma_3\ \sigma_4\ \sigma_5\ \sigma_6\ \sigma_7\ \sigma_8\ \sigma_9\ \sigma_{10}\ \sigma_{11}\ \sigma_{12}\ \sigma_{13}\ \sigma_{14}\ \sigma_{14}\ \sigma_{15}\ \cdots$

grace-based synchronization



$\pi = \sigma_1 \ \sigma_2 \ \sigma_3 \ \sigma_4 \ \sigma_5 \ \sigma_6 \ \sigma_7 \ \sigma_8 \ \sigma_9 \ \sigma_{10} \ \sigma_{11} \ \sigma_{12} \ \sigma_{13} \ \sigma_{14} \ \sigma_{14} \ \sigma_{15} \ \ldots$

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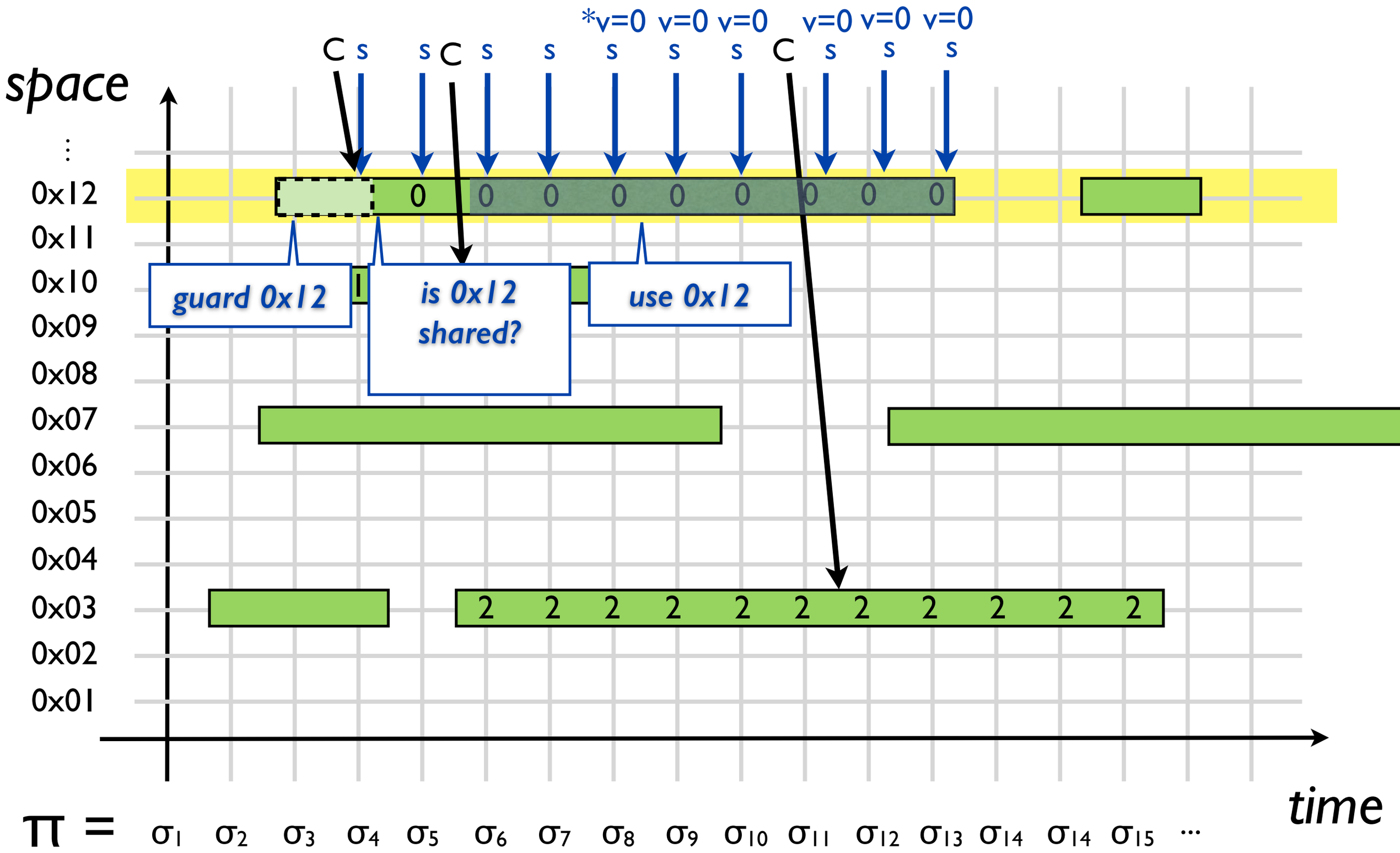

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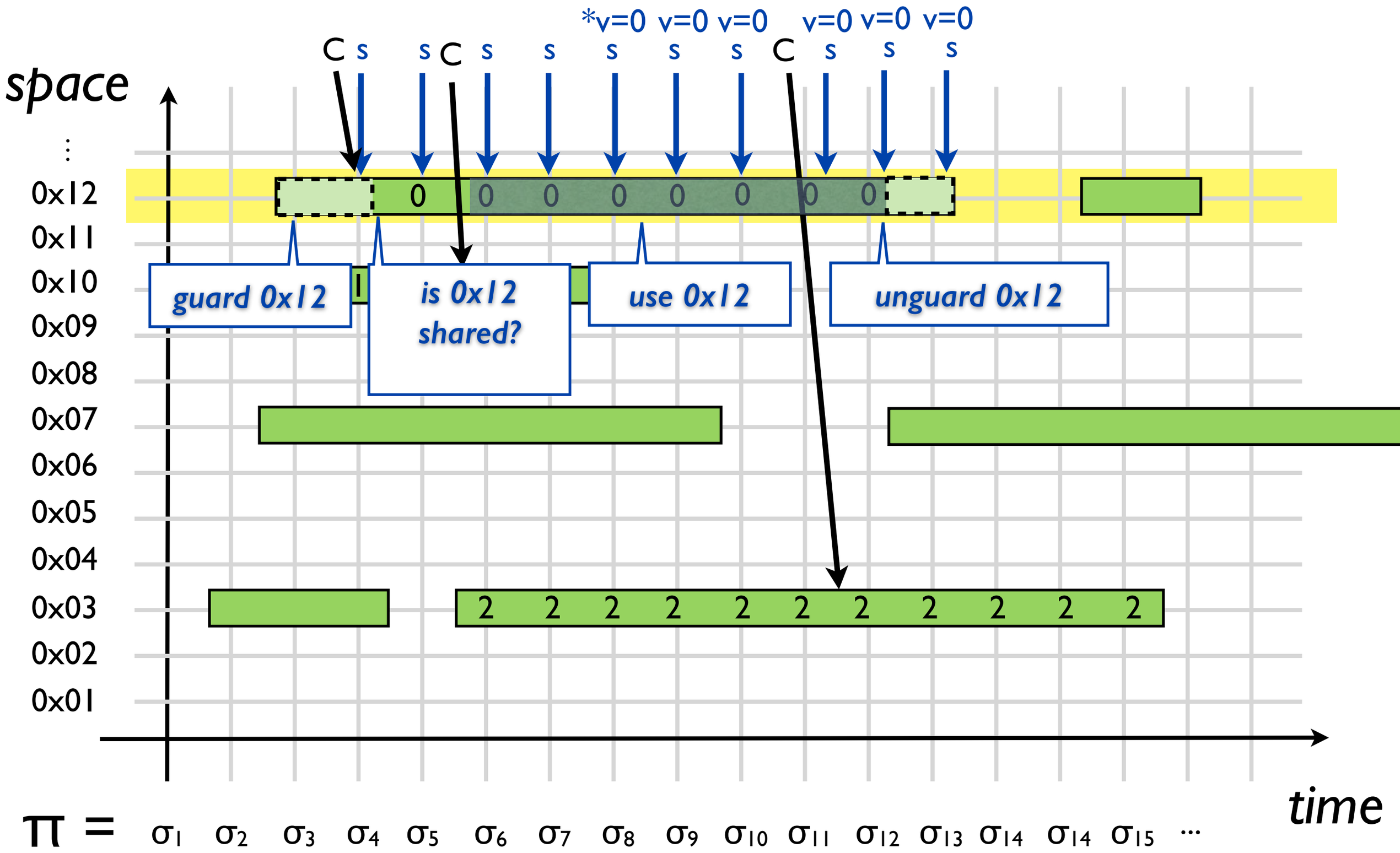
grace-based synchronization



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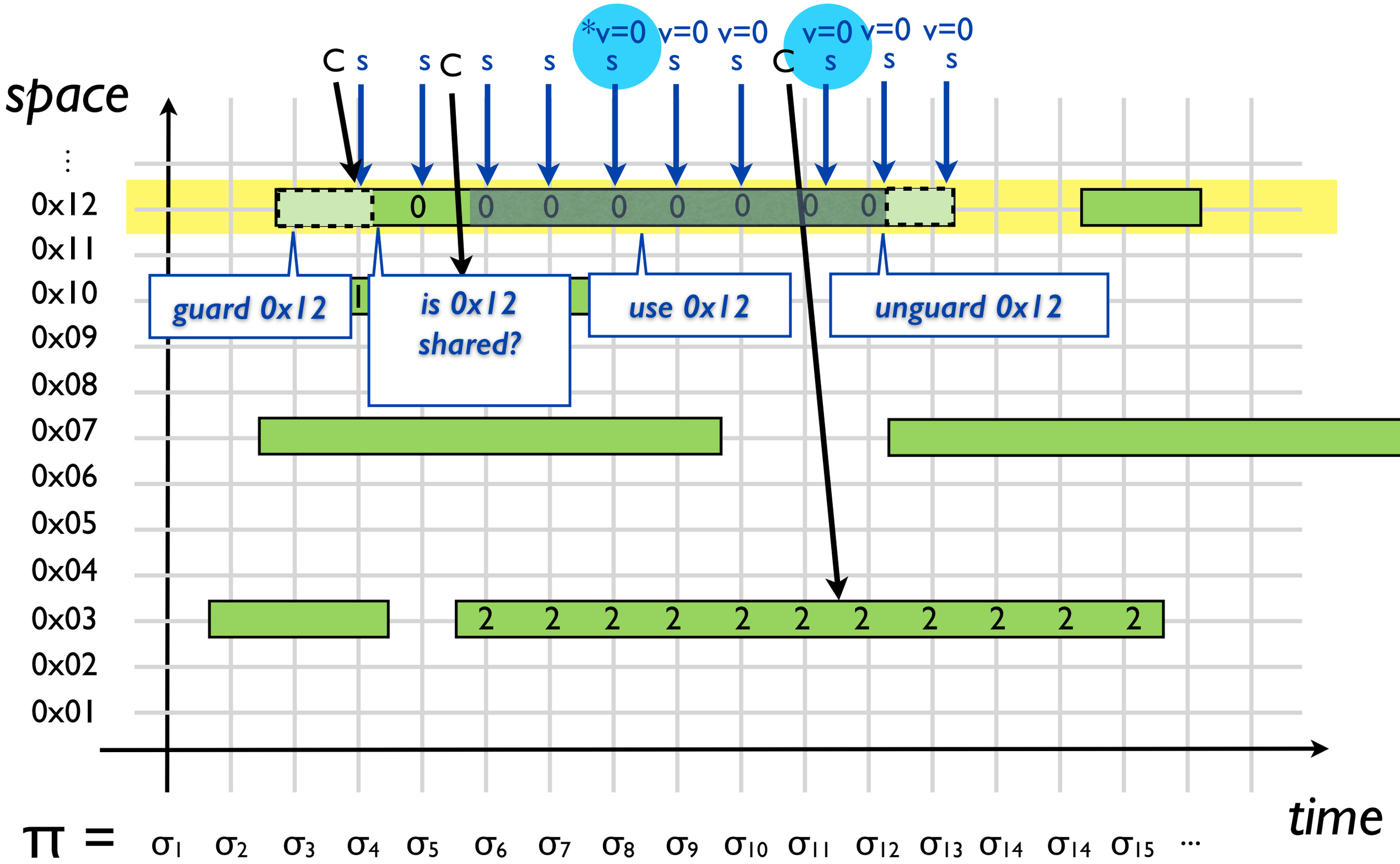
grace-based synchronization



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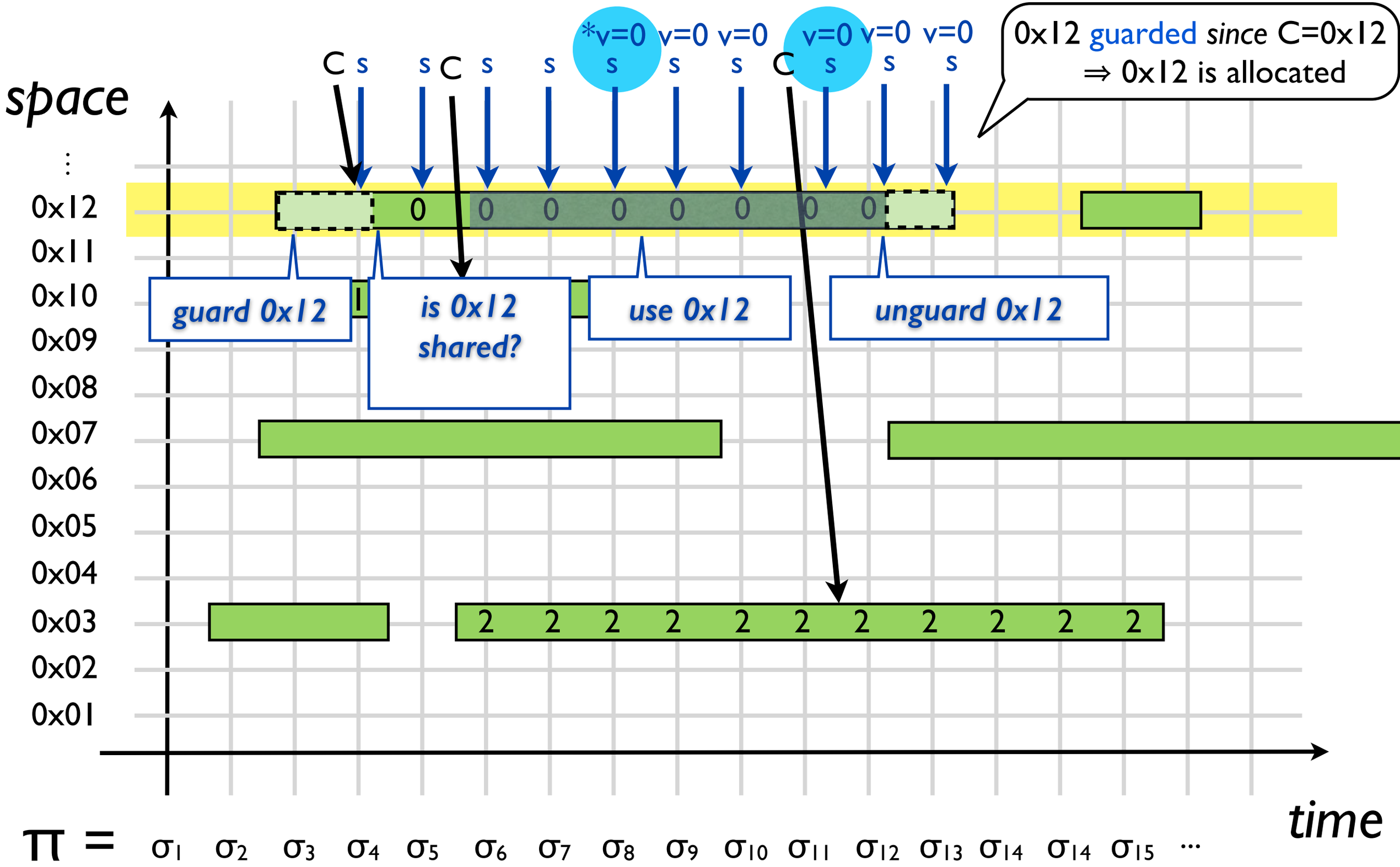
grace-based synchronization



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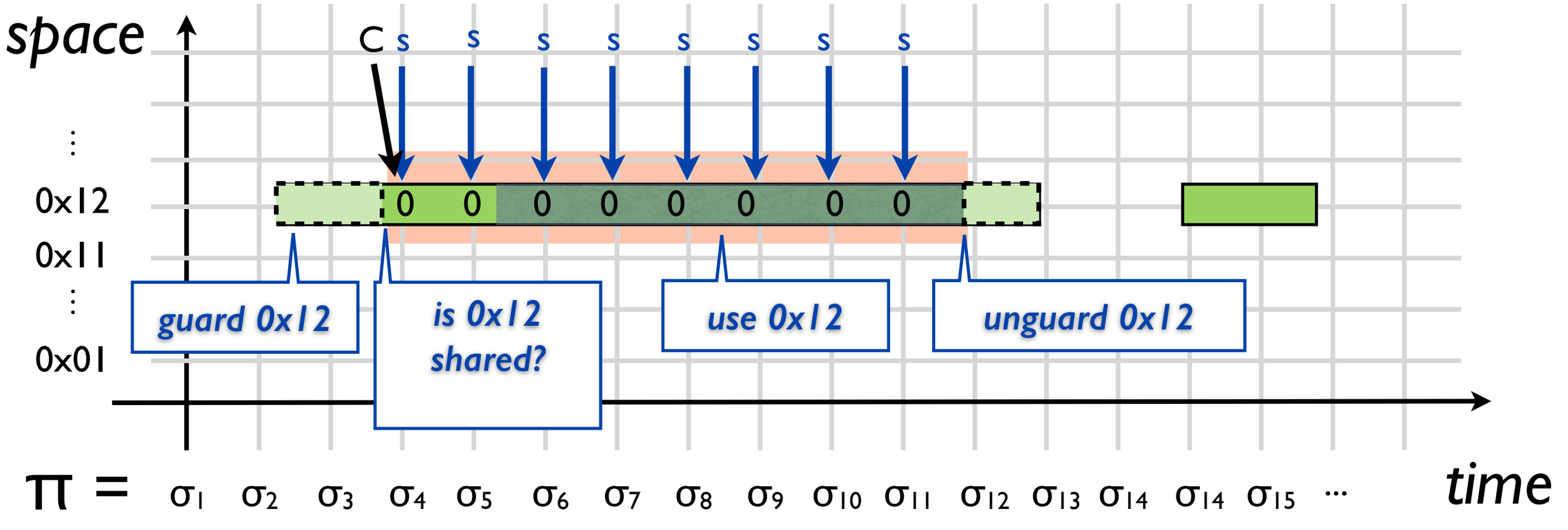
grace-based synchronization

- **grace period**$_{t,s}$ the period of time during which a thread *t* can access a shared node *s* without a fear that *s* might get deallocated



grace-based synchronization

- **grace period**$_{t,s}$ the period of time during which a thread *t* can access a shared node *s* without a fear that *s* might get deallocated

➡ *t* access node *s* during its grace period for *s*

➡ *t* reclaim node *s* after the grace periods of all *t'* for *s*

temporal invariant

- **grace period$_{t,s}$**
  - if t guards s *since* s was shared then s is allocated

temporal invariant

- **grace invariant:** ∀t,s. **grace period**$_{t,s}$
  - ∀t,s. t guards s *since* s was shared then s is allocated
  - ∀t,s. $G_{t,s}$ *since* R_s ⇒ A_s

temporal invariant

- **grace invariant:** ∀t,s. **grace period**$_{t,s}$
  - ∀t,s. t guards s *since* s was shared then s is allocated
  - ∀t,s. $G_{t,s}$ *since* R_s ⇒ A_s

- **Hazard Pointers** [Michael'02]
- **RCU** [McKenney+'98]
- **Epoch** [Fraser+'03]

temporal invariant

- **grace invariant:** ∀t,s. **grace period**$_{t,s}$
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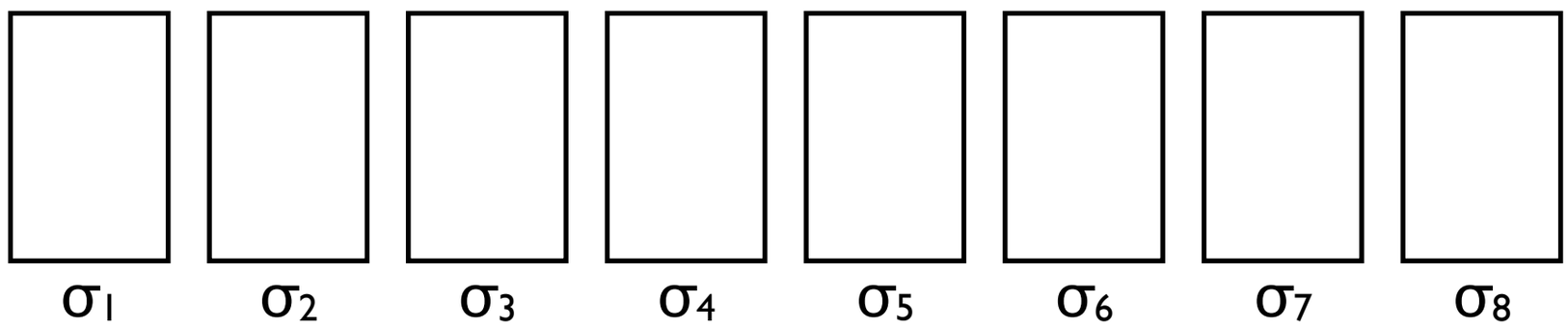
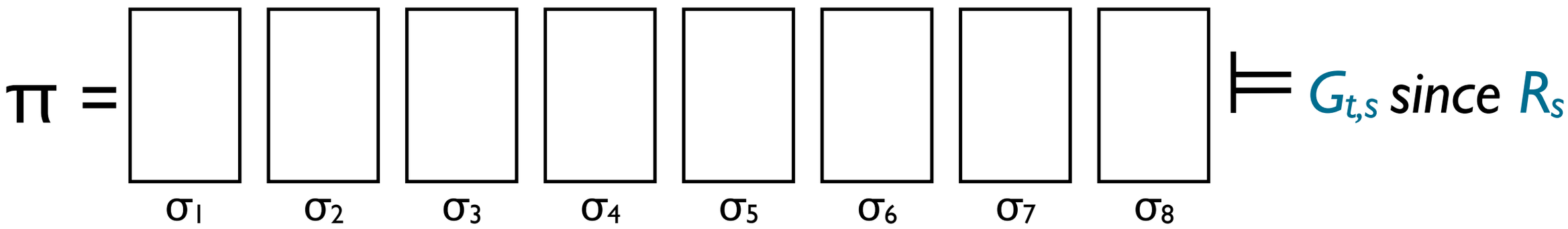
π = 

temporal invariant

- **grace invariant:** $\forall t,s.$ **grace period**$_{t,s}$
  - $\forall t,s.$ t guards s *since* s was shared then s is allocated
  - $\forall t,s.$ $G_{t,s}$ *since* R_s $\Rightarrow$ A_s



temporal invariant

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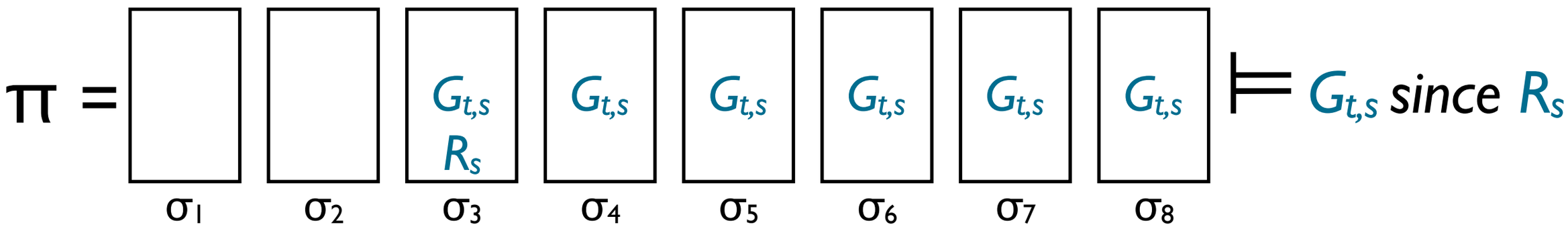

temporal invariant

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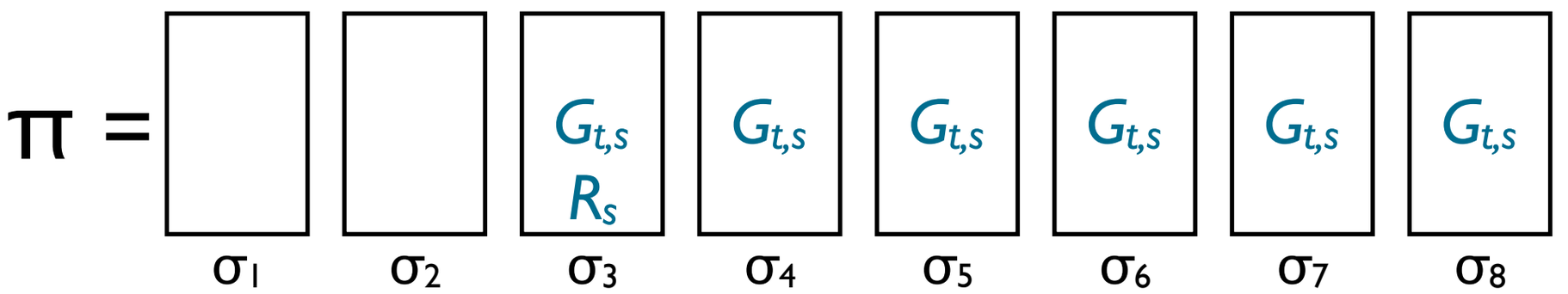
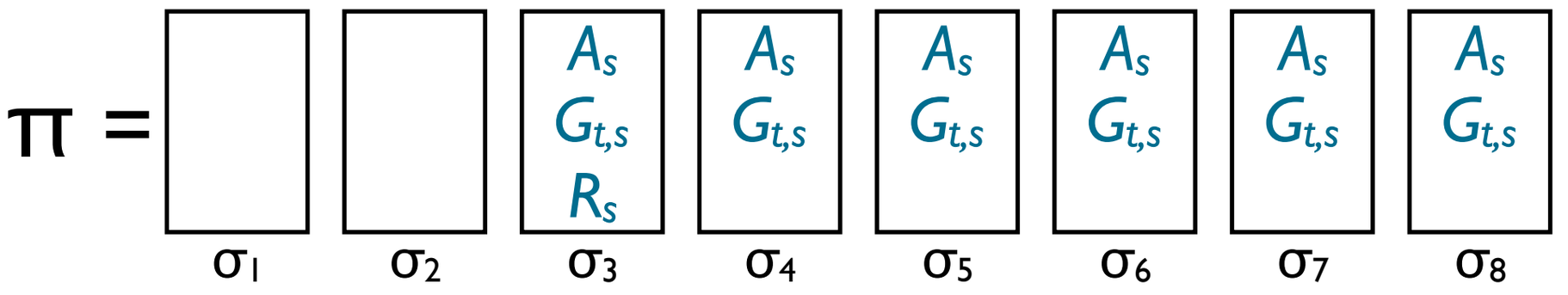


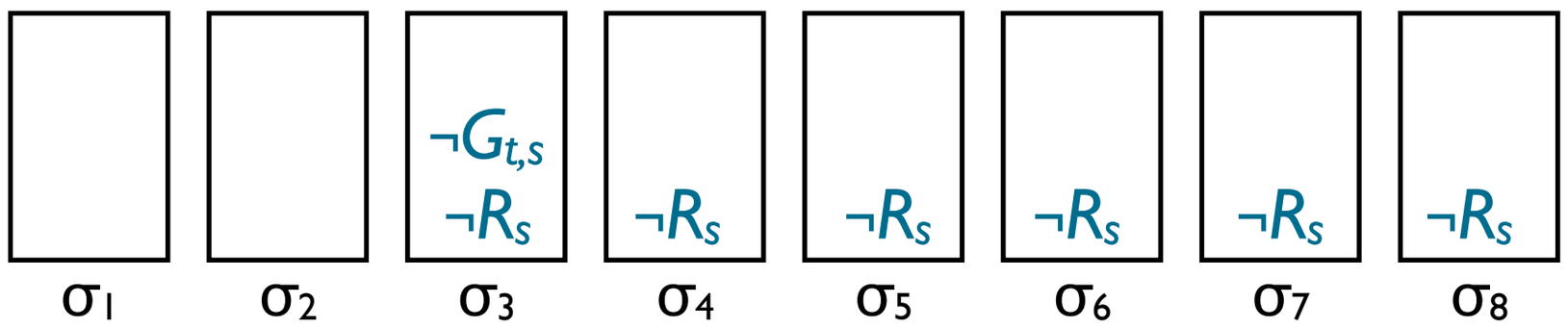
$\models G_{t,s}$ *since* R_s



$\models G_{t,s}$ *since* $R_s \Rightarrow A_s$

temporal invariant

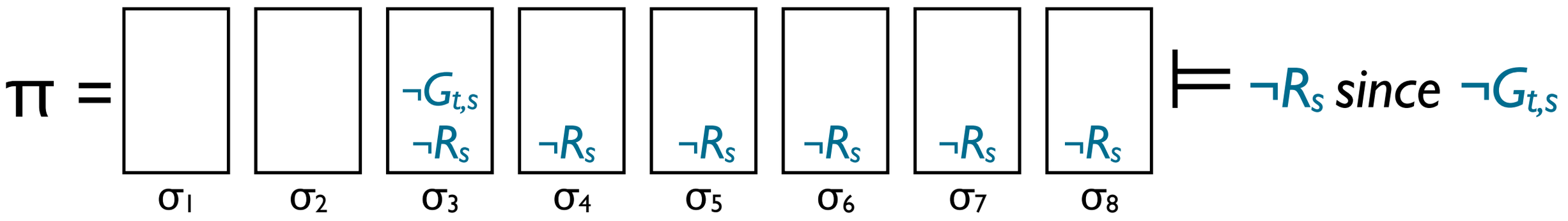
- **grace invariant:** $\forall t,s.$ **grace period**$_{t,s}$
  - $\forall t,s.$ t guards s *since* s was shared then s is allocated
  - $\forall t,s.$ $G_{t,s}$ *since* R_s $\Rightarrow$ A_s

$\pi =$ 

$\models \neg R_s$ *since* $\neg G_{t,s}$

temporal invariant

- **grace invariant:** $\forall t,s.$ **grace period**$_{t,s}$
  - $\forall t,s.$ t guards s *since* s was shared then s is allocated
  - $\forall t,s.$ $G_{t,s}$ *since* R_s $\Rightarrow$ A_s



tautology: $(\neg R_s \text{ since } \neg G_{t,s}) \Rightarrow \neg(G_{t,s} \text{ since } R_s)$

temporal invariant

- **grace invariant:** $\forall t,s.$ **grace period**$_{t,s}$
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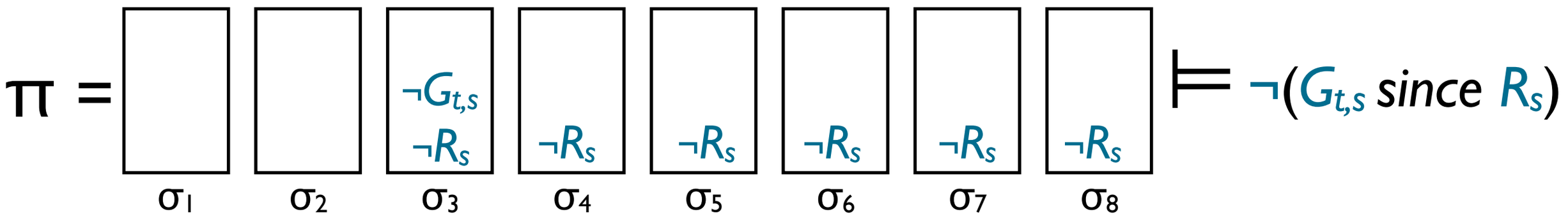

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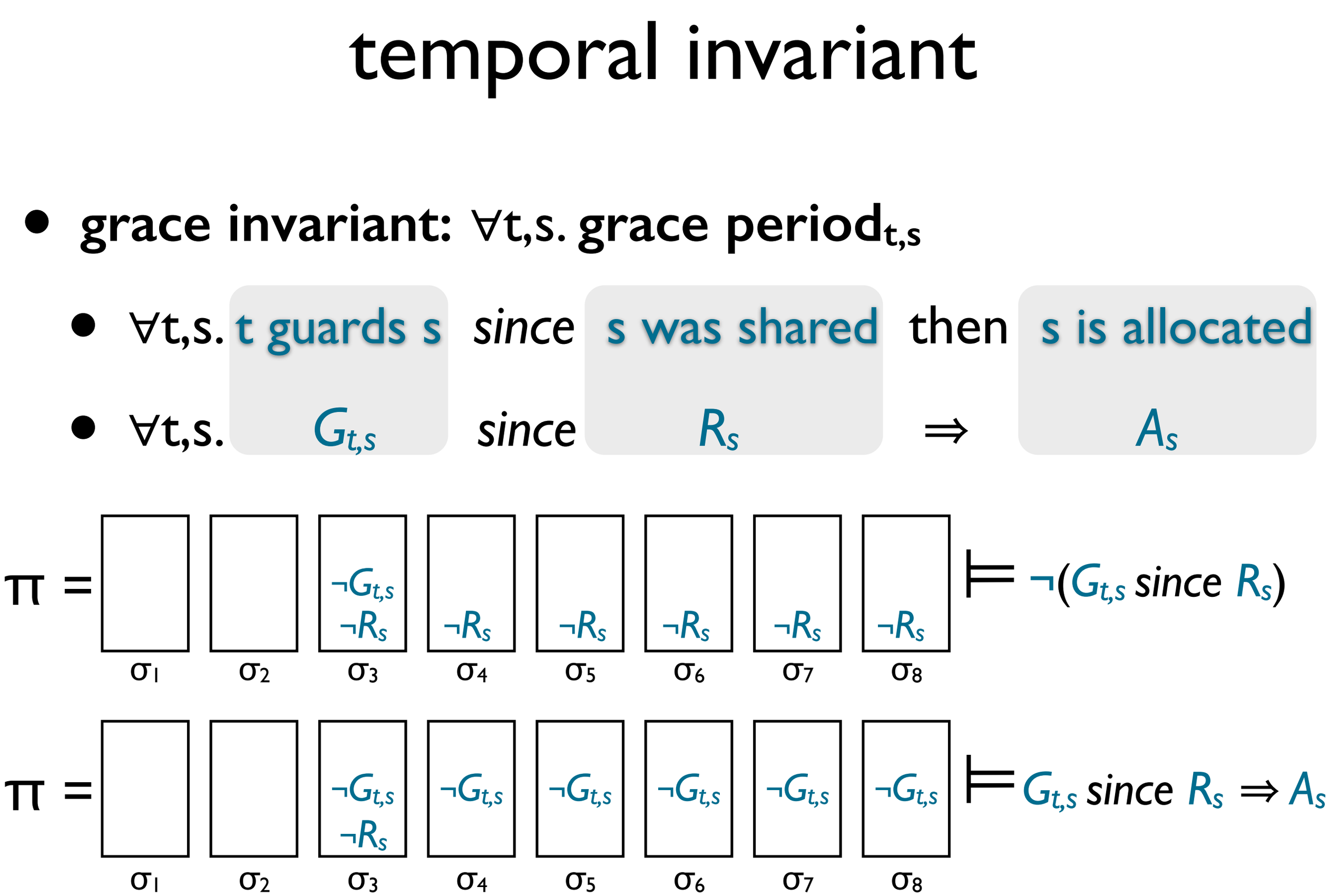

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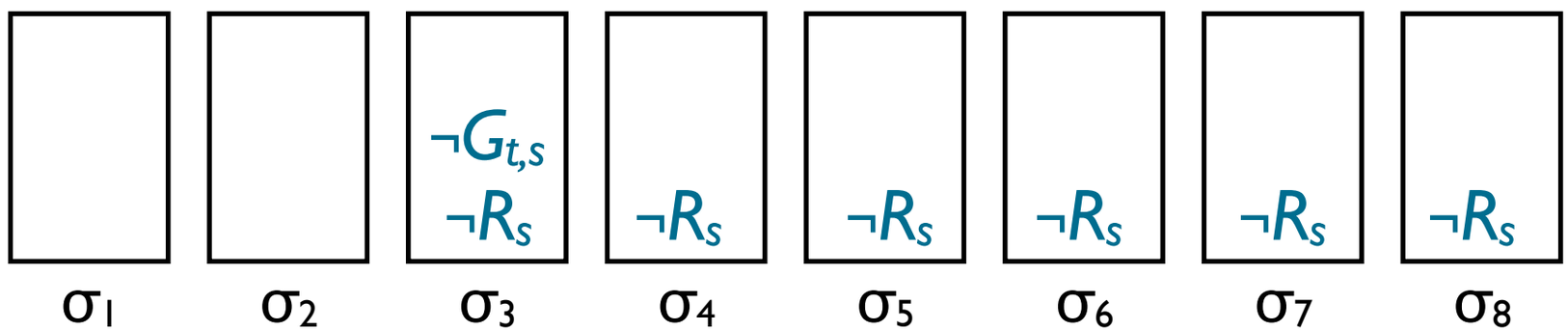
$\pi =$ 

$\models G_{t,s}$ *since* $R_s \Rightarrow A_s$

$\pi =$ 

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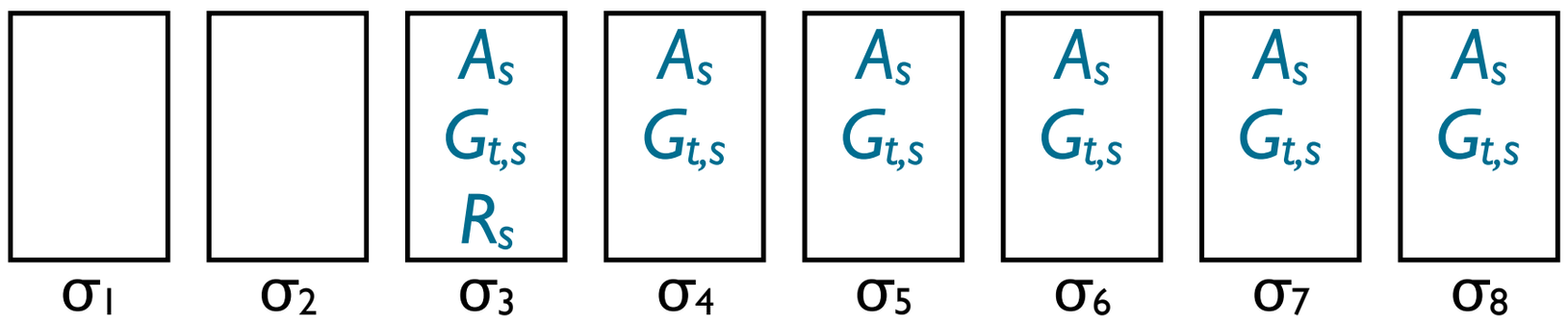
$\pi =$ 

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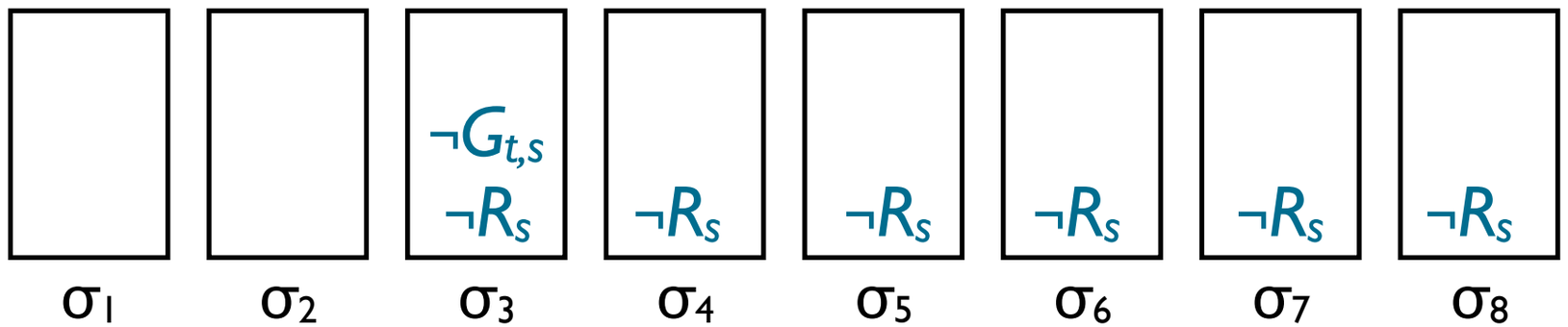
$(G_{t,s}$ *since* $R_s)$

$\pi =$ 

$\models G_{t,s}$ *since* $R_s \Rightarrow A_s$

$\neg(G_{t,s}$ *since* $R_s)$

grace-based synchronization

- **grace invariant:** $\forall t,s.$ **grace period**$_{t,s}$
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  - $\forall t,s.$ $G_{t,s}$ *since* R_s $\Rightarrow$ A_s

➡ ***t*** access ***s***: $G_{t,s}$ *since* R_s

➡ ***t*** reclaim ***s***: $\forall t'.\neg R_s$ *since* $\neg G_{t',s}$

grace-based synchronization

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  - $\forall t,s.$ $G_{t,s}$ *since* R_s $\Rightarrow$ A_s

➡ *t* access *s*: $G_{t,s}$ *since* R_s ($\Rightarrow$ s is allocated)

➡ *t* reclaim *s*: $\forall t'.\neg R_s$ *since* $\neg G_{t,s}$ ($\Rightarrow$ s can be deallocated)

★tautology: $(\neg R_s$ *since* $\neg G_{t,s}) \Rightarrow \neg(G_{t,s}$ *since* $R_s)$

reclamation algorithms (paradigms)

- **Hazard Pointers** [Michael'02]
- **RCU** [McKenney Slingwine'98]
- **Epoch** [Fraser Haris'03]

- pattern of temporal synchronization invariant
  - grace periods
- same idiom for invariant preservation
  - $(\neg R_s \ \mathit{since}\ \neg G_{t,s}) \Rightarrow \neg(G_{t,s}\ \mathit{since}\ R_s)$

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(simplified) counter with hazard pointers

```
int *C;
```

```
int *C;
int *HP[TNUM];
```

```
int inc() {
 int v, *s, *n;
 n = new int;
 do{
  s = C;



  v = *s;
  *n = v + 1;
 }while(!CAS(&C,s,n));
 free(s);

 return v;
}
```

```
int inc() {
 int v,*s1,*s2,*n;
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     s1 = C;
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 HP[tid] = null;
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counter with hazard pointers

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int *C;
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  }while(HP[i]== s);

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 free(s);
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is s1 shared?
(C=s1?)

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 free(s);
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use s1

```
int inc() {
 int v,*s1,*s2,*n;
 n = new int;
 do{
  do{
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  HP[tid] = null;
  reclaim(s1);
  return v;
}
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assuming C ≠ s

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 HP[tid] = null;
 reclaim(s1);
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guard s1

is s1 shared? (C=s1?)

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(simplified)

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 free(s);
}
```

assuming C ≠ s

t_i does not guard s

```
int inc() {
 int v,*s1,*s2,*n;
 n = new int;
 do{
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     s1 = C;
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  *n = v + 1;
 }while(!CAS(&C,s1,n));
 HP[tid] = null;
 reclaim(s1);
 return v;
}
```

guard s1

is s1 shared? (C=s1?)

use s1

unguard s1

partial ownership

- proof hinges on C ≠ s during retire(s)

- ensured by ownership annotations
  - the thread which removes s can
    - C = s
    - free (s)
  - other threads can only access s

(simplified)

counter with hazard pointers

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int *C;
```

```
int *C;
int *HP[TNUM];
```

$\forall t,s.\ (HP[t]=s\ \textit{since}\ C=s) \Rightarrow allocated(s)$

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reclaim(s) {
 i = 0;
 do{
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int *C;
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∀t,s. (HP[t]=s *since* C=s)⇒allocated(s)

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(simplified)

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HP[tid]=s1

HP[tid]=s1
∧ C=s2

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HP[tid]=s1

HP[tid]=s1
since C=s2

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s1 is allocated *(memory safety)*

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(simplified)

counter with hazard pointers

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∀t,s. (HP[t]=s *since* C=s)⇒allocated(s)

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HP[tid]=s1

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HP[tid]=s1 *since* C=s1

s1 is allocated *(memory safety)*

s1 is allocated since C=s *(no ABA)*

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counter with hazard pointers *(simplified)*

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C≠s

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C≠s

∀t<i.C≠s *since* HP[t] ≠s

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∀t<i.C≠s *since* HP[t] ≠s

∀t.C≠s *since* HP[t] ≠s

∀t.(C≠s since HP[t]≠s)⇒¬(HP[t]=s since C=s)

(Invariant is preserved)

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```

grace-based synchronization

- **grace invariant:** $\forall t,s.$ **grace period**$_{t,s}$
  - $\forall t,s.$ t guards s *since* s was shared then s is allocated
  - $\forall t,s.$ $G_{t,s}$ *since* R_s $\Rightarrow$ A_s

➡ *t* access *s*:

$G_{t,s}$	*since*	R_s
HP[t]=s	*since*	C=s

➡ t_1 reclaim *s*:

$\forall t.$	$\neg R_s$	*since*	$\neg G_{t,s}$
$\forall t.$	C≠s	*since*	HP[t]≠s

★tautology: $(\neg R_s \text{ since } \neg G_{t,s}) \Rightarrow \neg(G_{t,s} \text{ since } R_s)$

temporal separation logic

$$R, G, I \vdash \{P\}\ C\ \{Q\}$$

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$$\sigma = (\boxed{p}, \boxed{s}) \in \Sigma_p \times \Sigma_s$$

$$\omega = (\boxed{p}, \boxed{s}\boxed{s}\boxed{s}) \in \Sigma_p \times \Sigma_s^*$$

temporal separation logic

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$$P, Q = \{(p, s\,s\,s), \ldots\} \subseteq \Sigma_p \times \Sigma_s^*$$

$$I = \{s\,s\,s, \ldots\} \subseteq \Sigma_s^*$$

$$R_t, G_t = \{(s, s), \ldots\} \subseteq \Sigma_s \times \Sigma_s$$

temporal separation logic

temporal logic, separation logic, rely/guarantee, local actions, stability, temporal assertions, spatial resource invariants, local/shared state partitioning, permissions, temporal invariant

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$$I = \{\boxed{s}\boxed{s}\boxed{s}, \dots\} \subseteq \Sigma_s^*$$

$$R_t, G_t = \{(\boxed{s}, \boxed{s}), \dots\} \subseteq \Sigma_s \times \Sigma_s$$

proof (retire)

1 $\{V \Vdash p \mapsto_m \_ * F_{\mathtt{tid}} \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}} \wedge \boxed{p \mapsto_e \_ * \mathsf{true}}\}$
2 `void retire(int* p) {`
3 $\quad\{V \Vdash p \mapsto_m \_ * \exists A.\, \mathtt{detached} \mapsto A * D(A) \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}} \wedge \boxed{p \mapsto_e \_ * \mathsf{true}}\}$
4 `insert(detached,p);`
5 $\quad\{V \Vdash \exists A.\, \mathtt{detached} \mapsto A * D(A) \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
6 `  if (nondet())`
7 $\quad\quad\{V \Vdash F_{\mathtt{tid}} \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
8 `    return;`
9 `  Set used;`
10 $\quad\{V \Vdash \exists A.\, \mathtt{detached} \mapsto A * D(A) \wedge \mathit{used} = \emptyset \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
11 `  while (!isEmpty(detached)) {`
12 $\quad\quad\{V \Vdash \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) \wedge A \neq \emptyset \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
13 `    bool my = true;`
14 `    Node *n = pop(detached);`
15 $\quad\quad\{V \Vdash \mathit{my} \wedge \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) * n \mapsto_m \_ * \boxed{n \mapsto_e \_ * \mathsf{true}} * \boxed{H * \exists y.\, y \neq n \wedge \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
16 `    for (int i = 0; i < N && my; i++) {`
17 $\quad\quad\quad\{V \Vdash \mathit{my} \wedge 0 \leq i < \mathtt{N} \wedge \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) * n \mapsto_m \_ * \boxed{n \mapsto_e \_ * \mathsf{true}} *$
18 $\quad\quad\quad\boxed{H * \exists y.\, y \neq n \wedge \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}} \wedge \forall 0 \leq j < i.\, \boxed{\exists y.\, y \neq n \wedge \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}$ since $\boxed{\mathtt{HP}[i] \neq n * \mathsf{true}}\}$
19 `      if (⟨HP[i] == n⟩Id)`
20 `        my = false;`
21 $\quad\quad\quad\{V \Vdash 0 \leq i < \mathtt{N} \wedge \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) * n \mapsto_m \_ * \boxed{n \mapsto_e \_ * \mathsf{true}} *$
22 $\quad\quad\quad\boxed{H * \exists y.\, y \neq n \wedge \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}} \wedge (\mathit{my} \Rightarrow \forall 0 \leq j \leq i.\, \boxed{\exists y.\, y \neq n \wedge \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}$ since $\boxed{\mathtt{HP}[j] \neq n * \mathsf{true}})\}$
23 `    }`
24 $\quad\quad\{V \Vdash \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) * n \mapsto_m \_ * \boxed{n \mapsto_e \_ * \mathsf{true}} \wedge$
25 $\quad\quad\boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}} \wedge (\mathit{my} \Rightarrow \forall 0 \leq j \leq \mathtt{N}.\, \boxed{\exists y.\, y \neq n \wedge \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}$ since $\boxed{\mathtt{HP}[j] \neq n * \mathsf{true}})\}$
26 `    if (my) {`
27 `      ⟨ ; ⟩Take;`
28 $\quad\quad\{V \Vdash \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) * n \mapsto \_ * \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
29 `      free(n);`
30 `    } else {`
31 `      insert(used, n);`
32 `    }`
33 $\quad\quad\{V \Vdash \exists A.\, \mathtt{detached} \mapsto A * D(A) * D(\mathit{used}) \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
34 `  }`
35 $\quad\{V \Vdash \mathtt{detached} \mapsto \emptyset * D(\mathit{used}) \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
36 `  moveAll(detached, used);`
37 $\quad\{V \Vdash F_{\mathtt{tid}} \wedge \boxed{H * \exists y.\, \mathtt{C} \mapsto y * y \mapsto \_ * \mathsf{true}}\}$
38 `}`

summary

- **Hazard Pointers** [Michael'02]
  - non-blocking reclaim, single reading of hazard array, dynamic allocation of hazard pointers, non-blocking stack, reuse of next-pointer
- **RCU** [McKenney Slingwine'98]
  - first formalization
- **Epoch** [Fraser Haris'03]

★ invariant: $\forall t,s.\ G_{t,s}\ \mathit{since}\ R_s \Rightarrow A_s$

★ tautology: $(\neg R_s\ \mathit{since}\ \neg G_{t,s}) \Rightarrow \neg(G_{t,s}\ \mathit{since}\ R_s)$

related work

- separation logic [Reynolds, LICS'02] [O'Hearn+ POPL'01]
- CSL [O'Hearn, TCS'07]
- Stack + HP in CSL [Parkinson+, POPL'07]
- reductions [Elmas+, POPL'09]
- R/G Separation Logic [Vefiadis, PhD.'08]
- temporal separation logic [Fu+, CONCUR'10]
- interval temporal logic [Tofan+, ICTAC'11]

future work

- **Pass the Buck** [Herlihy+, DISC'02]
- weak memory

conclusions

- *since* operator as abstraction of histories
  - natural specification
  - complicates logic

- programming patterns ⇒ proof patterns
  - simplify proofs using specialized rules

thank you!

“weirdnesses”

```
int *C;
```

```
int *C;
int *HP[TNUM];
```

```
reclaim(s) {
 i = 0;
 do{
   skip
 }while(HP[i]== s);

 i = i + 1;
}

free(s);
}
```

C ≠ s

t_i does not guard s

```
int inc() {
 int v,*s1,*s2,*n;
 n = new int;
 do{
  do{
     s1 = C;
     HP[tid] = s1;
     s2 = C;
  }while(s1 != s2);
   v = *s1;
   *n = v + 1;
 }while(!CAS(&C,s1,n));
 HP[tid] = null;
 reclaim(s1);
 return v;
}
```

guard s1

is s1 shared? (C=s1?)

use s1

unguard s1

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  *n = v + 1;
 }while(!CAS(&C,s1,n));
 HP[tid] = null;
 reclaim(s1);
 return v;
}
```

guard s1

s1 might get deallocated

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  *n = v + 1;
 }while(!CAS(&C,s1,n));
 HP[tid] = null;
 reclaim(s1);
 return v;
}
```

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  v = *s1;
  *n = v + 1;
 }while(!CAS(&C,s1,n));
 HP[tid] = null;
 reclaim(s1);
 return v;
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  v = *s1;
  *n = v + 1
 }while(!CA
 HP[tid] =
 reclaim(s1);
 return v;
}
```

s1 might get deallocated

s1 reallocated ⇒ “reguarded”