Formal Verification of Skiplists with Arbitrarily Many Levels

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ATVA'14, Sydney, 5 November 2014

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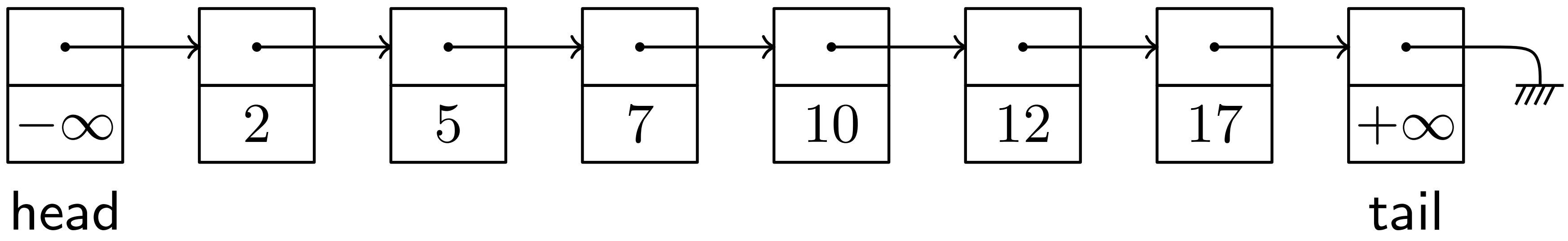

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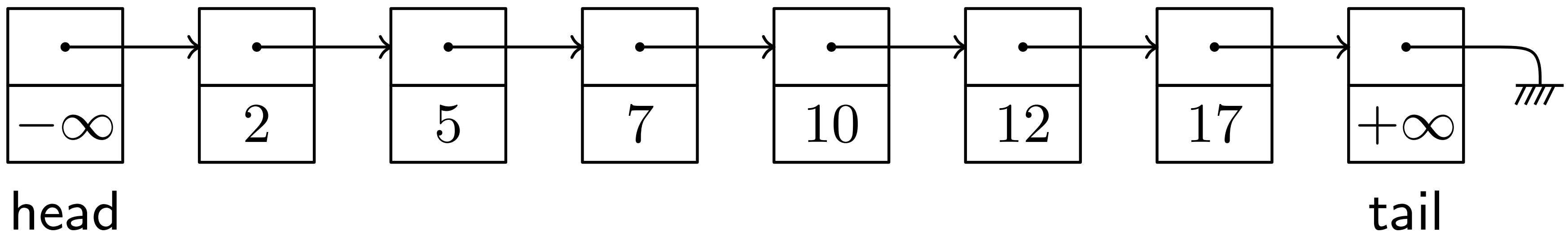
- Hierarchy of linked lists which **skip** nodes



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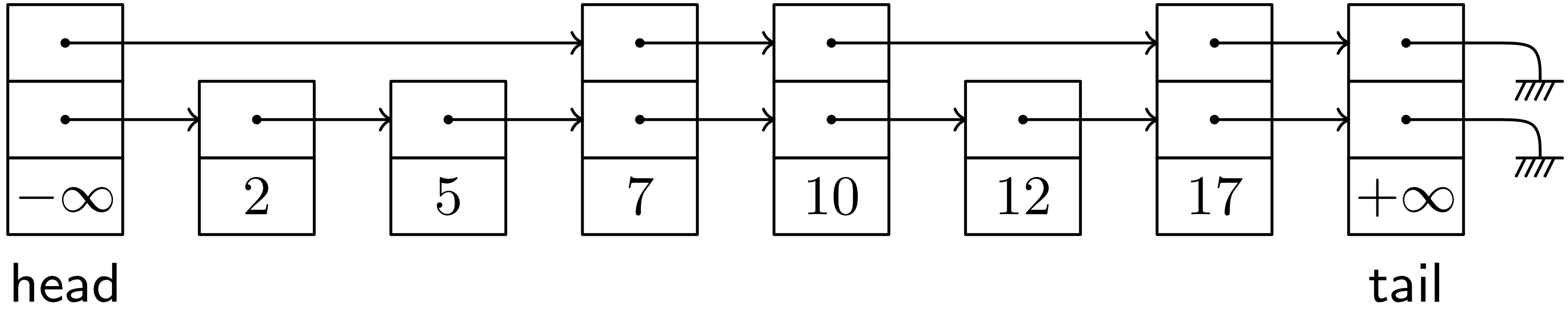

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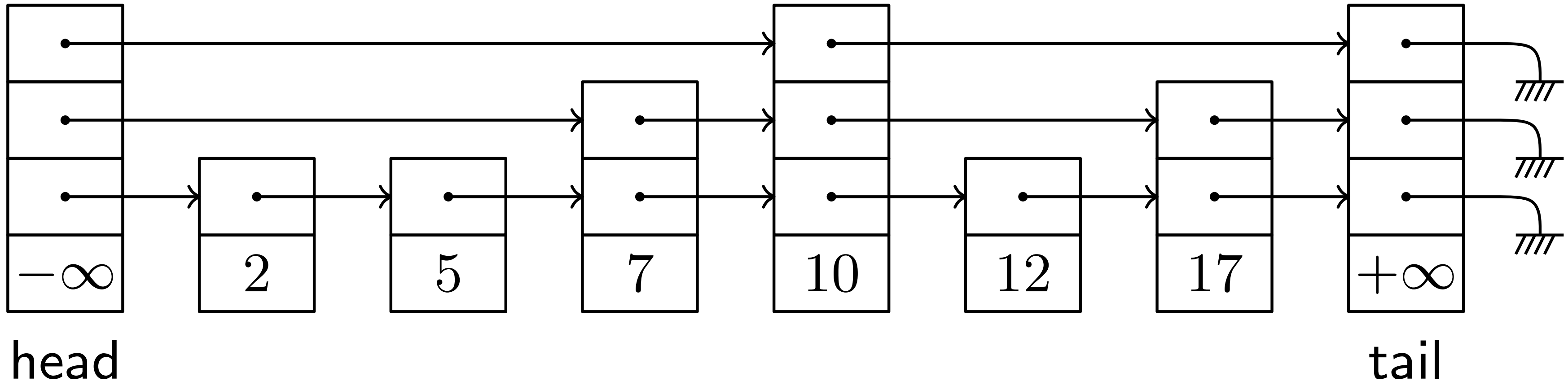

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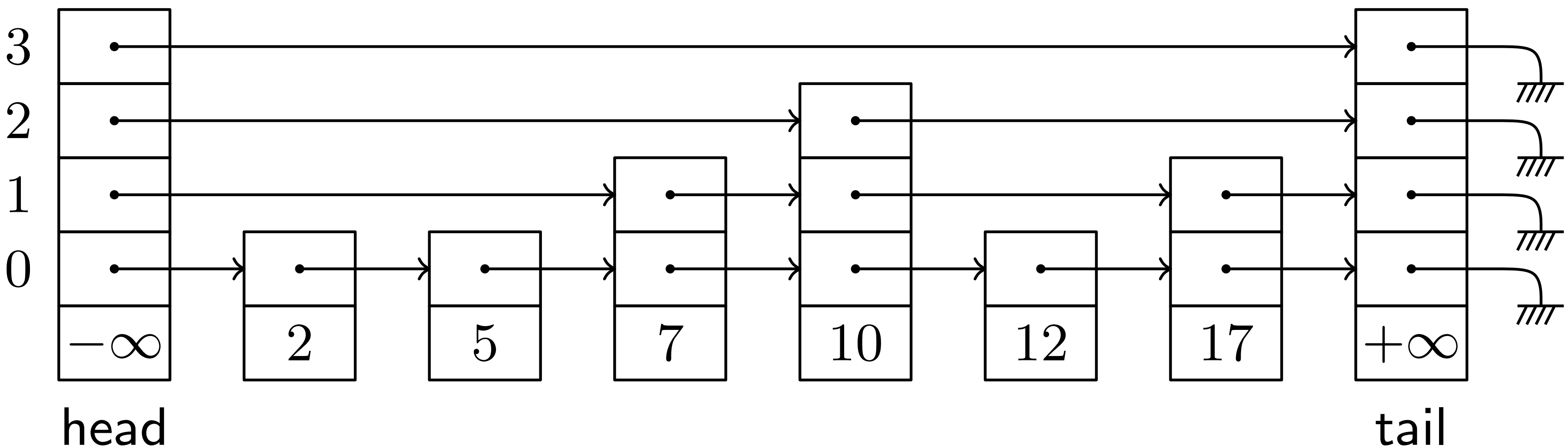

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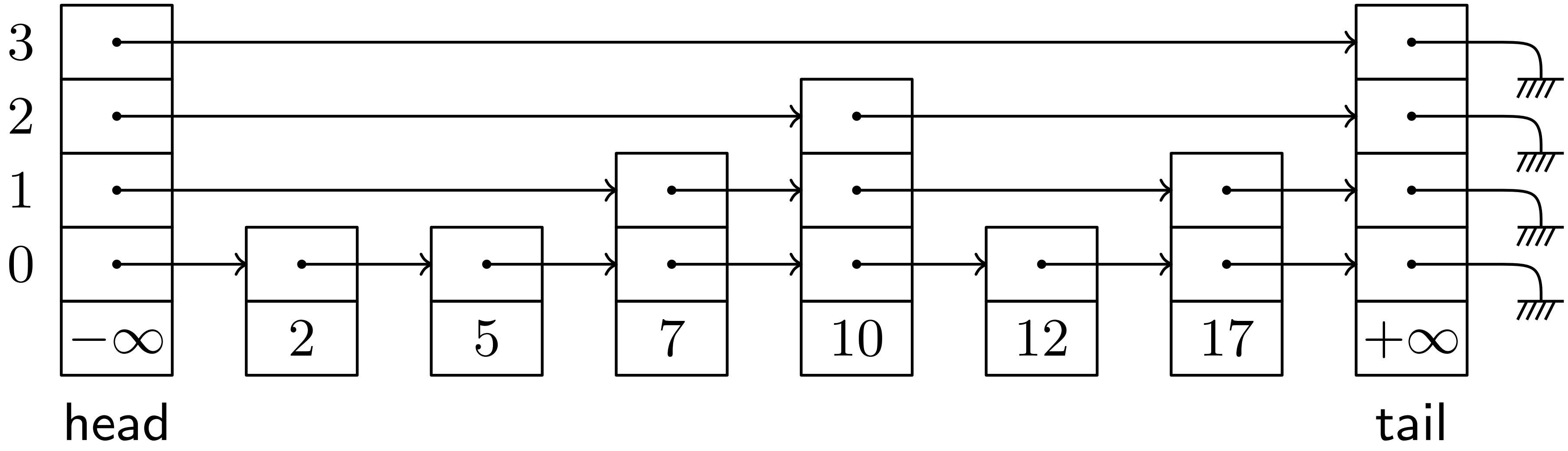

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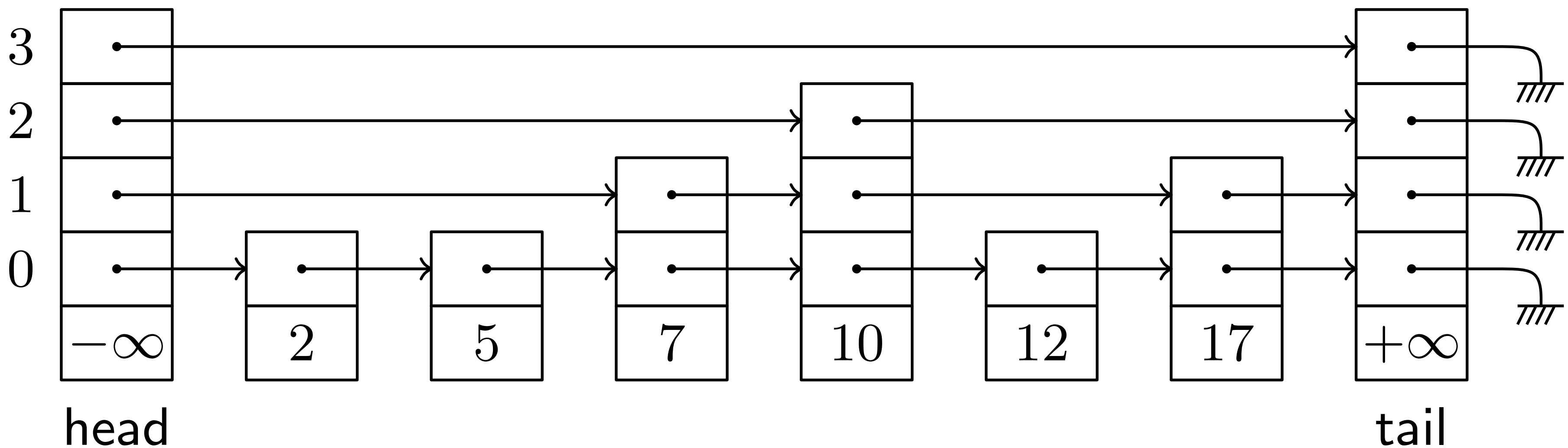

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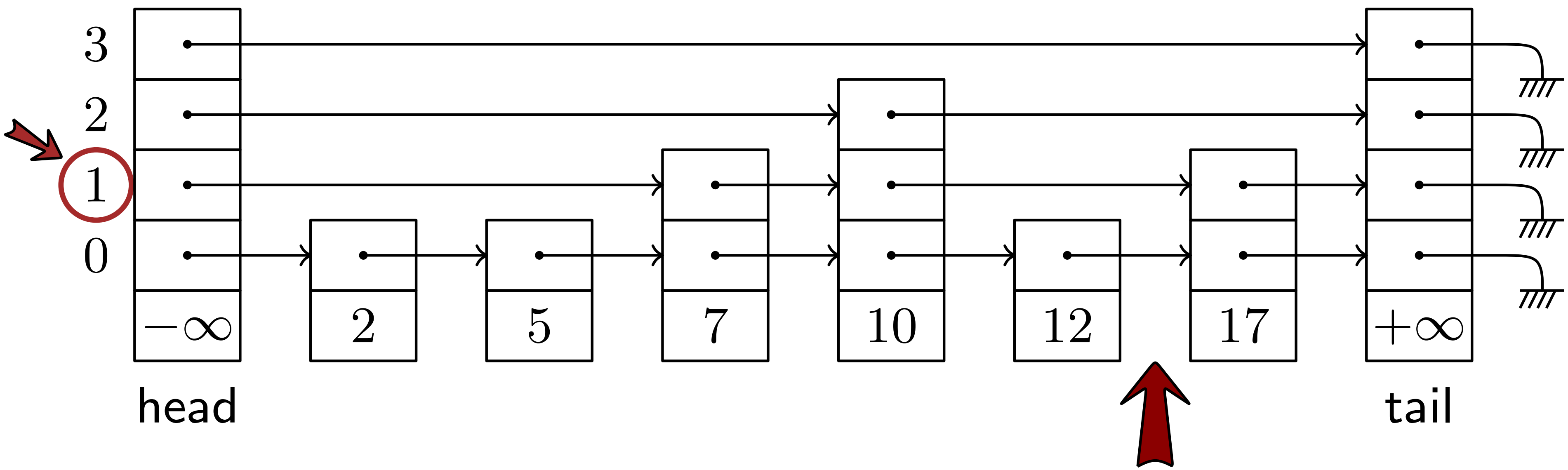

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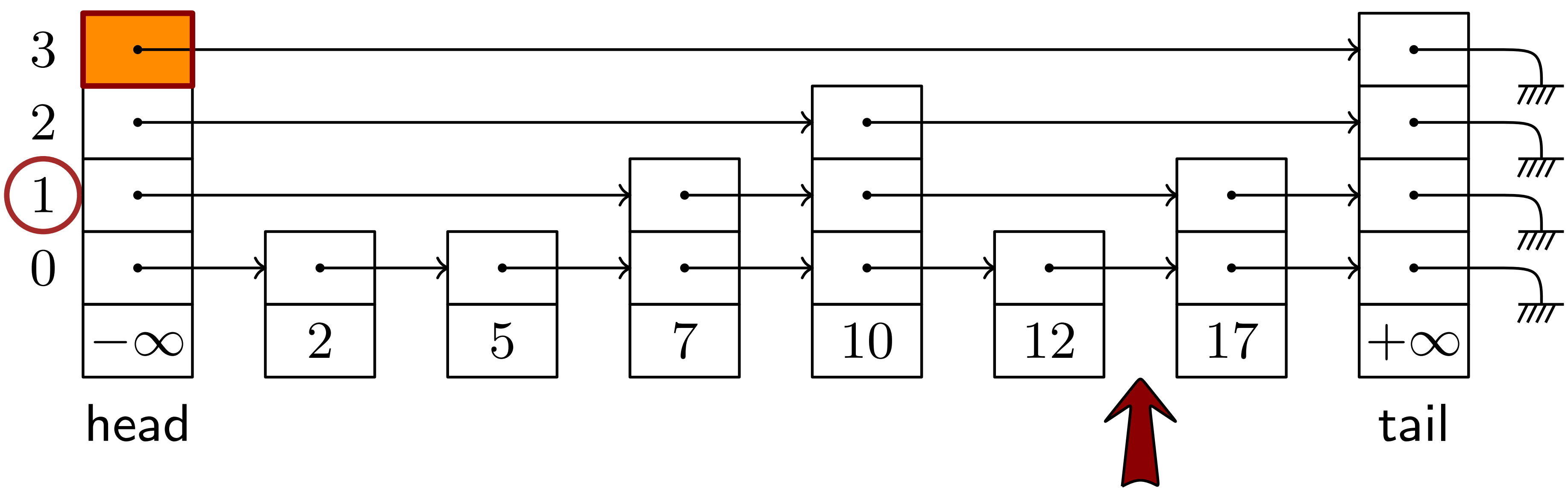

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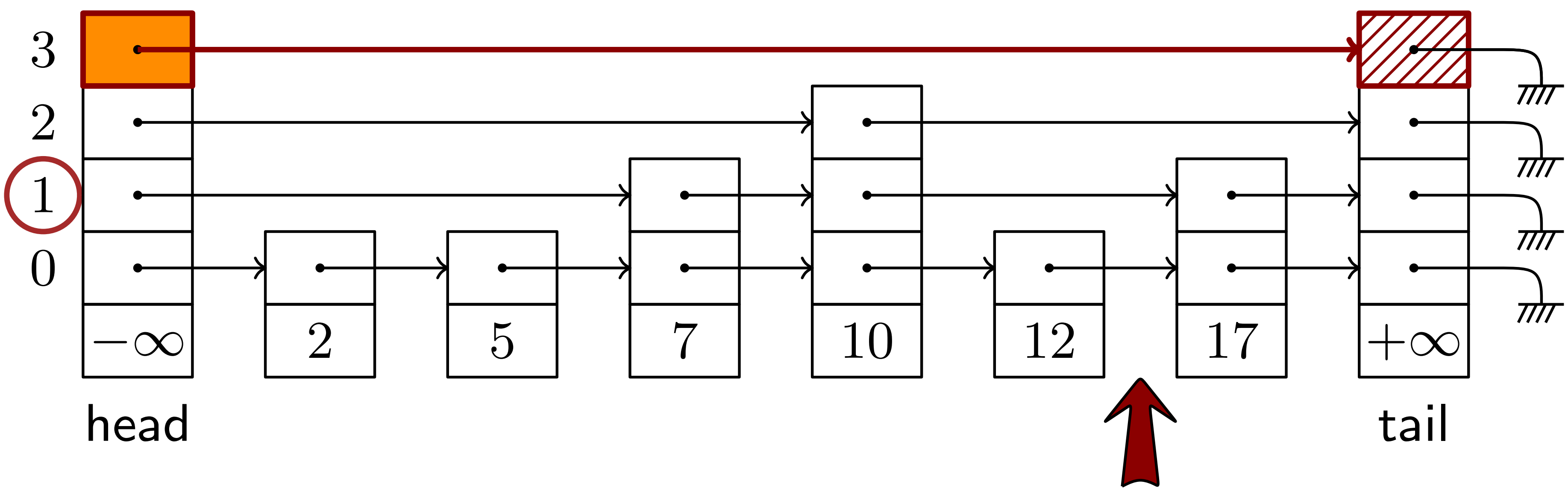

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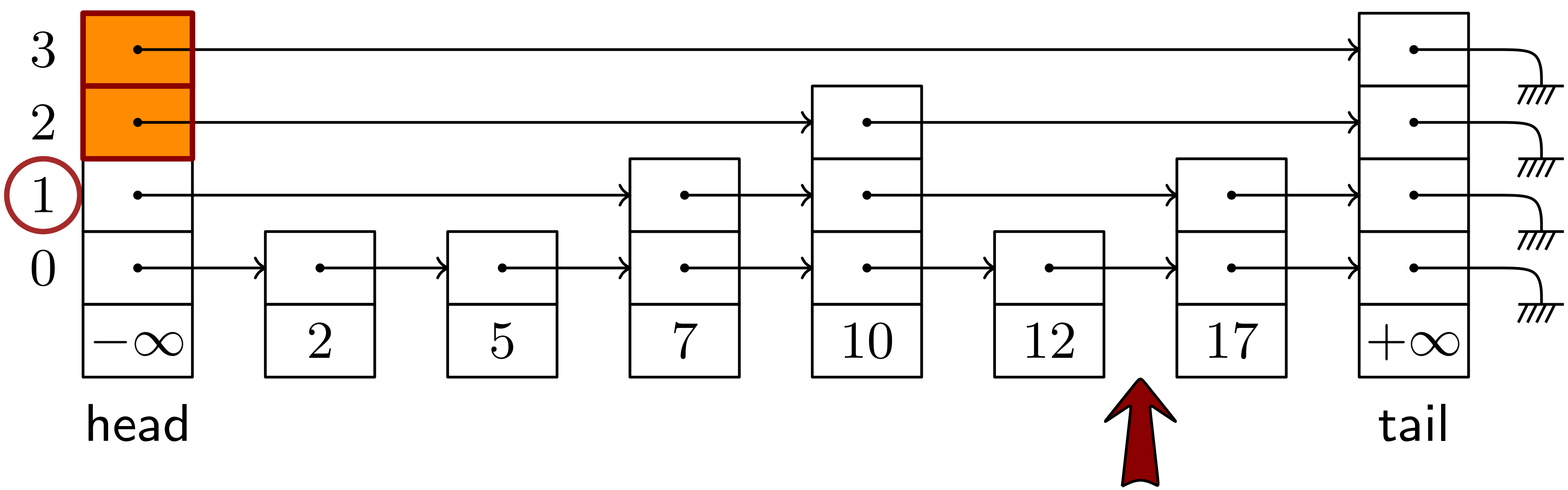

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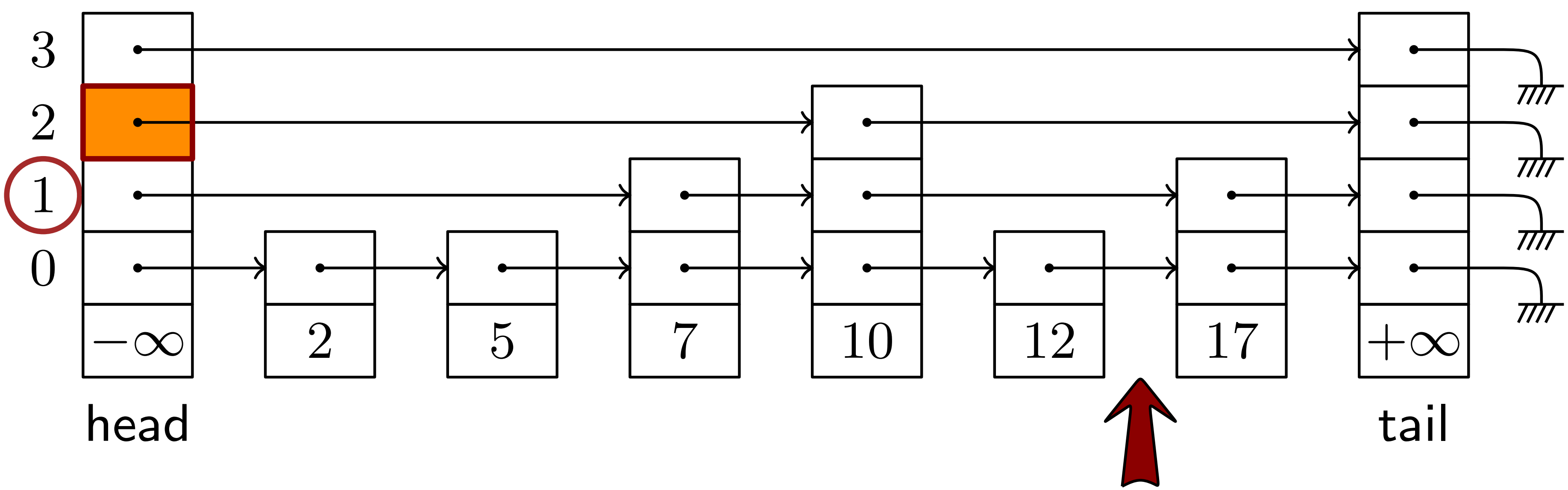

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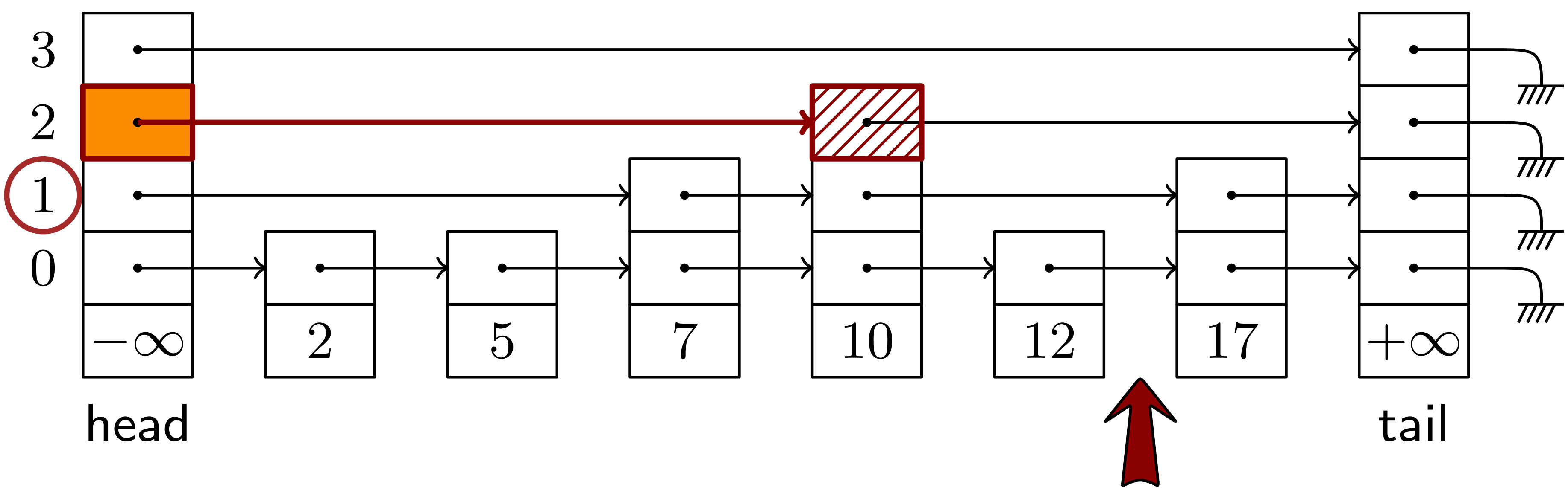

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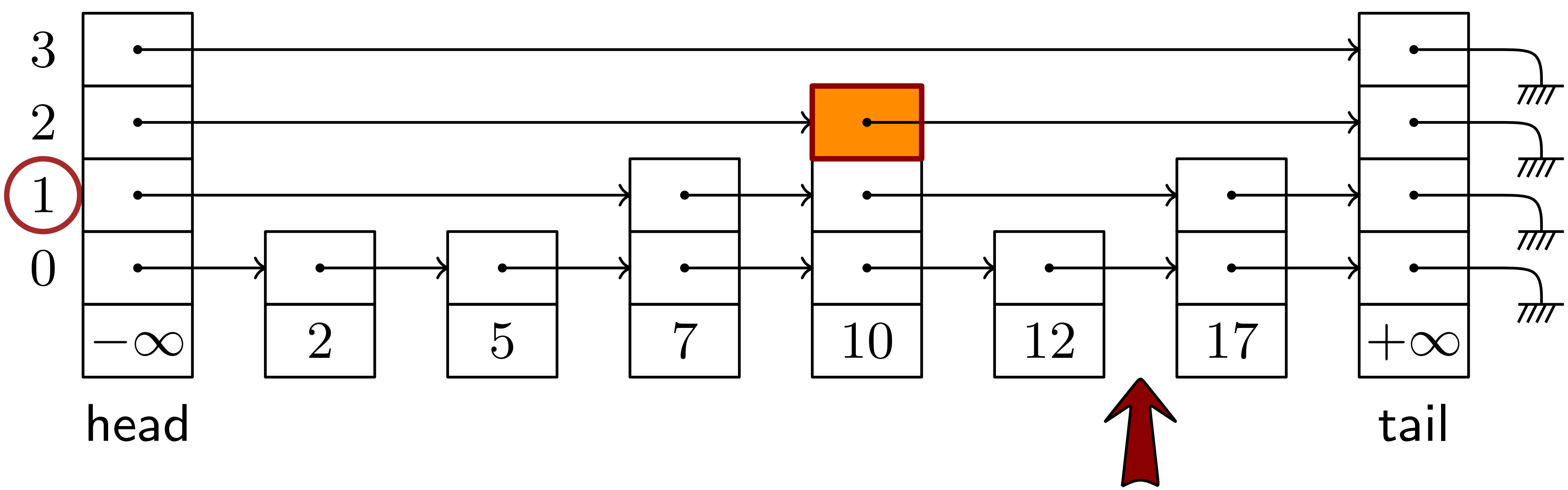

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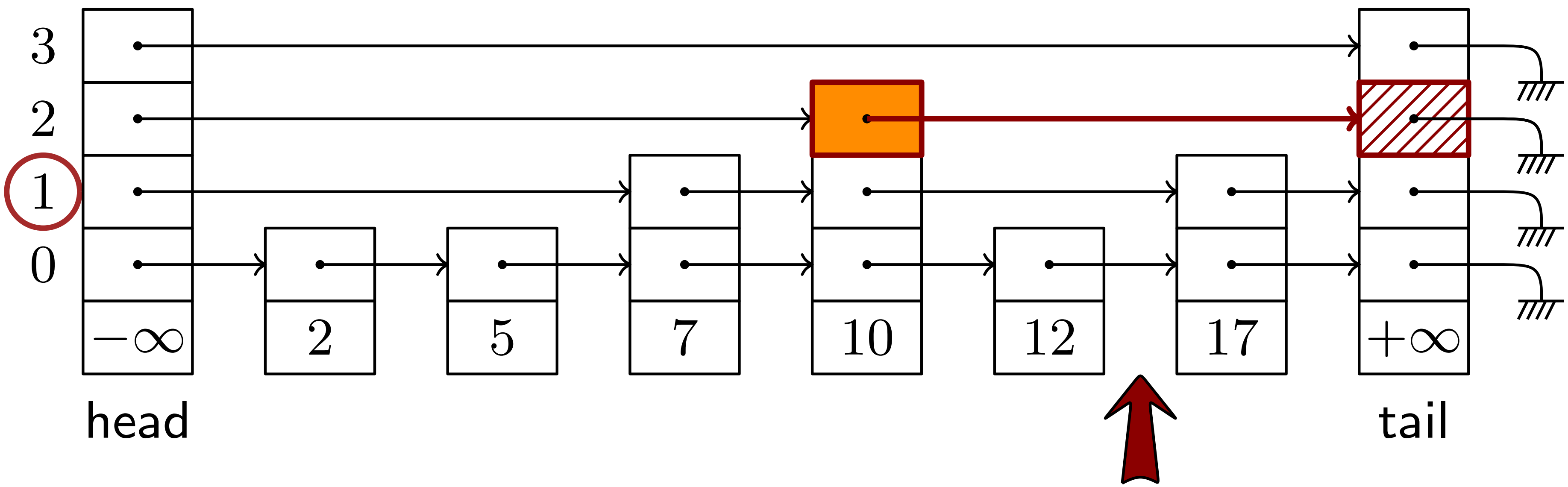

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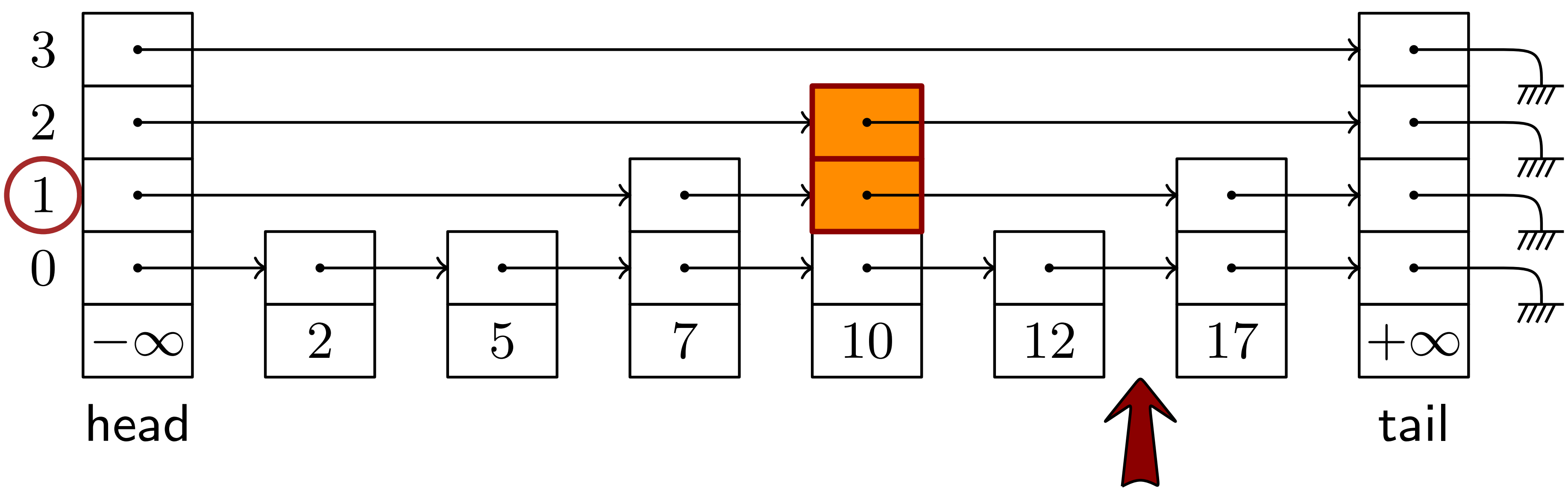

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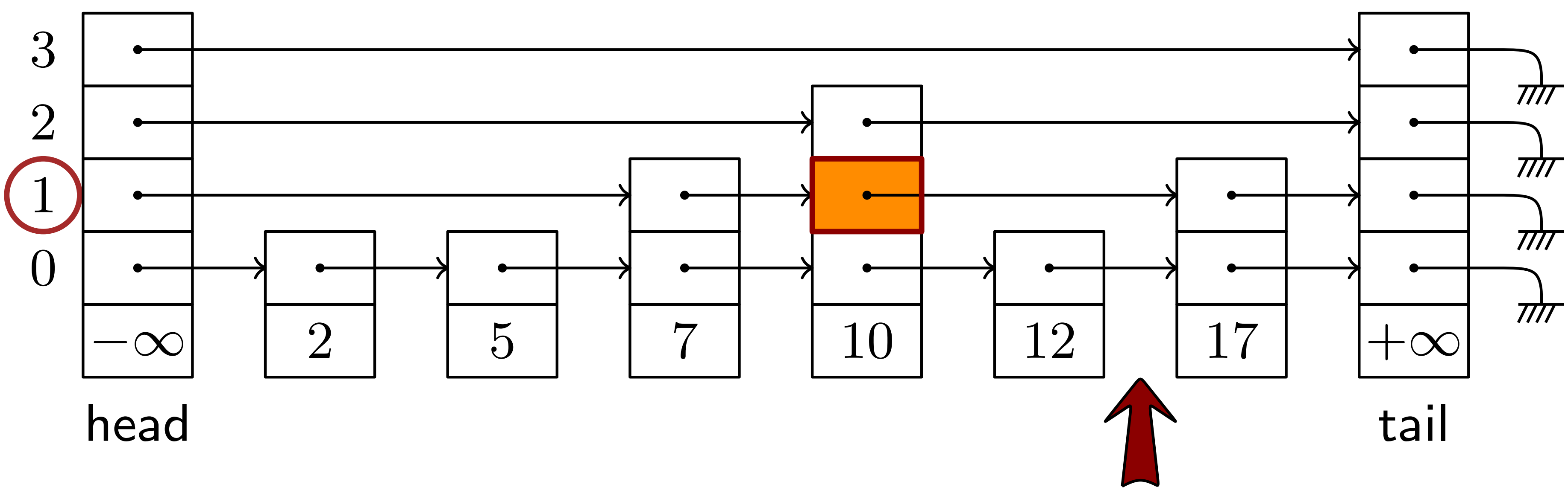

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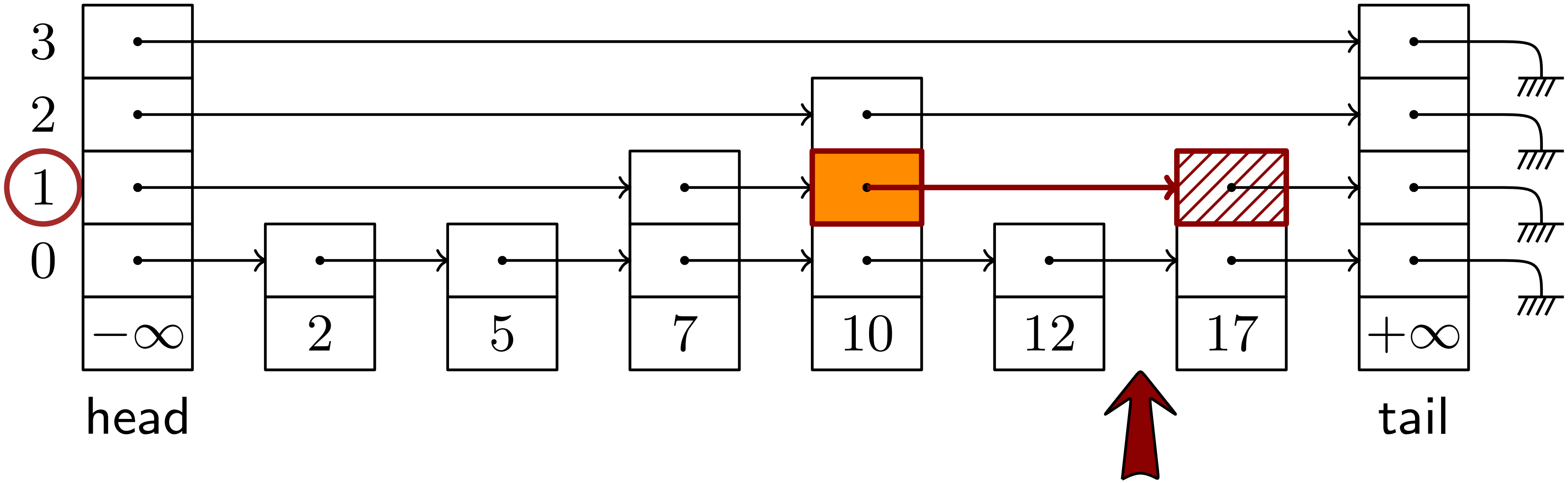

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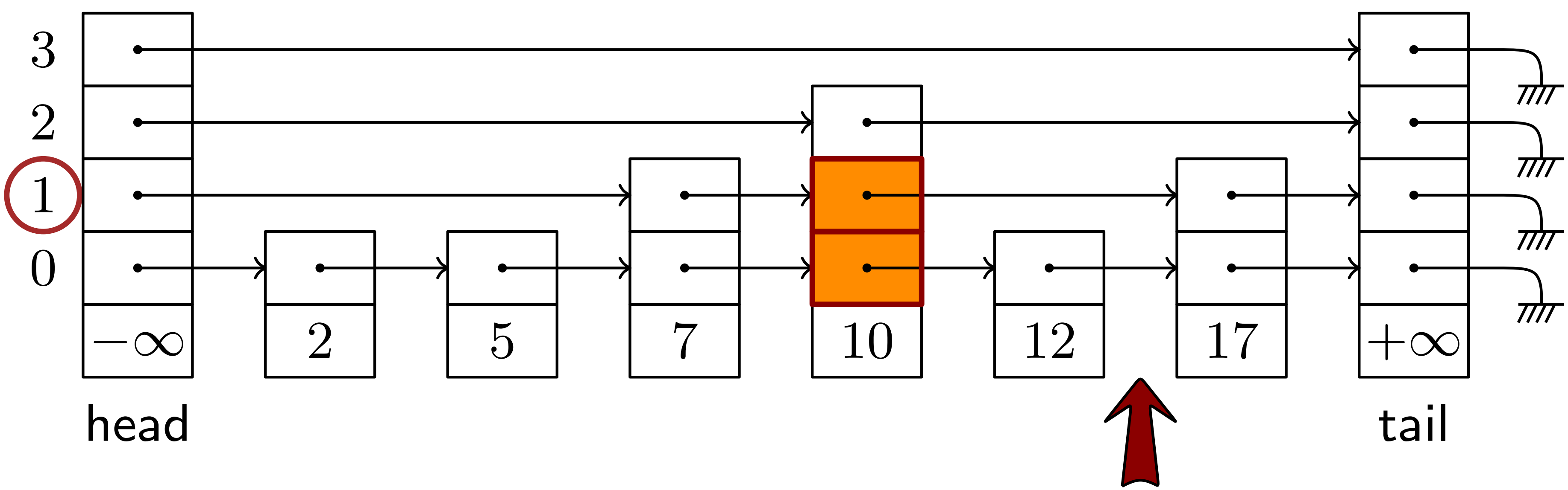

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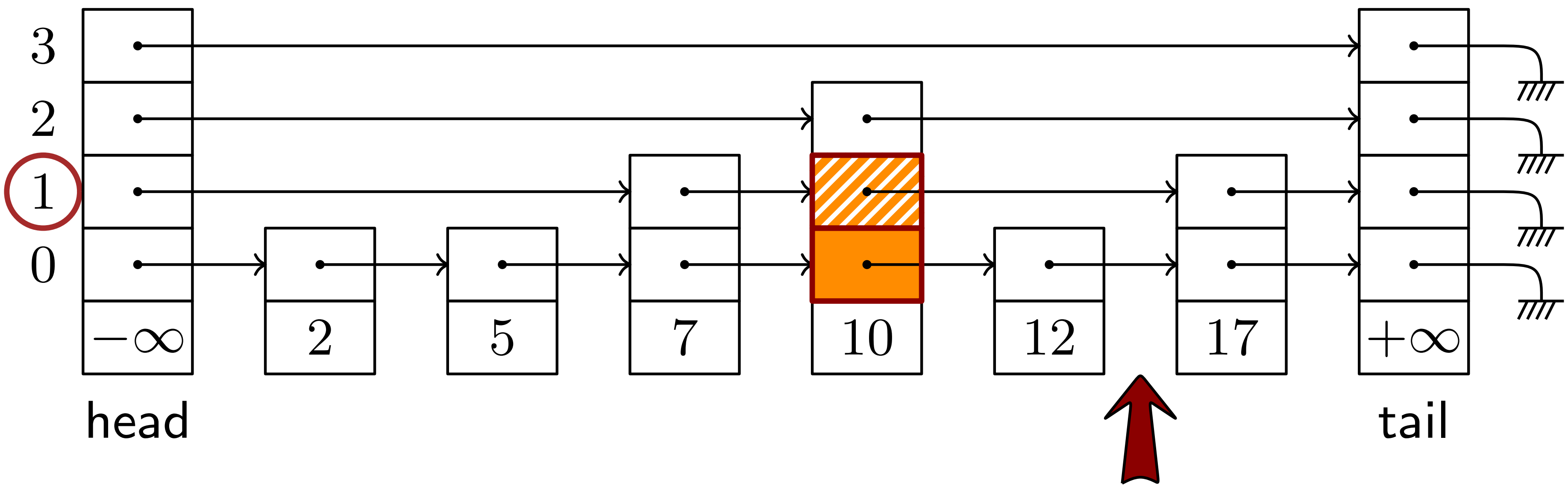

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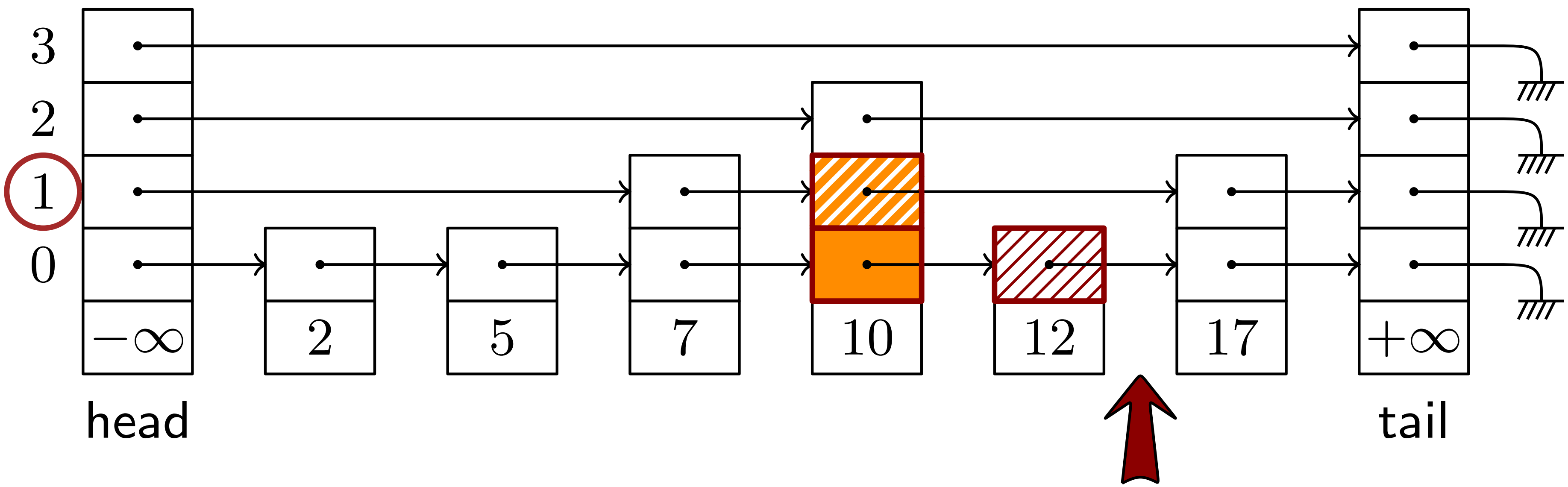

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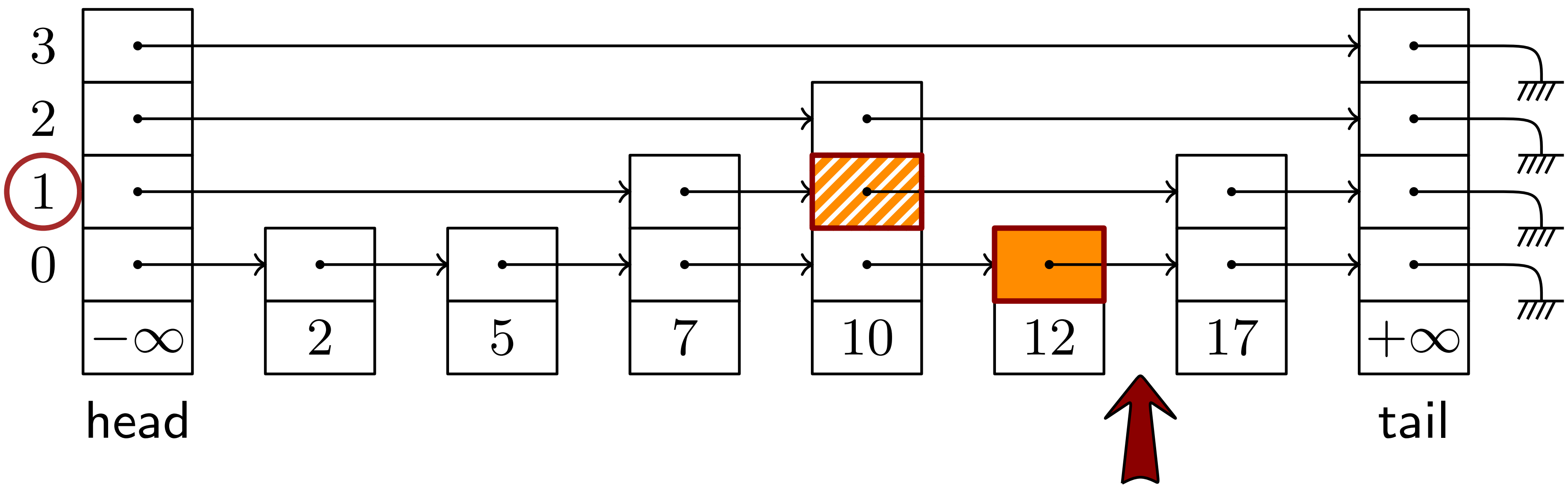

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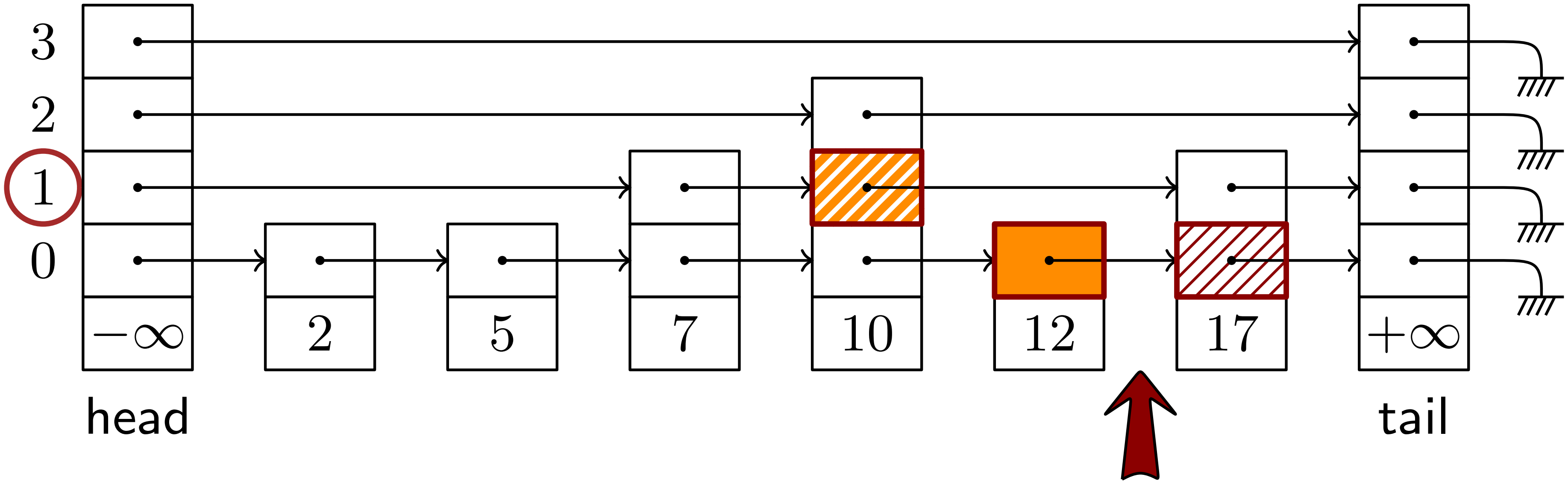

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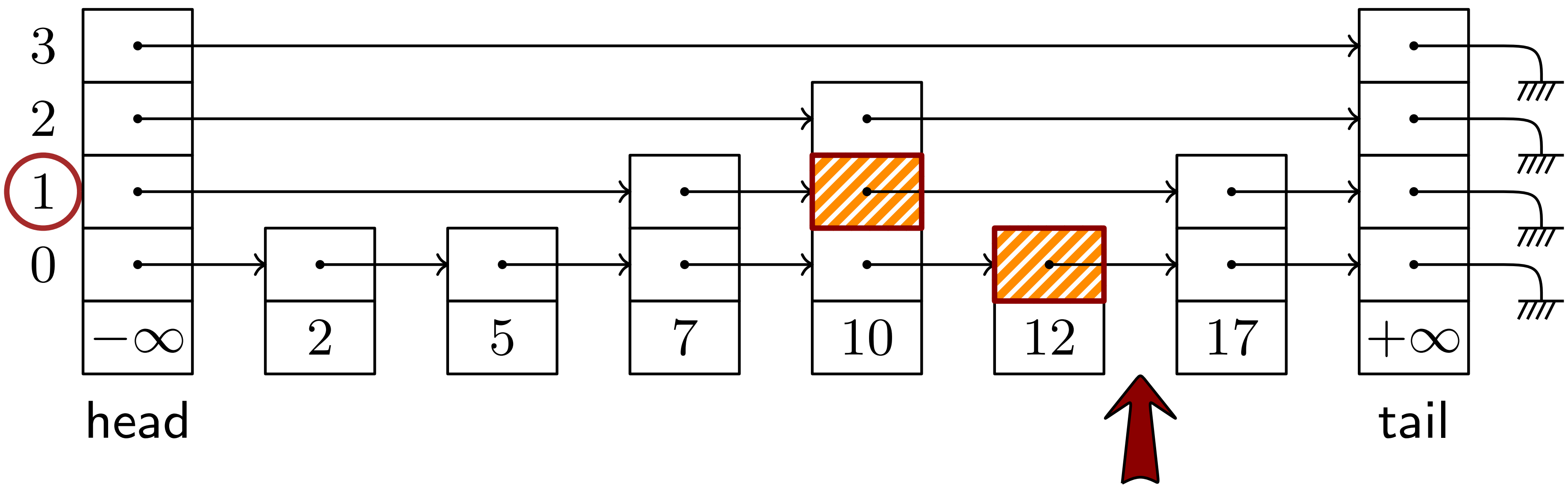

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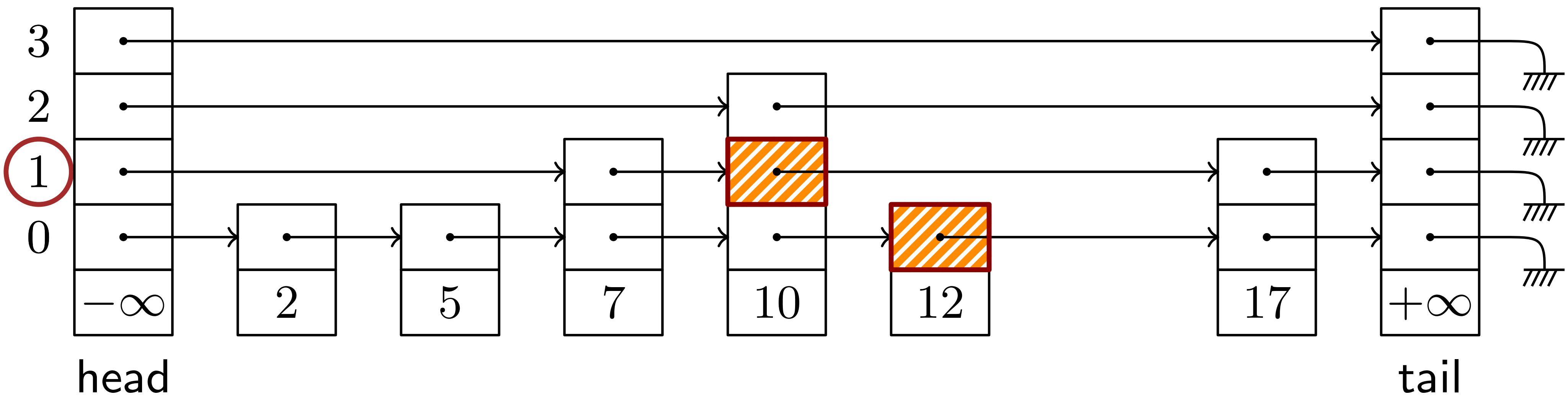

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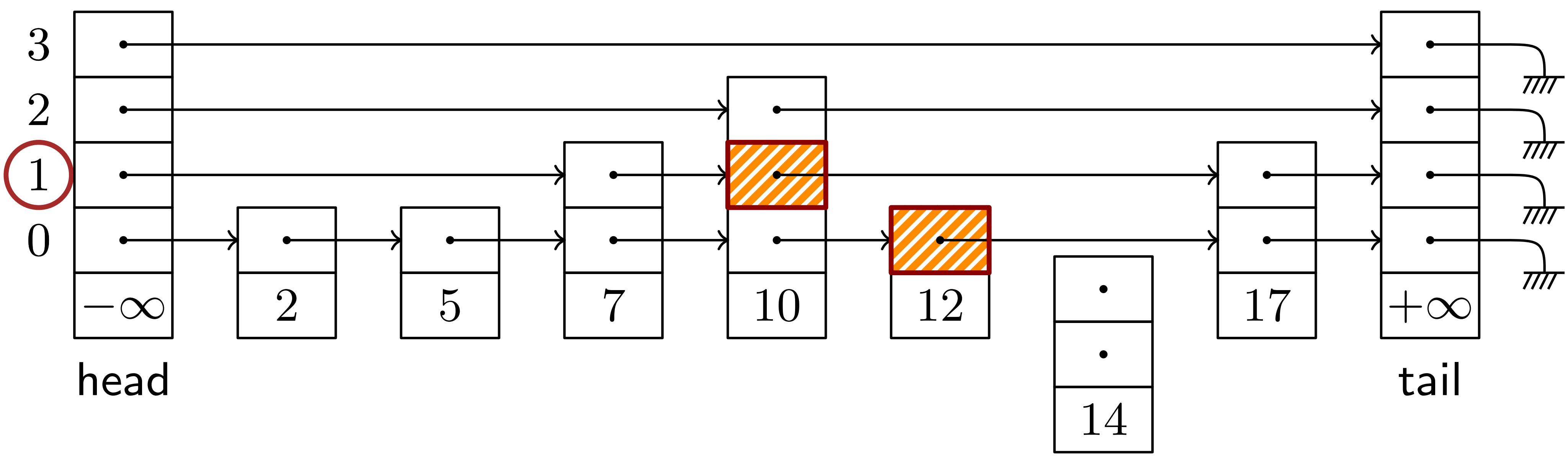

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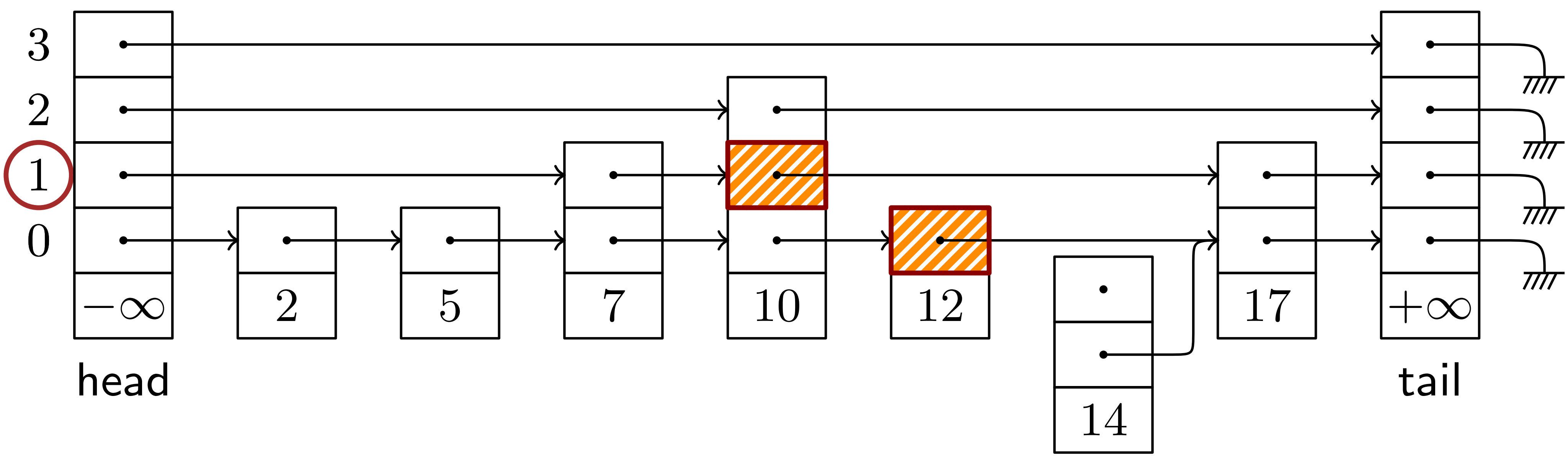

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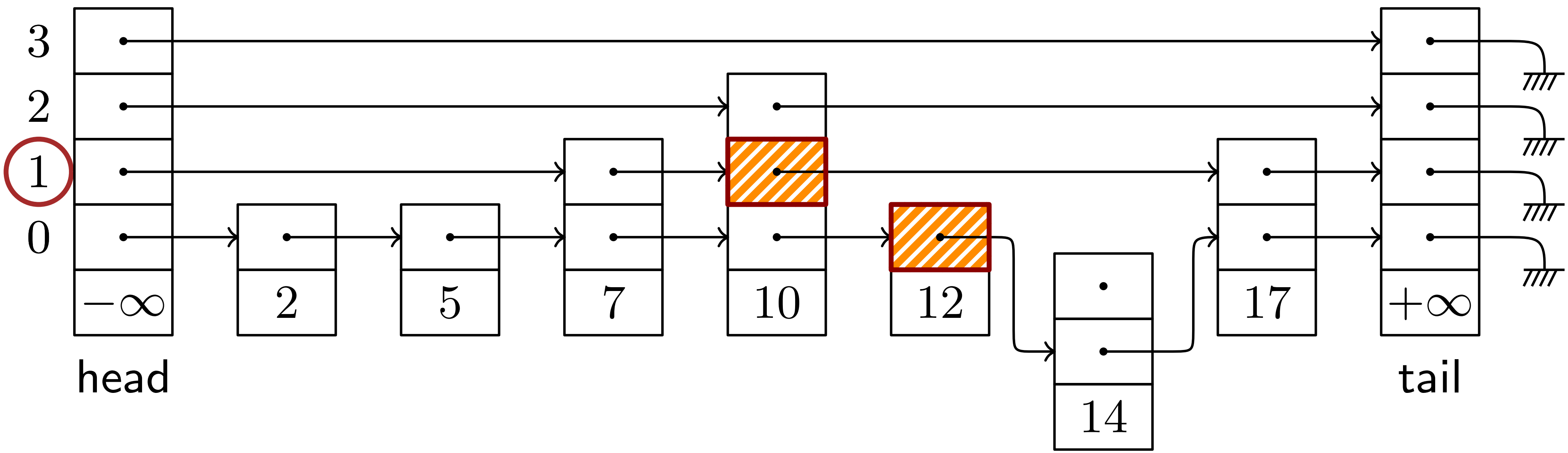

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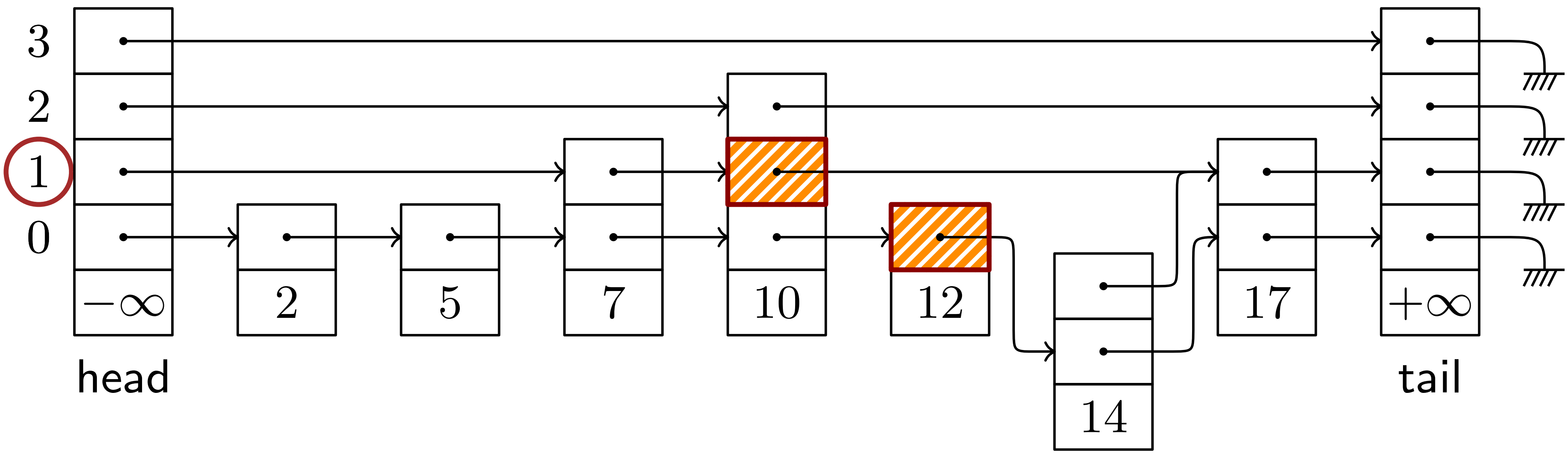

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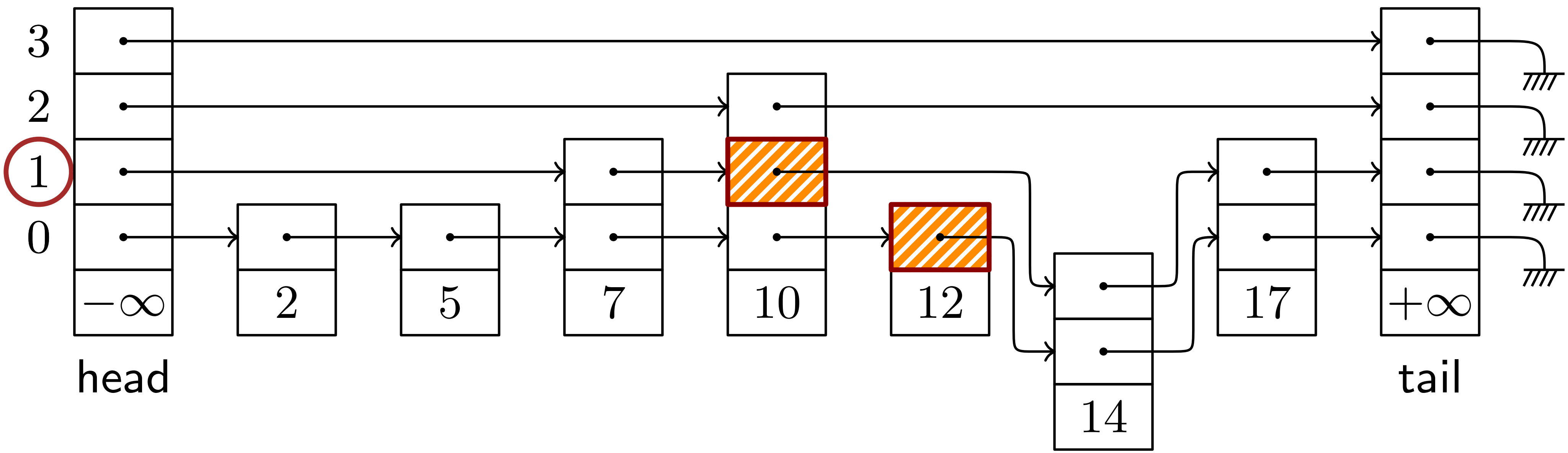

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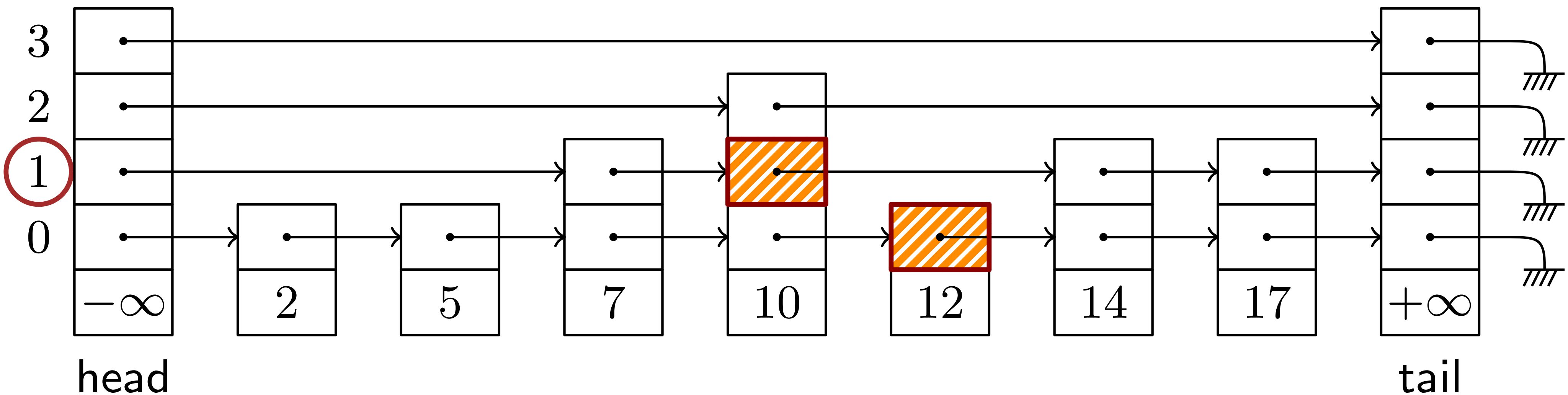

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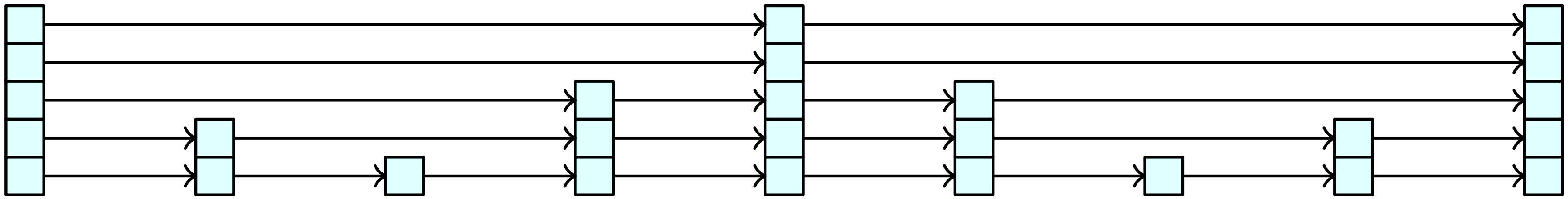

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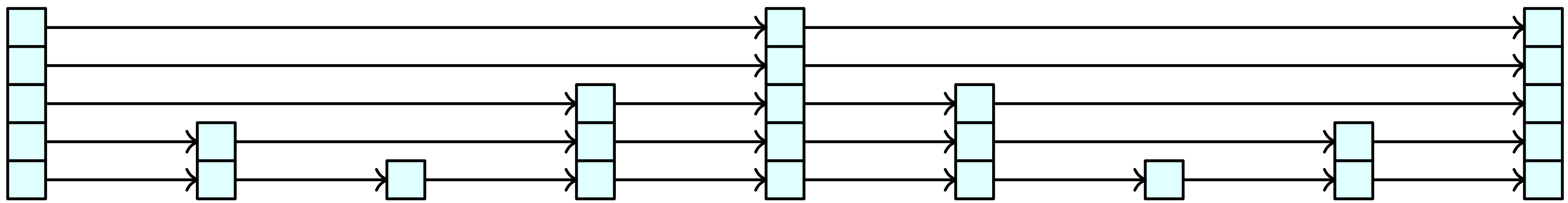

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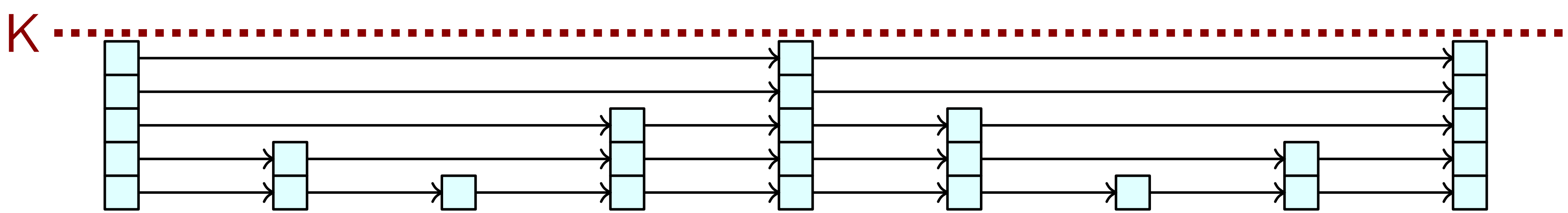

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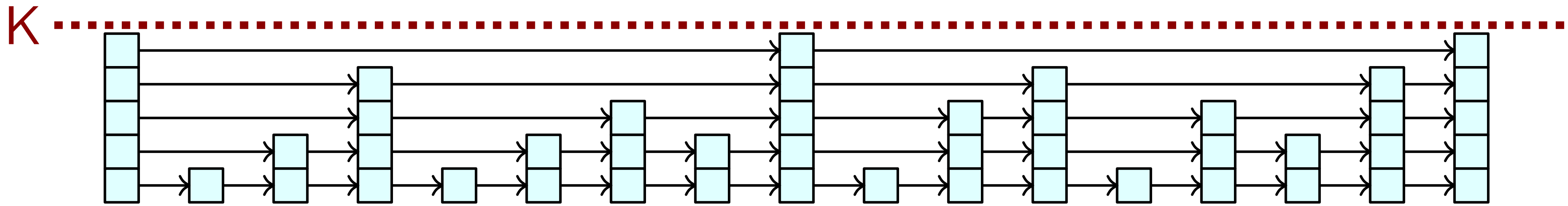

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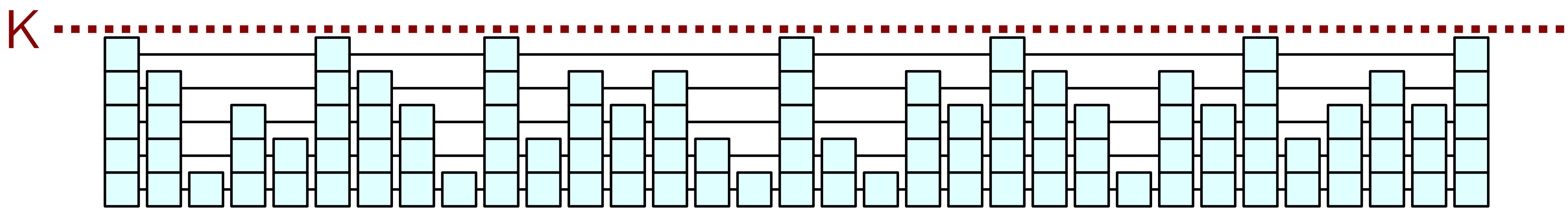

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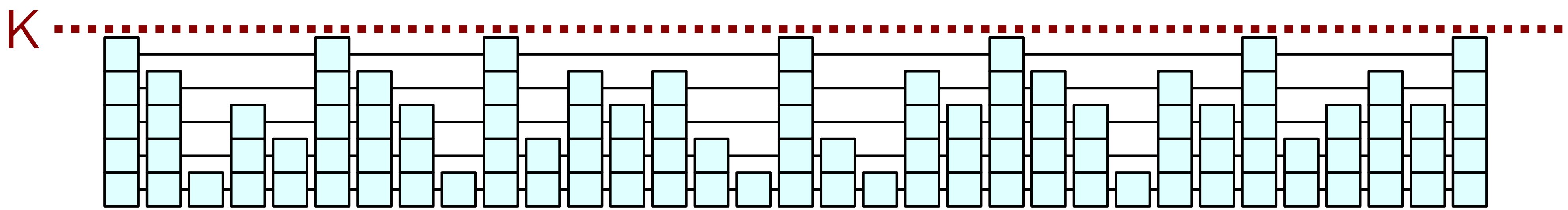

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TSL

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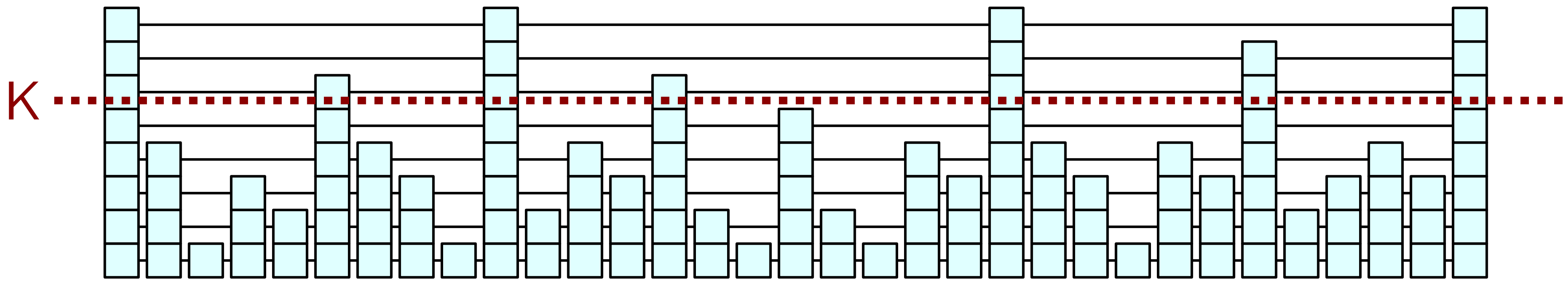
Dynamic height is required



Verification of Skiplists

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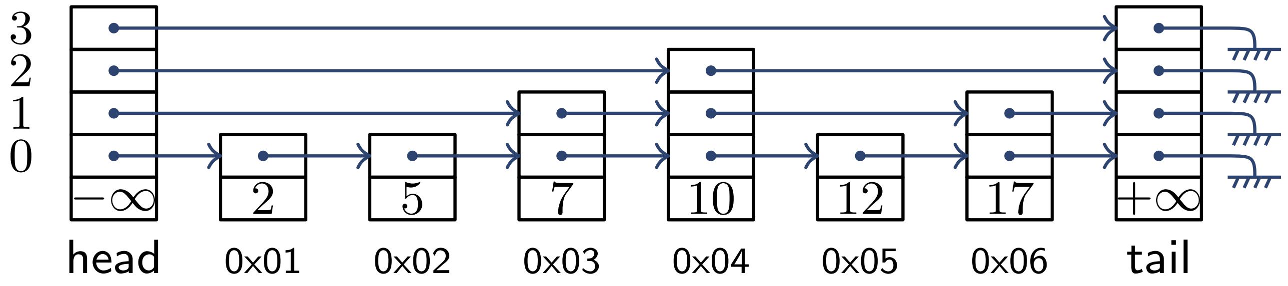

Verification of Skiplists

- **Skiplist** shape **preservation** : $\Box SkipList(heap, sl, r)$

skiplist

$$SkipList(heap, sl, r) \hat{=}$$

heap

region

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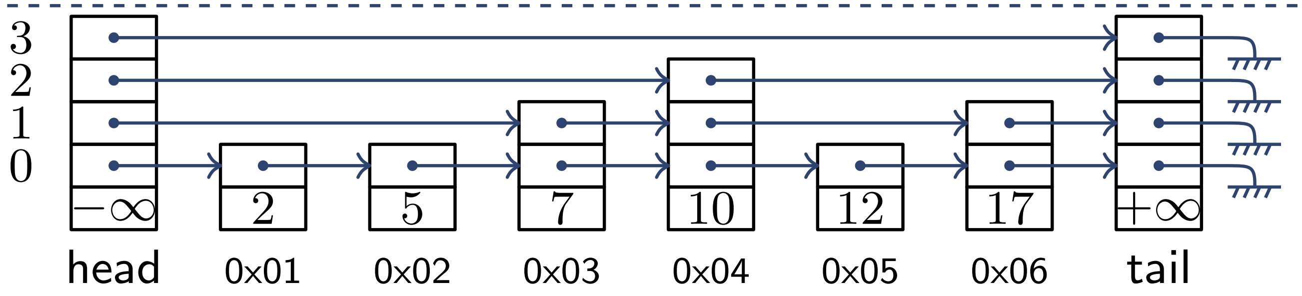

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$$SkipList(heap, sl, r) \hat{=} \left(ordList(heap, head, tail, 0) \right)$$

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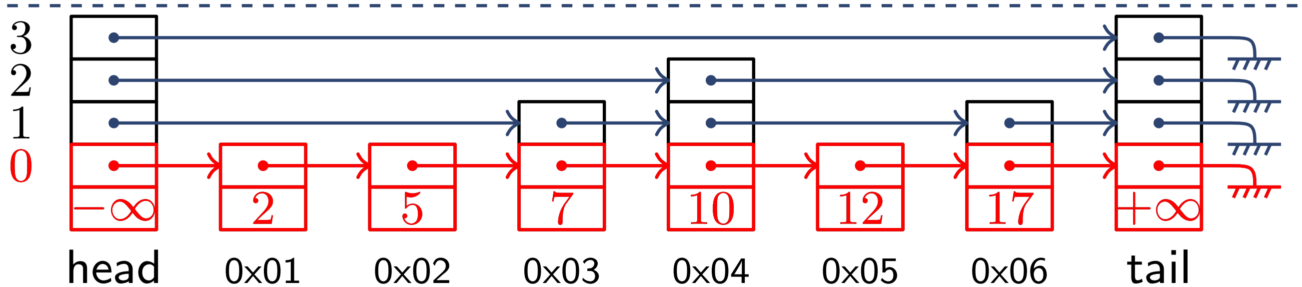

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$$SkipList(heap, sl, r) \hat{=} \left(\begin{array}{c} ordList(heap, head, tail, 0) \quad \wedge \\ r = region(heap, head, tail) \end{array} \right)$$

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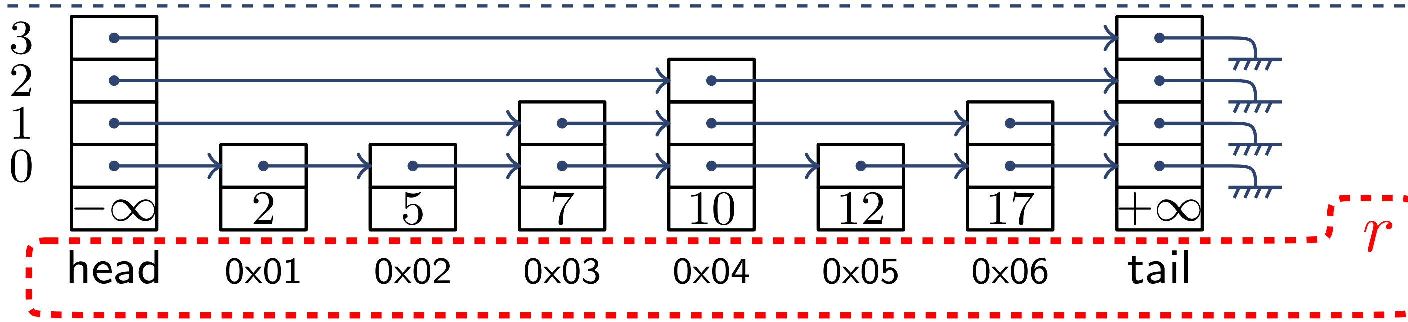

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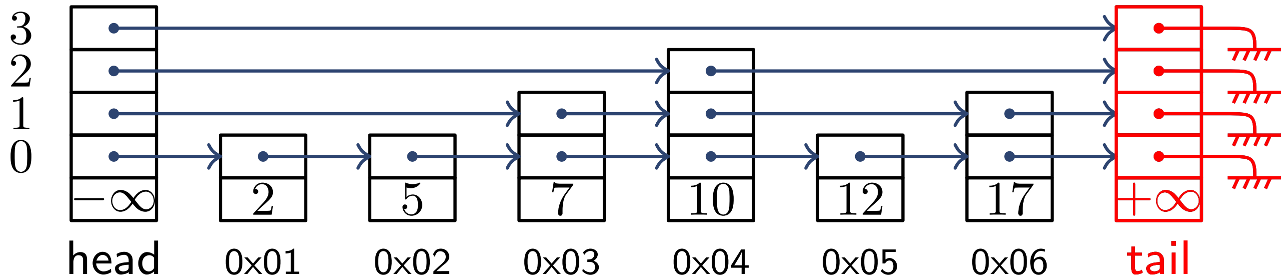
$M = 3$



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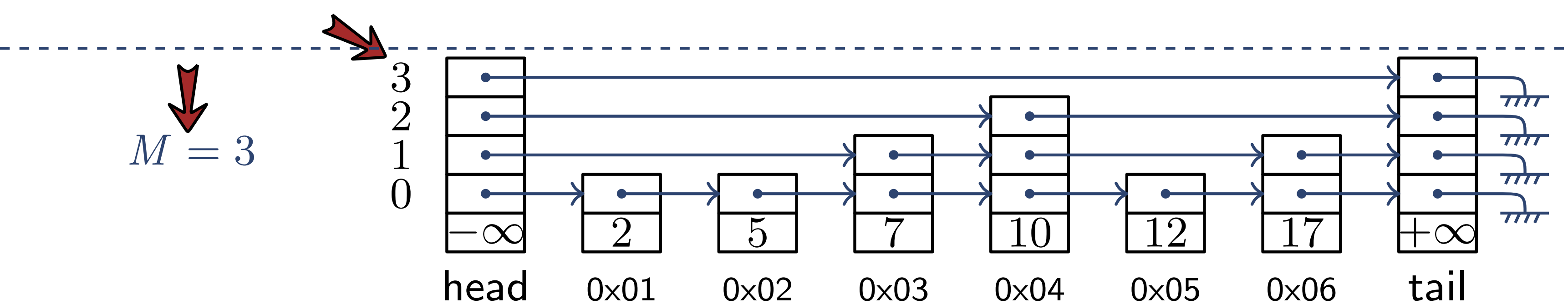

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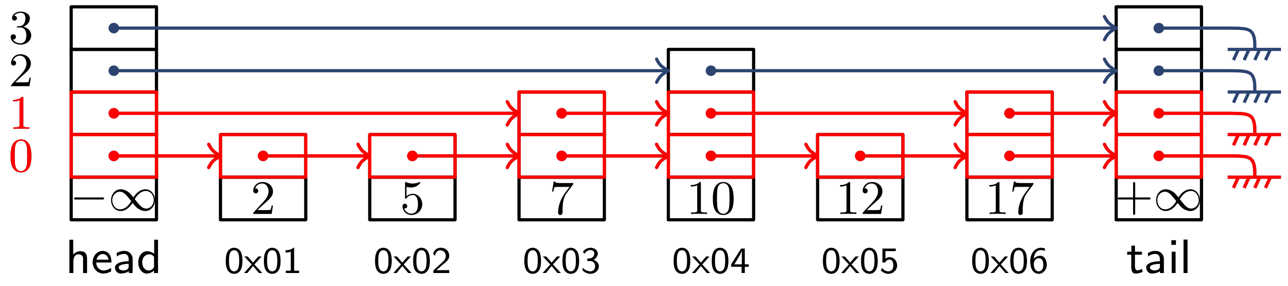

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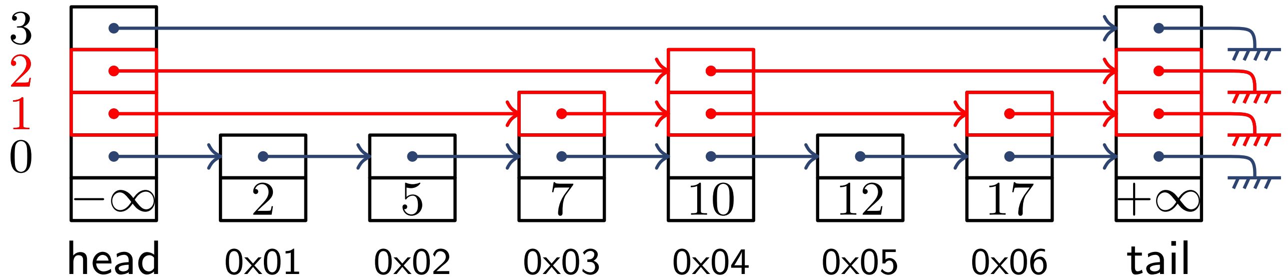

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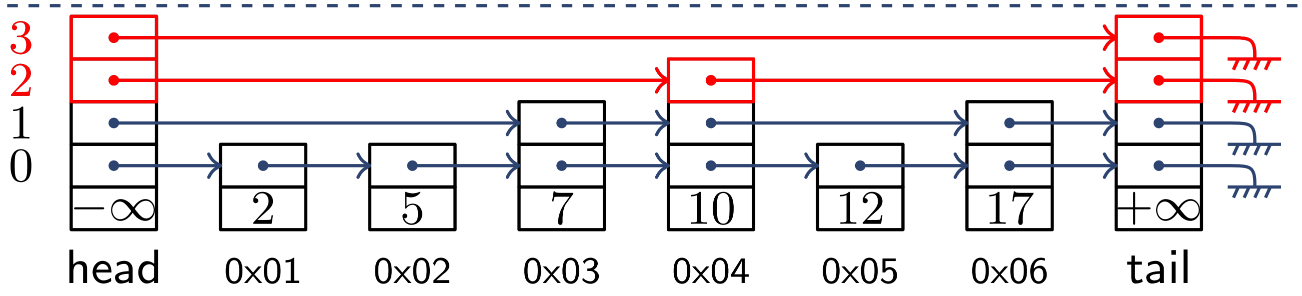

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```
 9: . . .
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```

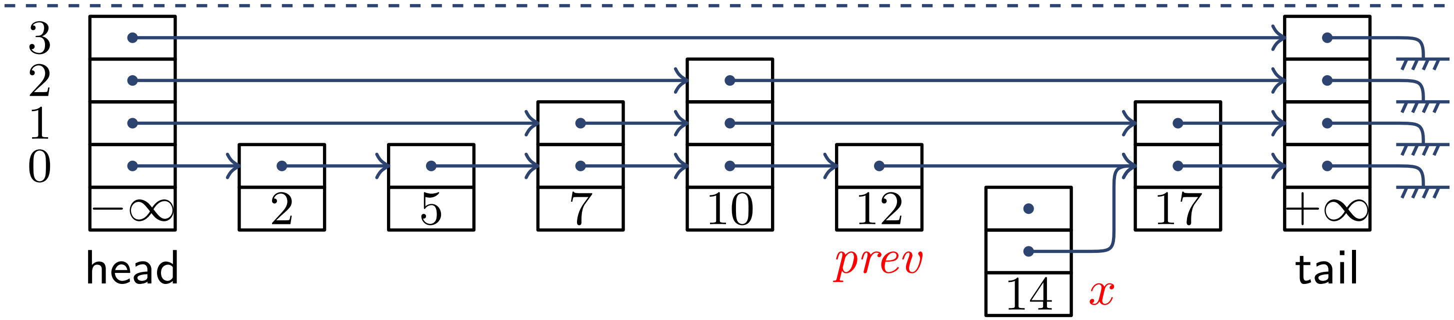

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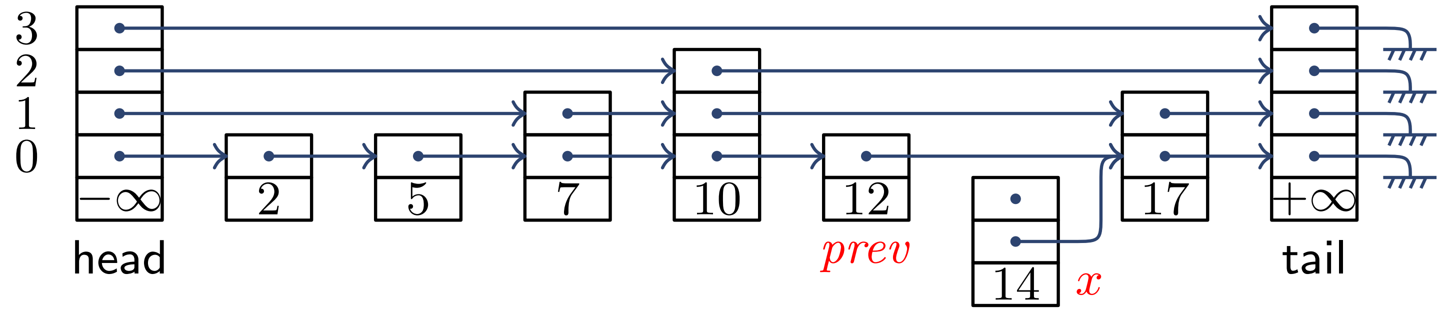

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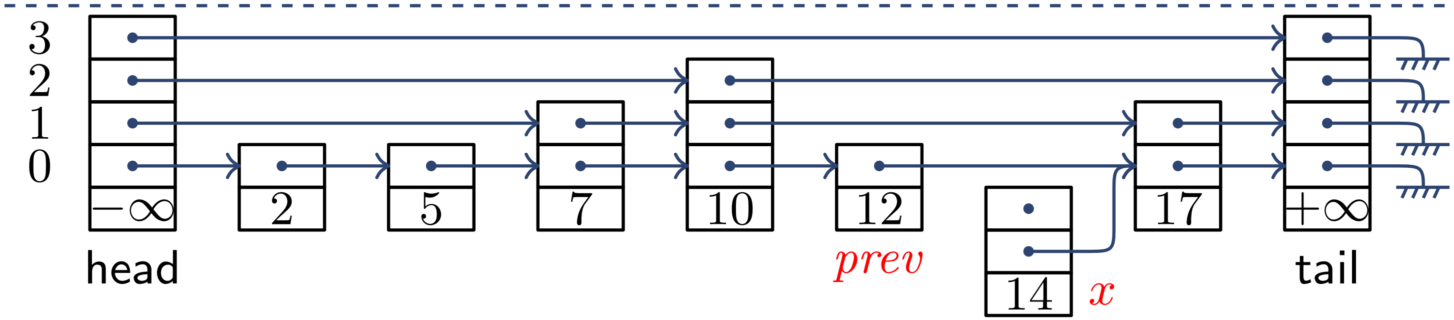

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$$SkipList(h, sl, r) \wedge \left(\begin{array}{rcll} x.val & = & 14 & \wedge \\ prev.val & < & 14 & \wedge \\ x.arr[0].val & > & 14 & \wedge \\ prev.arr[0] & = & x.arr[0] & \wedge \\ & x \notin r & & \end{array} \right) \wedge \left(\begin{array}{cl} at_{10} & \wedge \\ prev'.arr[0] = x & \wedge \\ at'_{11} & \wedge \\ h' = h \wedge sl = sl' & \wedge \\ r' = r \cup \{x\} \wedge x' = x & \ldots \end{array} \right)$$

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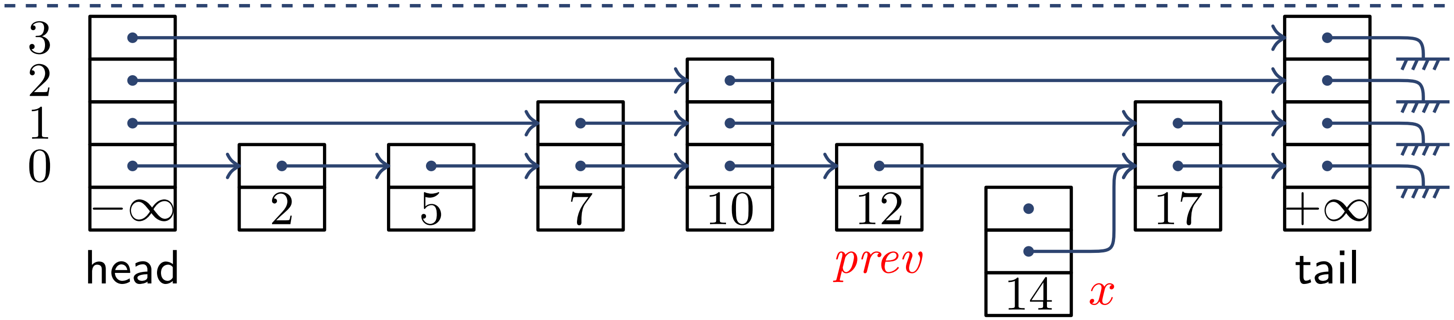

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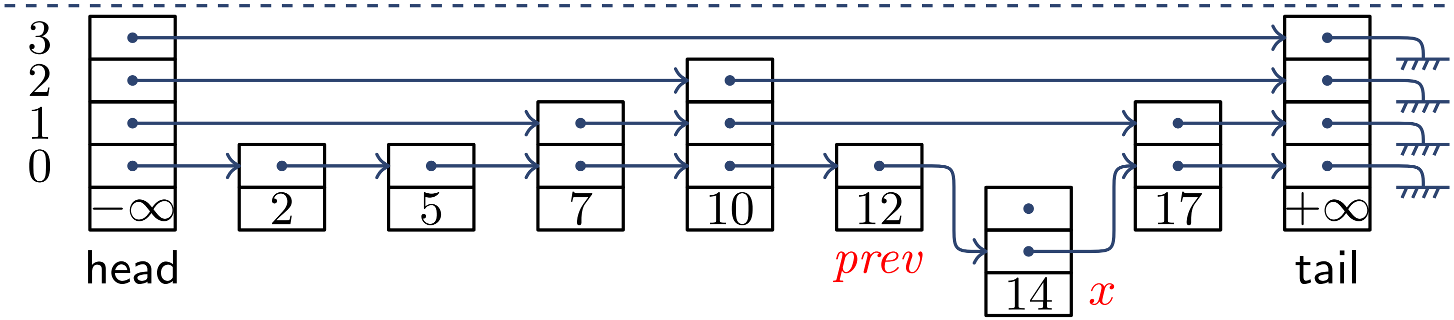

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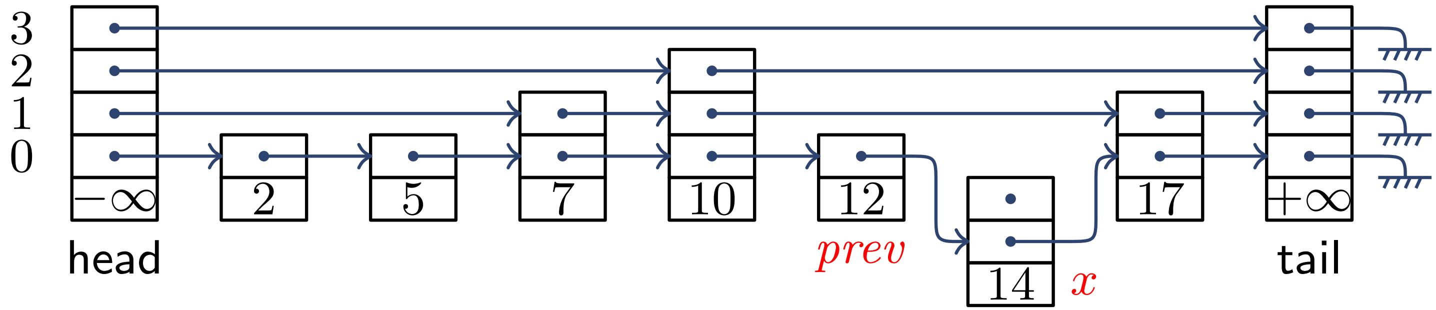

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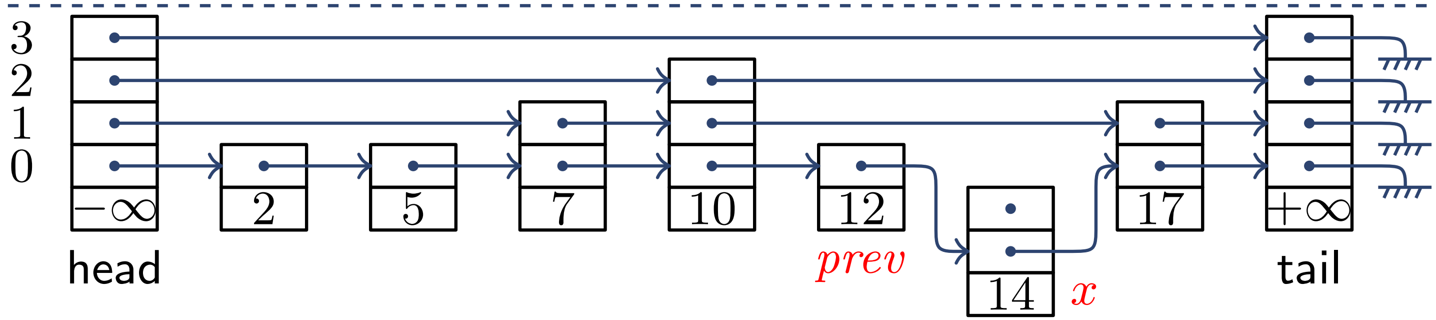

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reason about

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reason about

ordered values + notion of ordered list

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reason about

levels

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reason about

arrays

Verification of Skiplists

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reason about

regions (sets)

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reason about

memory, cells

Our Contribution

- **TSL**, a theory for skiplists of **arbitrary length and height**
- We show TSL **decidable**...
- ...by reducing **TSL satisfiability** to **TSL_K satisfiability**.
- Show it suitable for verifying **real world implementations**

TSL:Theory of Skiplist of Arbitrary Height

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- TSL, like TSL_K, is a **union of other theories**

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Σ_{addr}

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$$\Sigma_{\mathsf{addr}} \cup \Sigma_{\mathsf{elem}}$$

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TSL:Theory of Skiplist of Arbitrary Height

- TSL, like TSL_K, is a **union of other theories**

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TSL_K	TSL

TSL:Theory of Skiplist of Arbitrary Height

- TSL, like TSL_K, is a **union of other theories**

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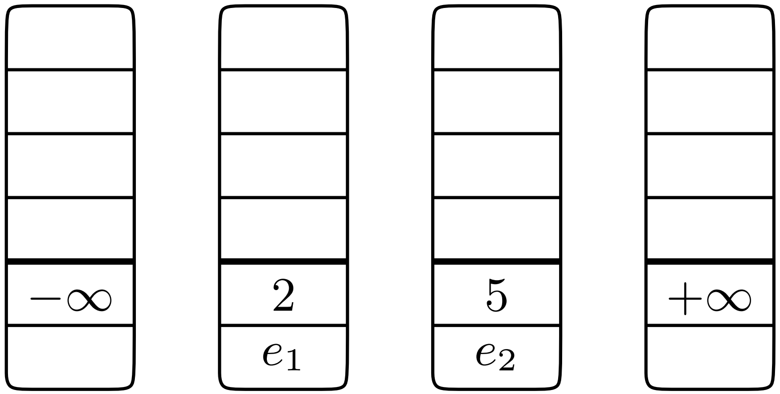
TSL_K | **TSL**



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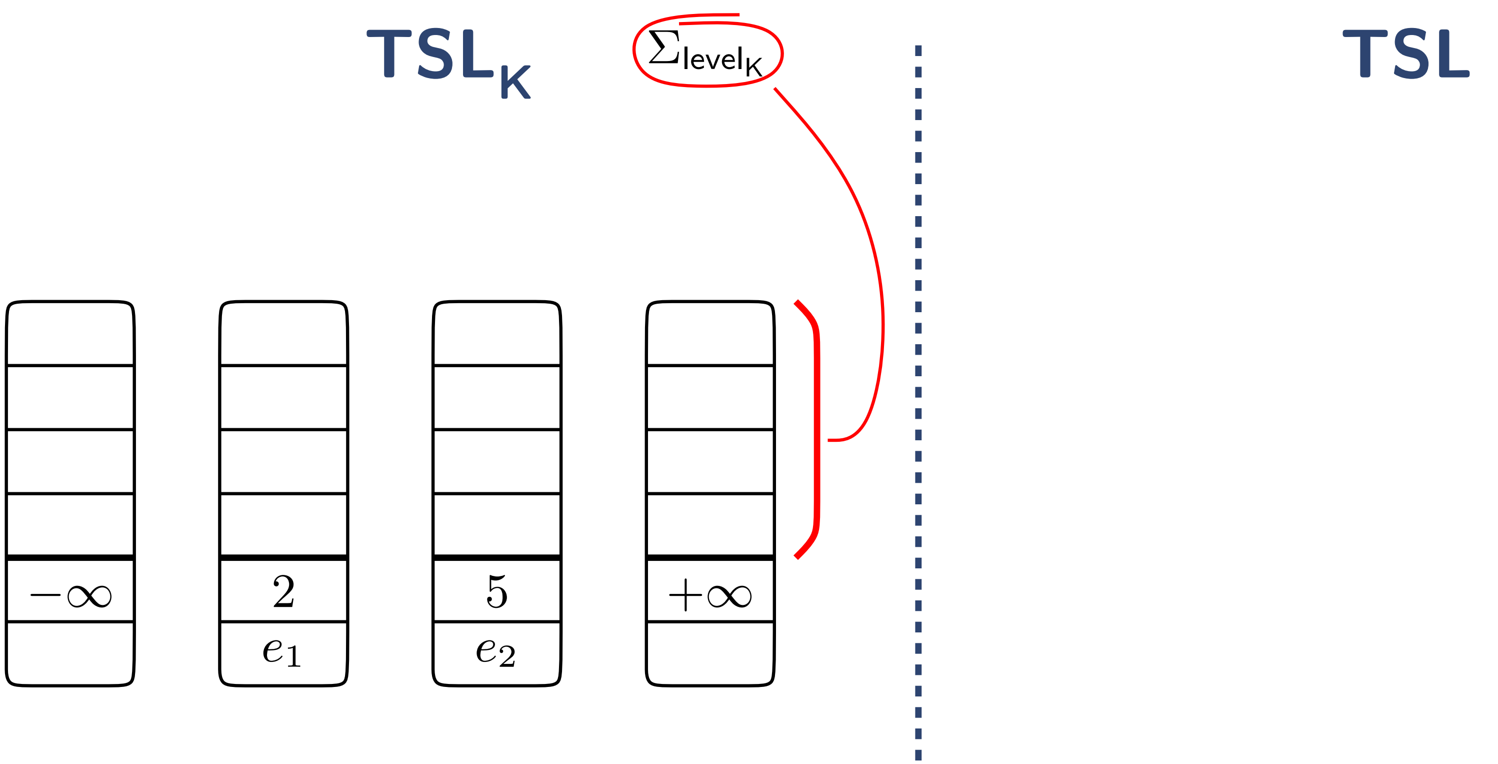

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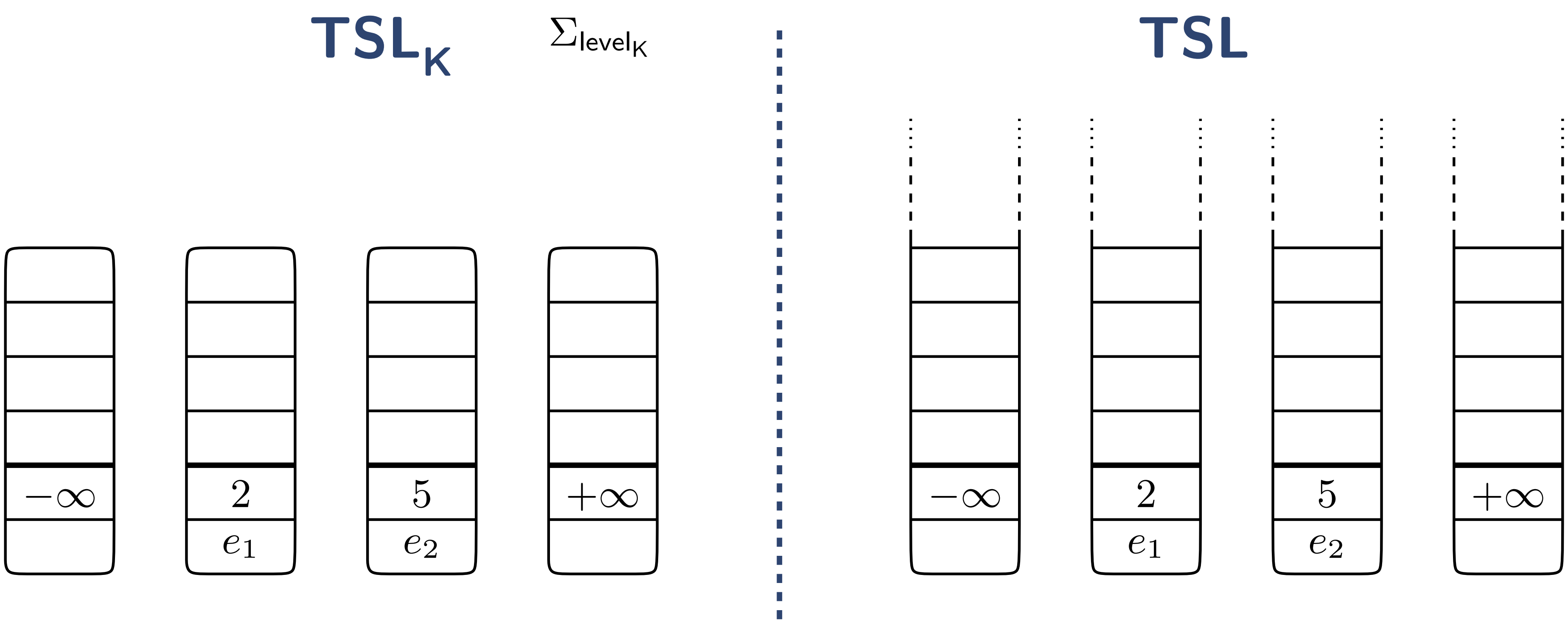

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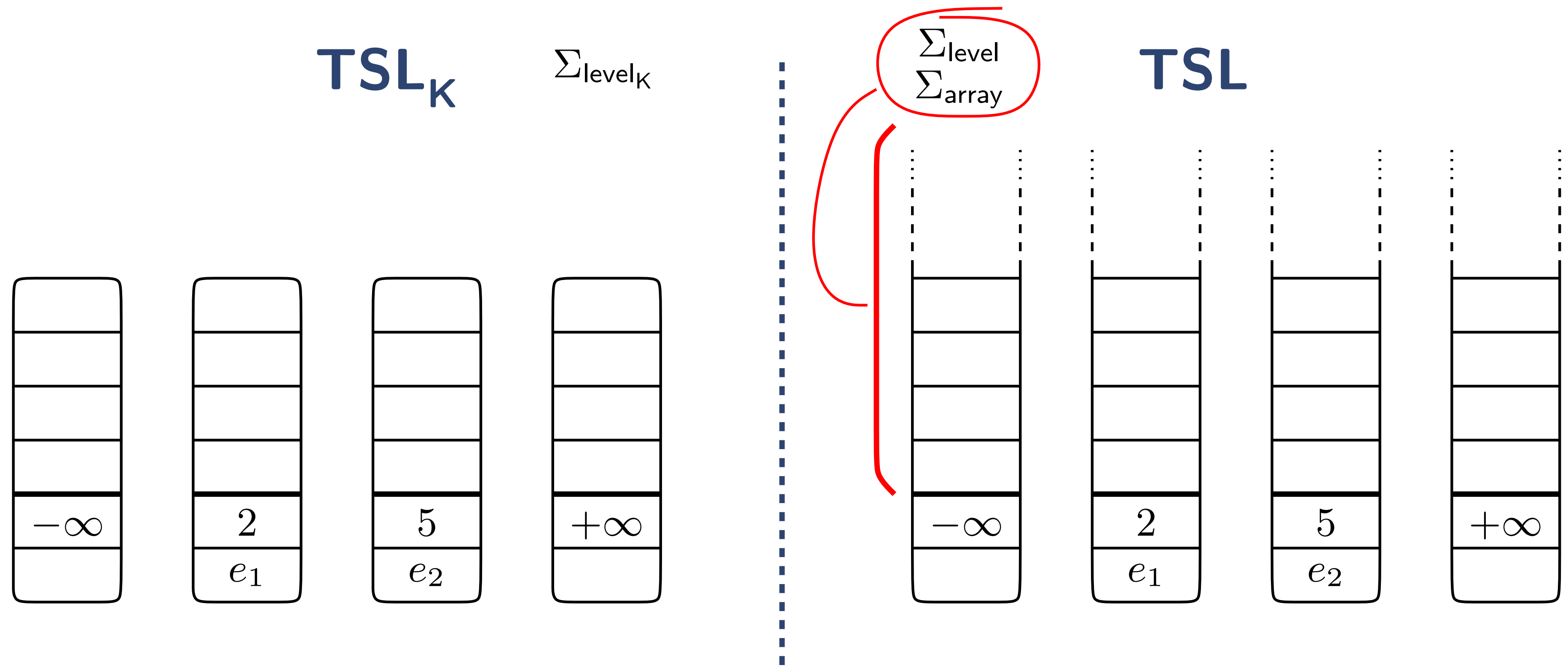

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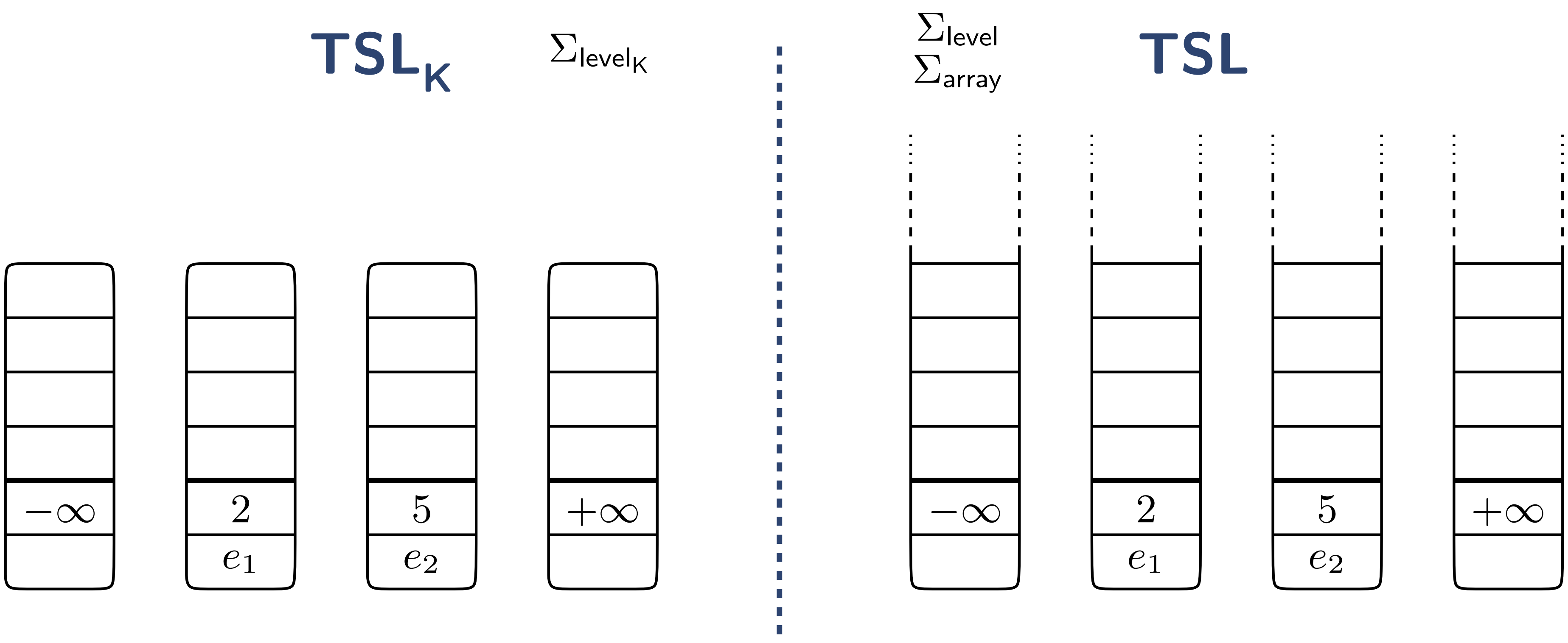

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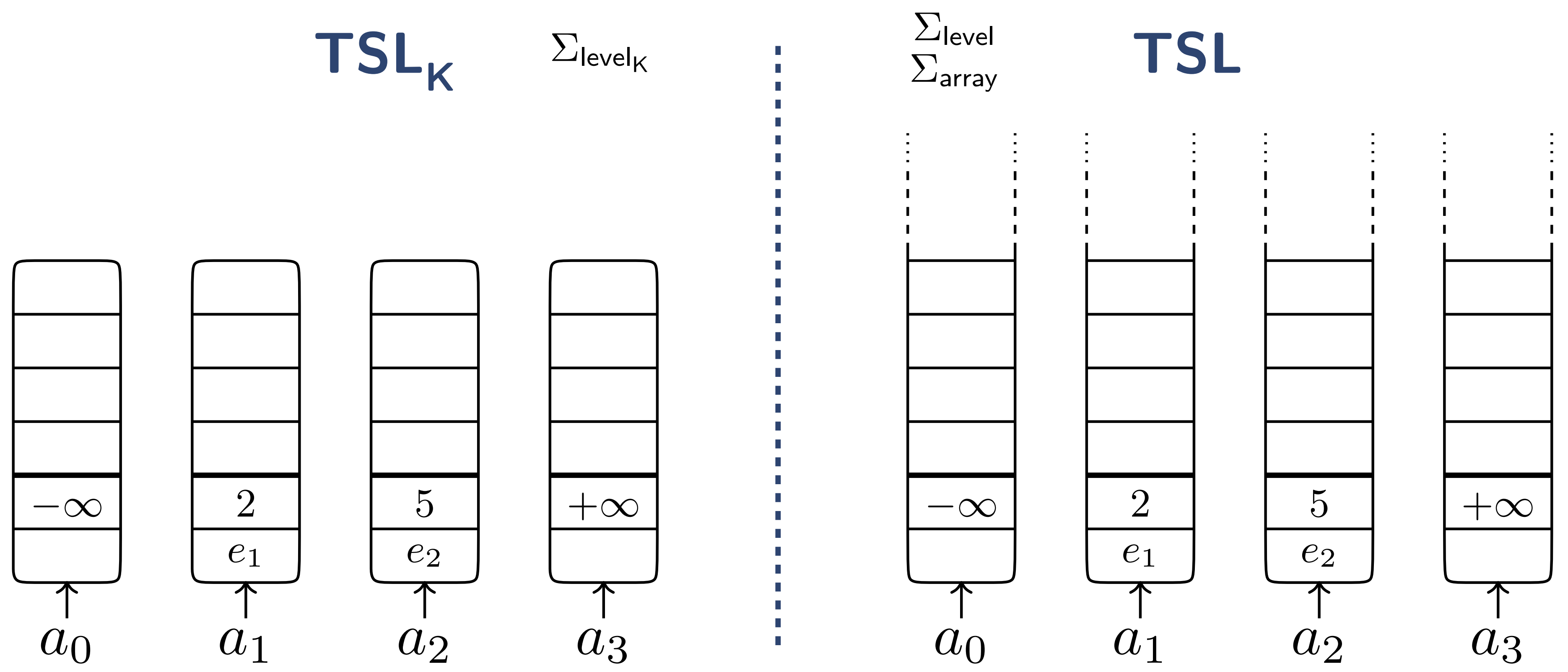

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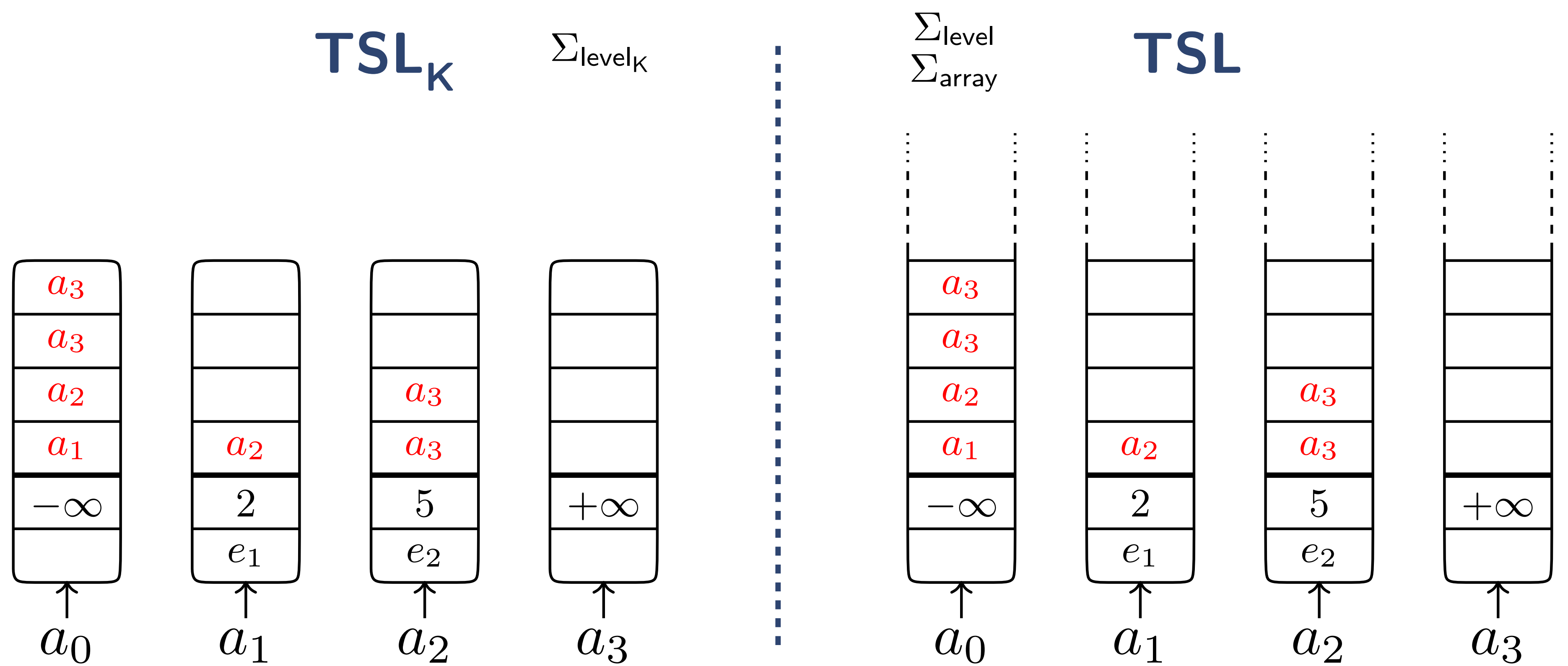

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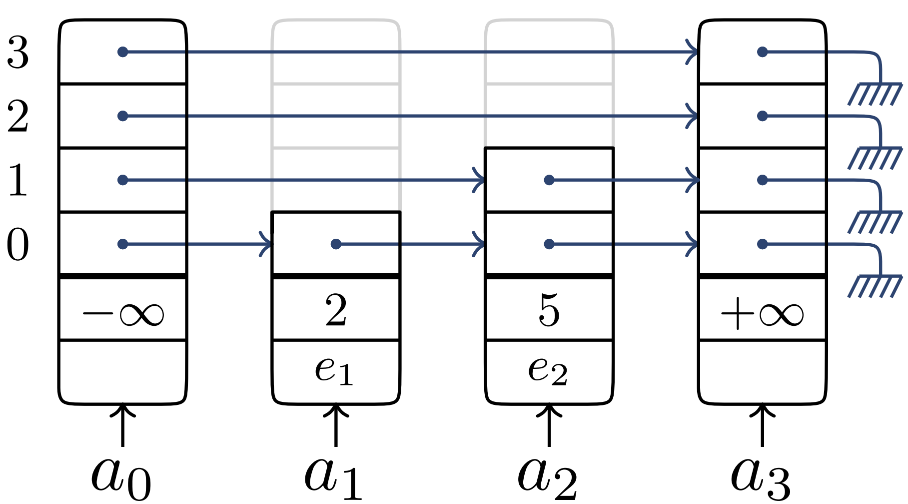
TSL_K

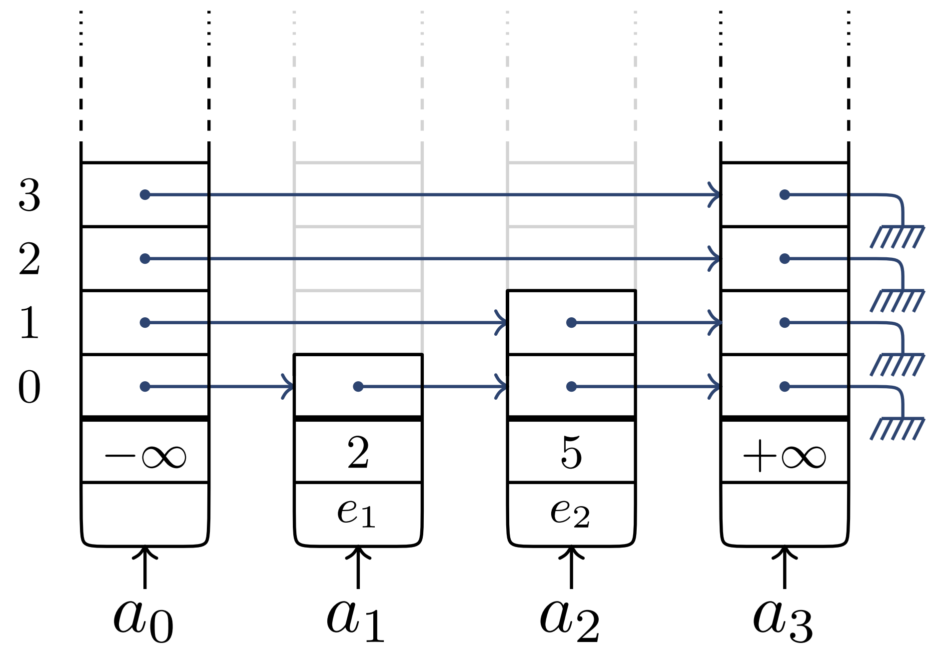
$\Sigma_{\text{level}_\text{K}}$



Σ_{level}

Σ_{array}

TSL



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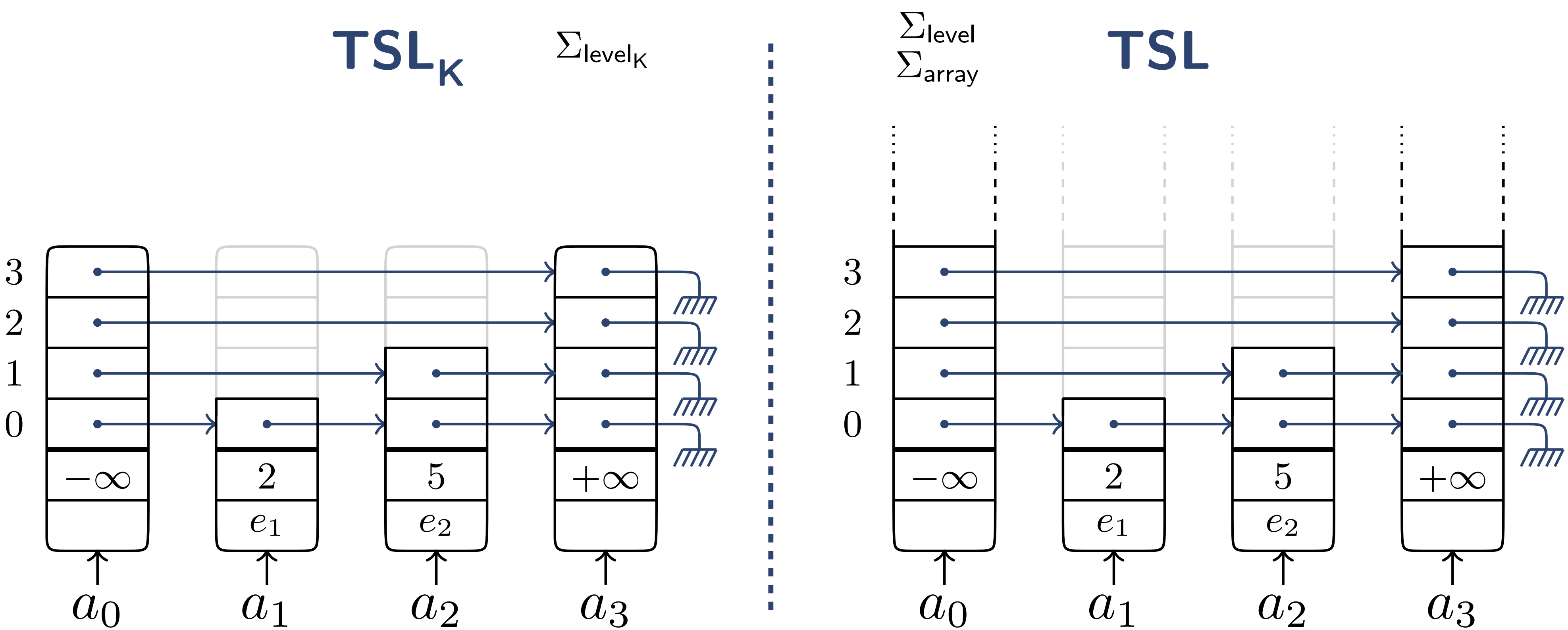

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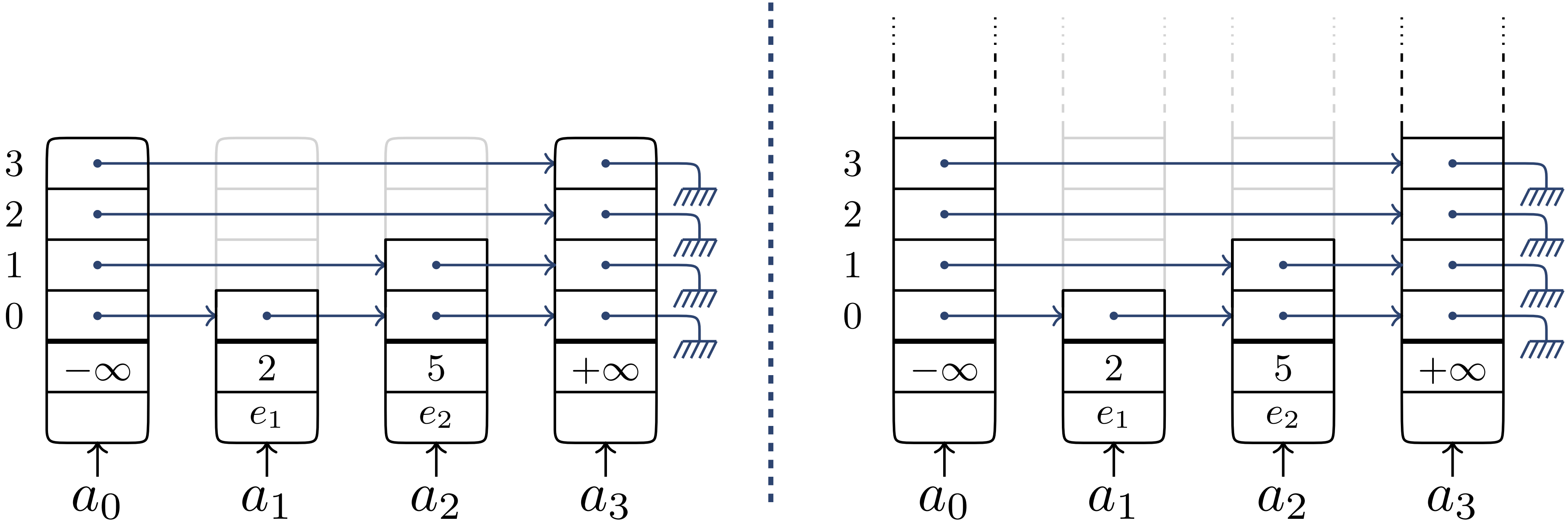
path = a non-repeating sequence of addresses

$$[a_1, a_2, a_3]$$

TSL_K $\Sigma_{\mathsf{level}_\mathsf{K}}$

Σ_{level} Σ_{array} **TSL**



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$$append([a_1, a_2], [a_3], [a_1, a_2, a_3])$$

TSL_K $\Sigma_{\mathsf{level}_\mathsf{K}}$

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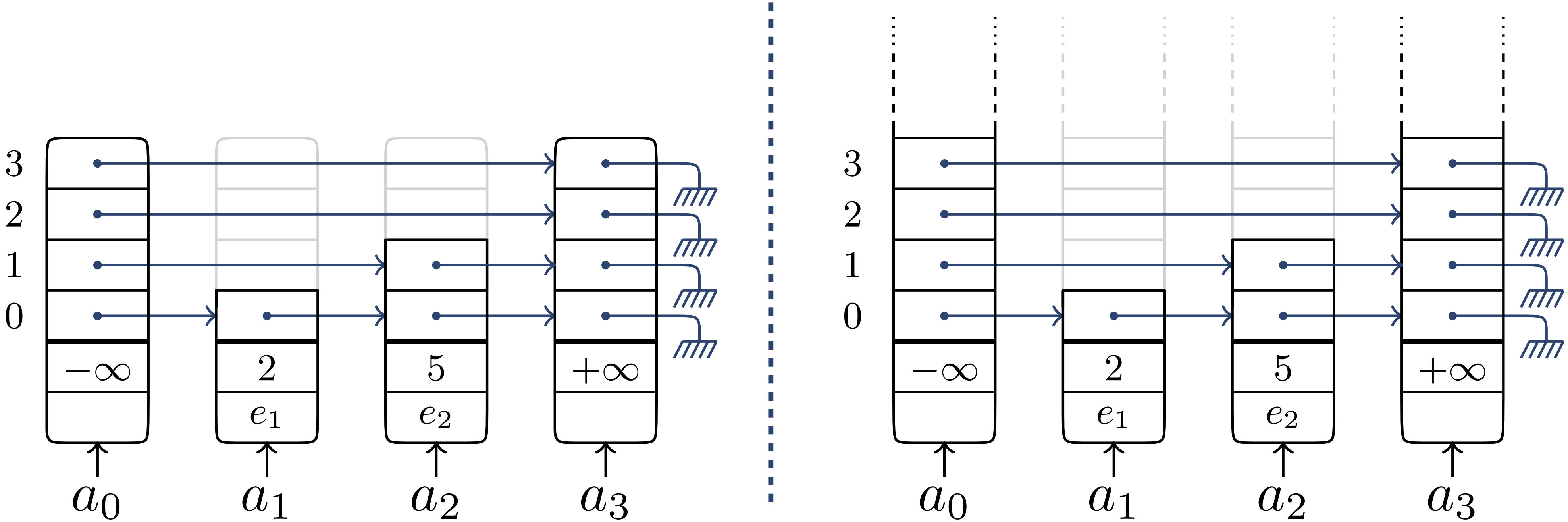

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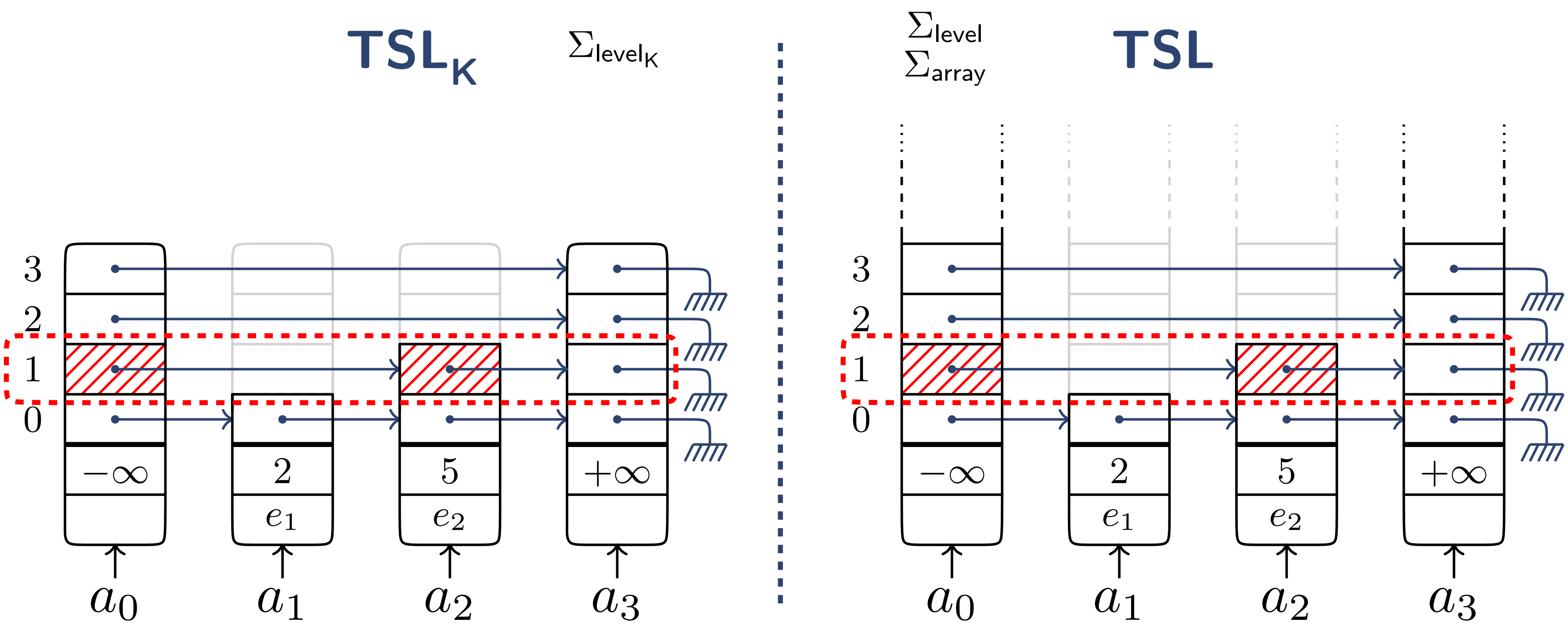
$$reach(a_0, a_3, 1, [a_0, a_2])$$



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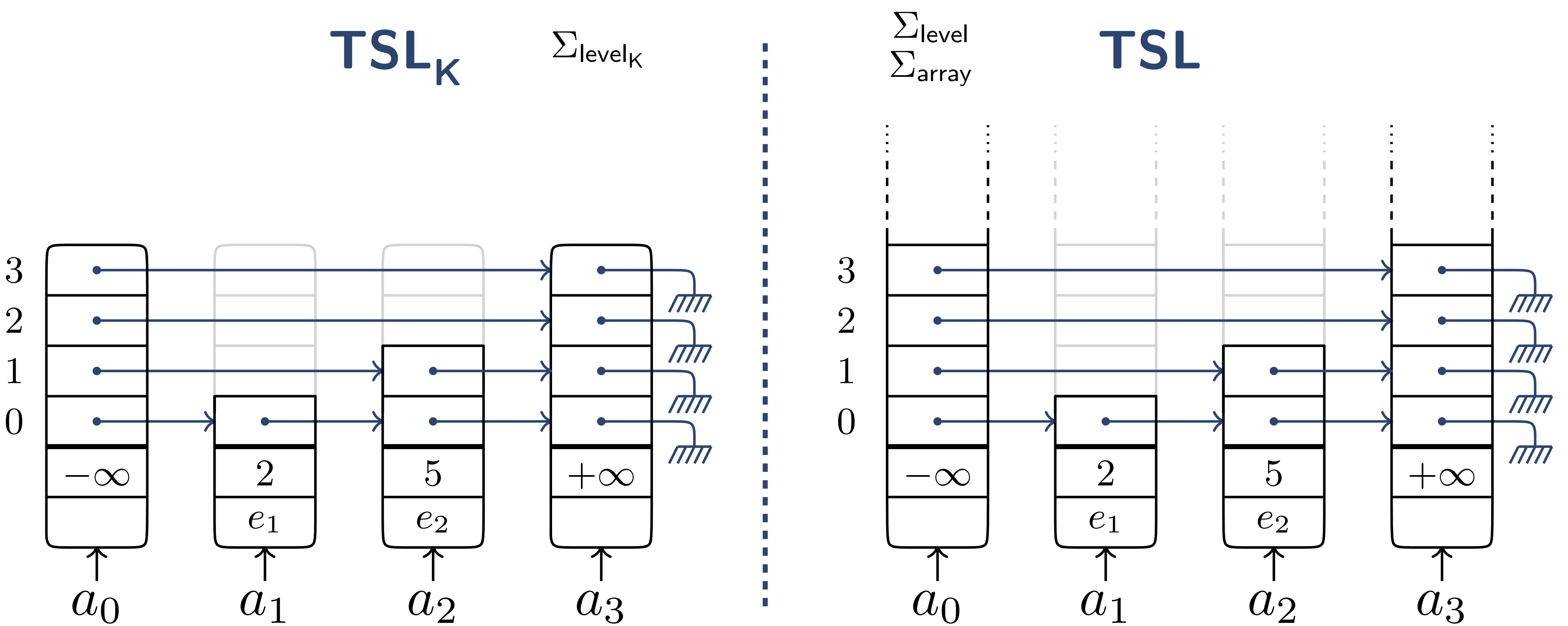

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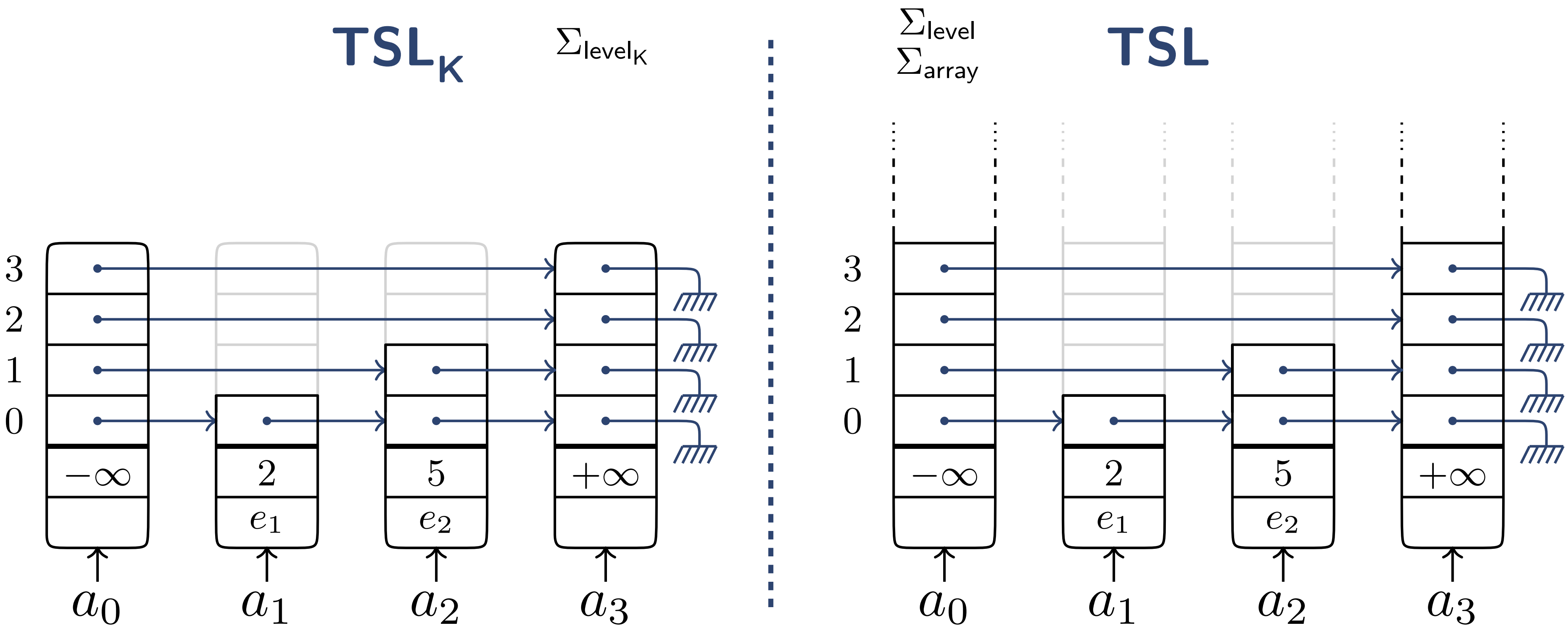
$$path2set([a_2, a_3]) = \{a_2, a_3\}$$



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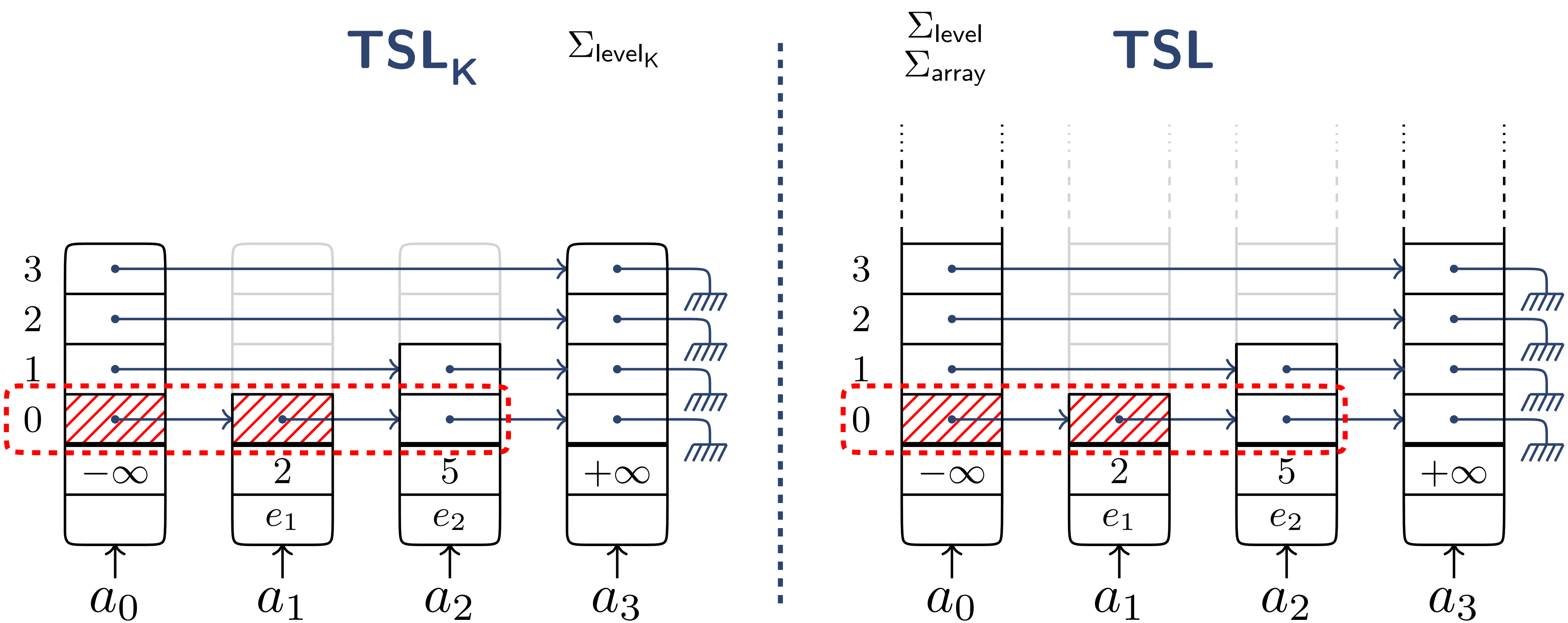
$$getp(a_0, a_2, 0) = [a_0, a_1]$$



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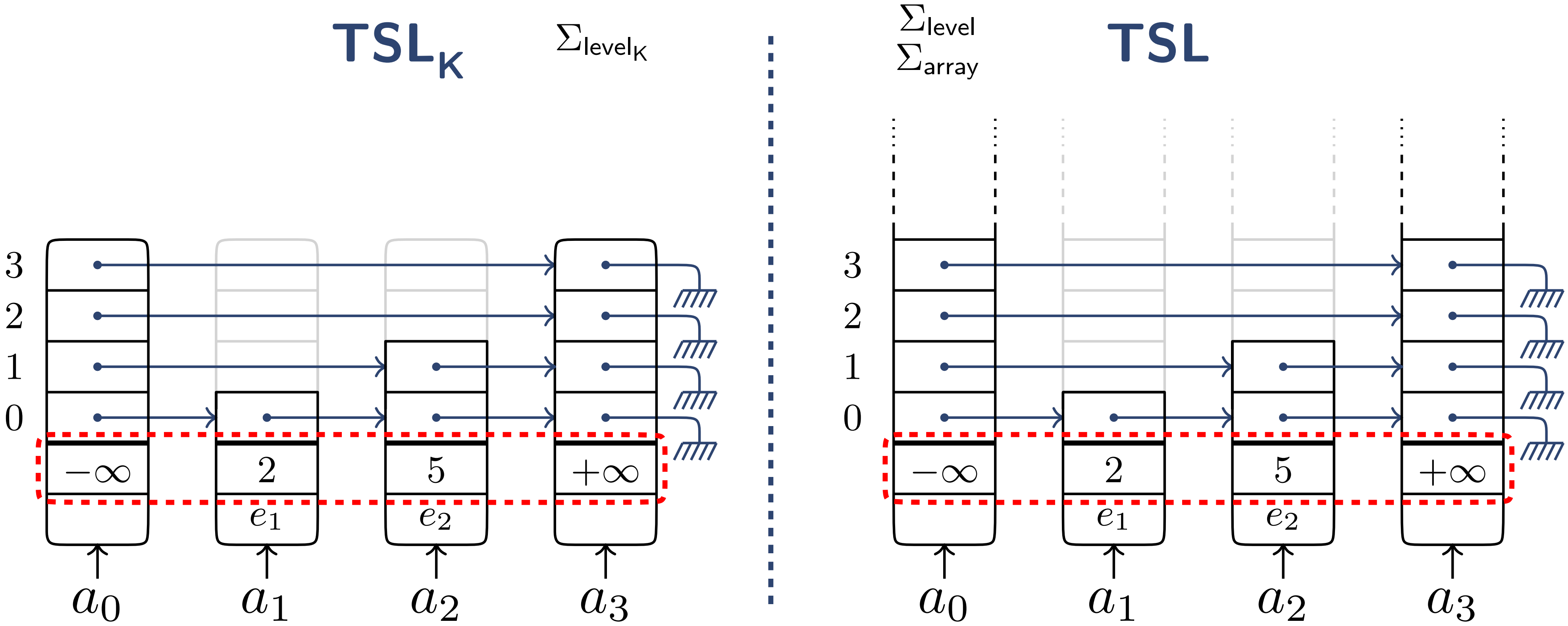
$$ordList([a_0, a_1, a_2, a_3])$$



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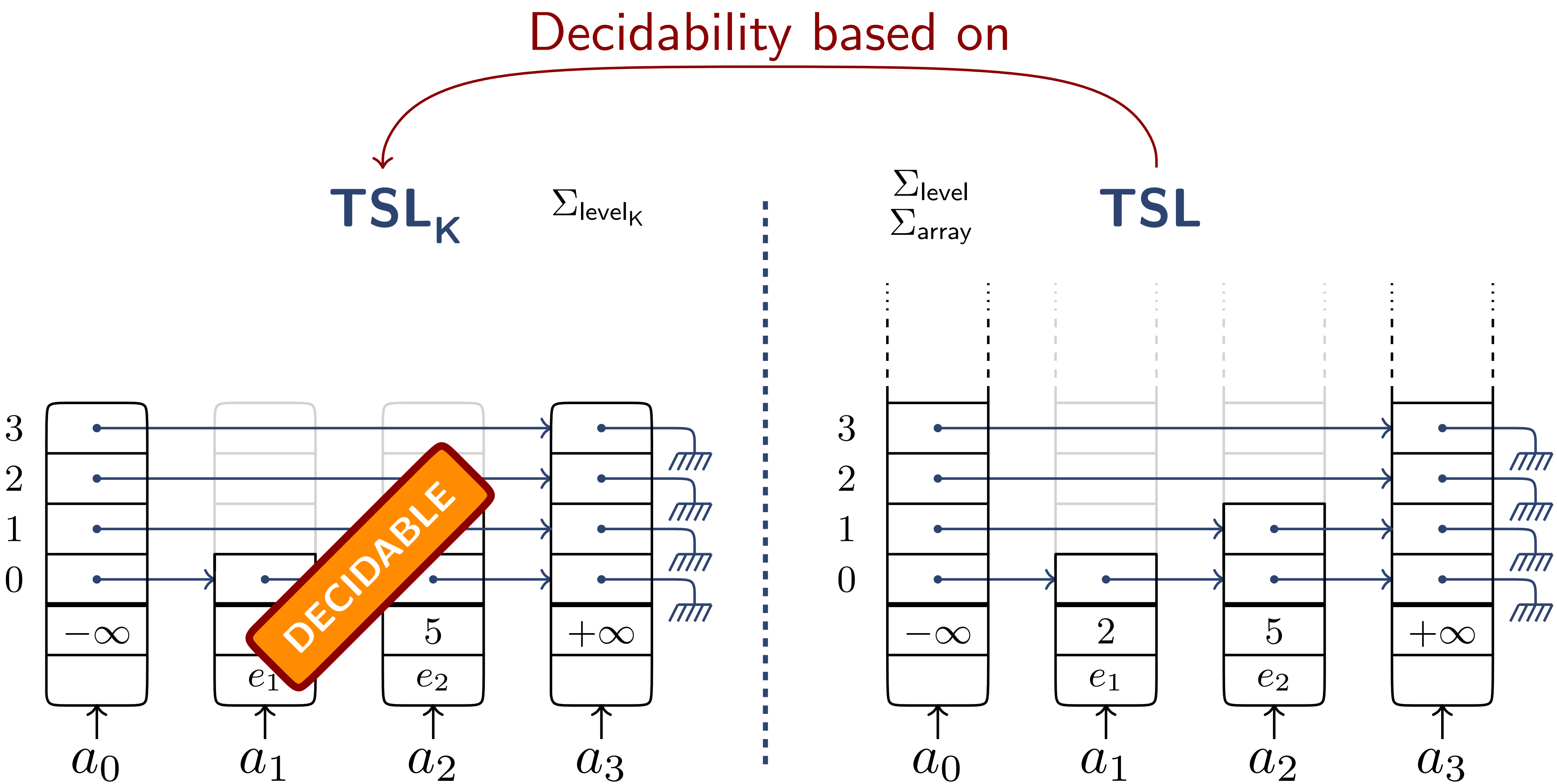

Decision Procedure for TSL

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- Let φ_{norm} be a normalized TSL formula

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$$\varphi \quad : \quad i = 0 \wedge \begin{pmatrix} A = heap[head].arr \quad \wedge \\ heap[head].max = 3 \end{pmatrix} \wedge B = A\{i \leftarrow tail\}$$

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Normalization

$$\varphi_{\mathsf{norm}} \quad : \quad i = 0 \wedge \begin{pmatrix} c = heap[head] & \wedge \\ c = mkcell(e, k, A, l) & \wedge \\ l = 3 & \end{pmatrix} \wedge B = A\{i \leftarrow tail\}$$

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- **STEP 1.** Sanitize

$$\varphi_{\mathsf{sanit}} := \varphi_{\mathsf{norm}} \wedge \bigwedge_{B=A\{l\leftarrow a\} \in \varphi_{\mathsf{norm}}} (l_{new} = l + 1)$$

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Why sanitization? Soon will be clear

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A possible arrangement: $\{i < l_{new} \ , \ i < l \ , \ l_{new} < l\}$

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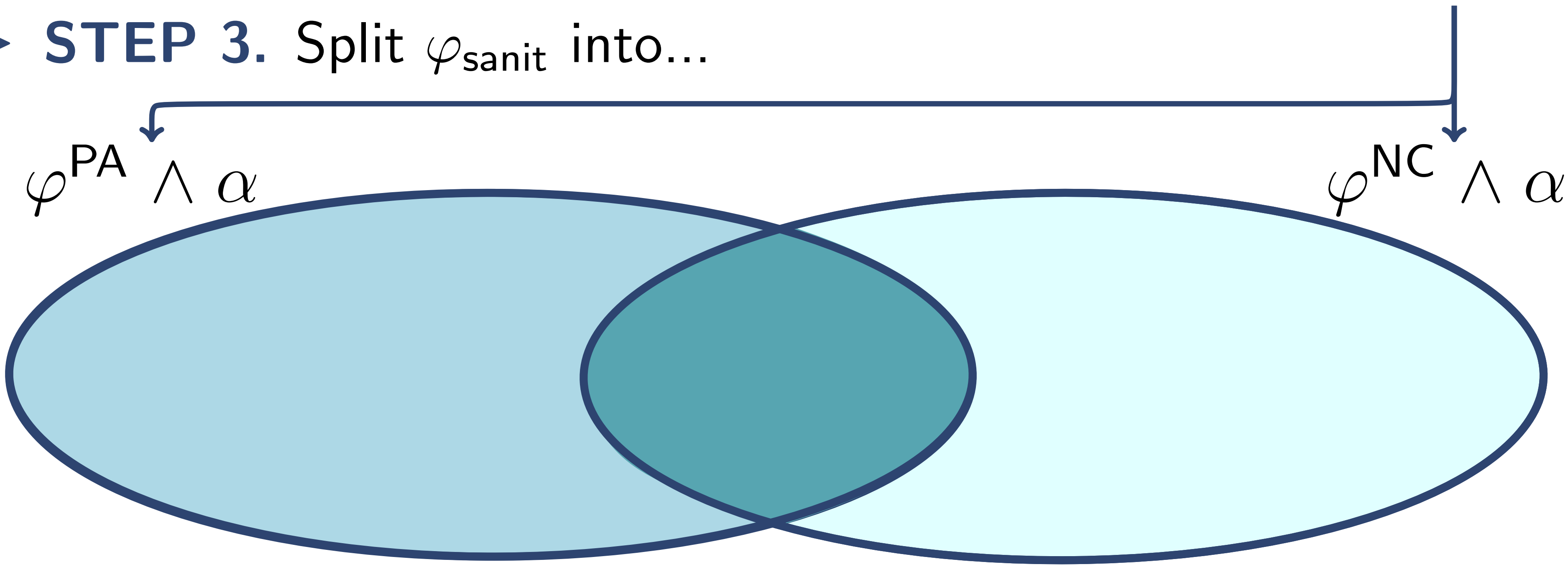

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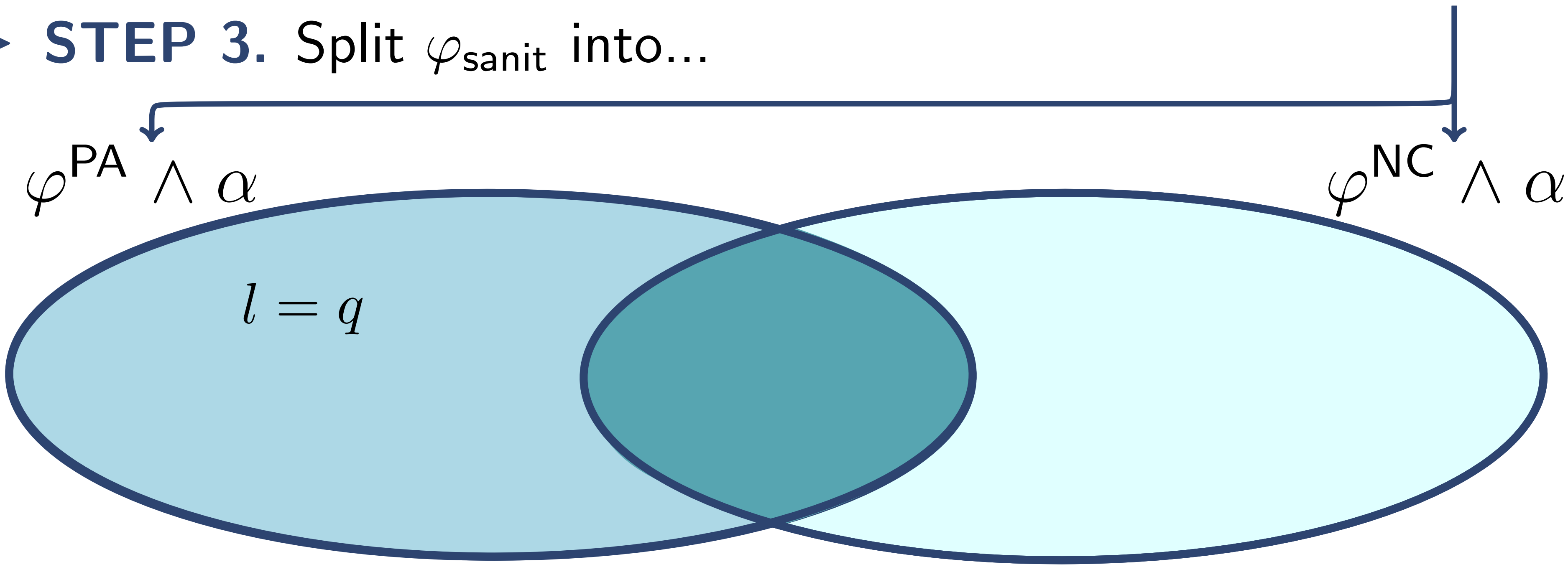

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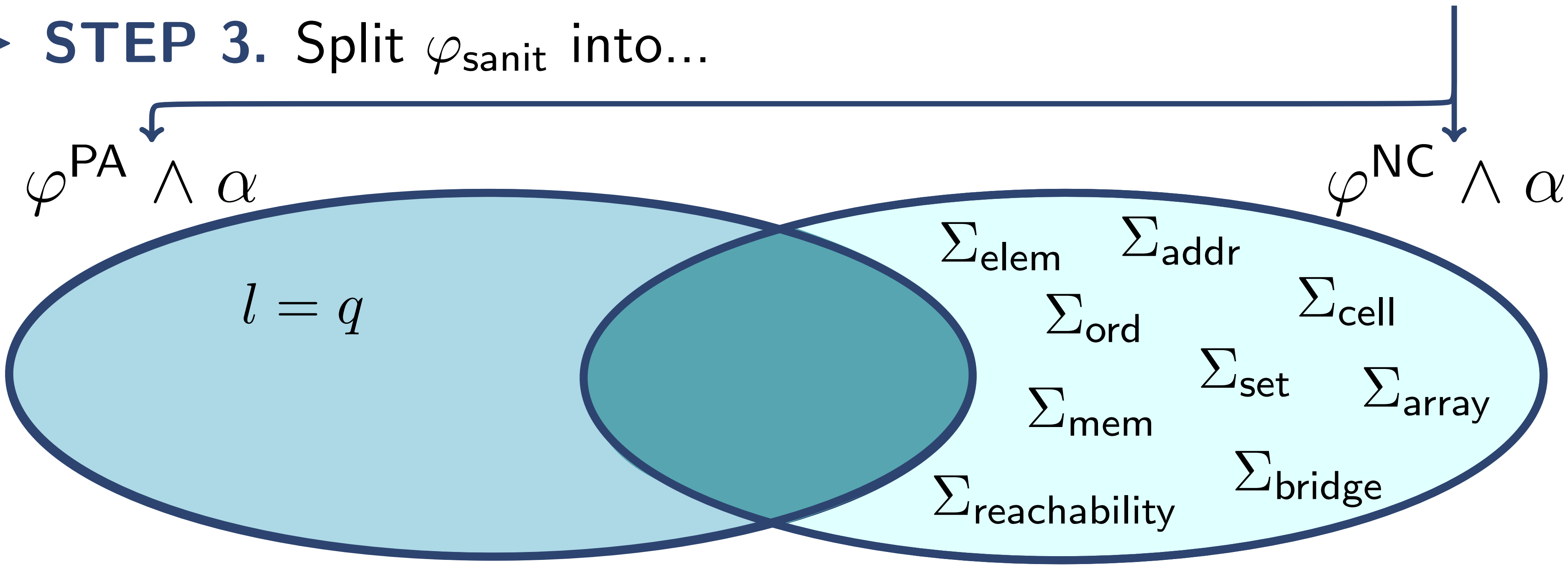

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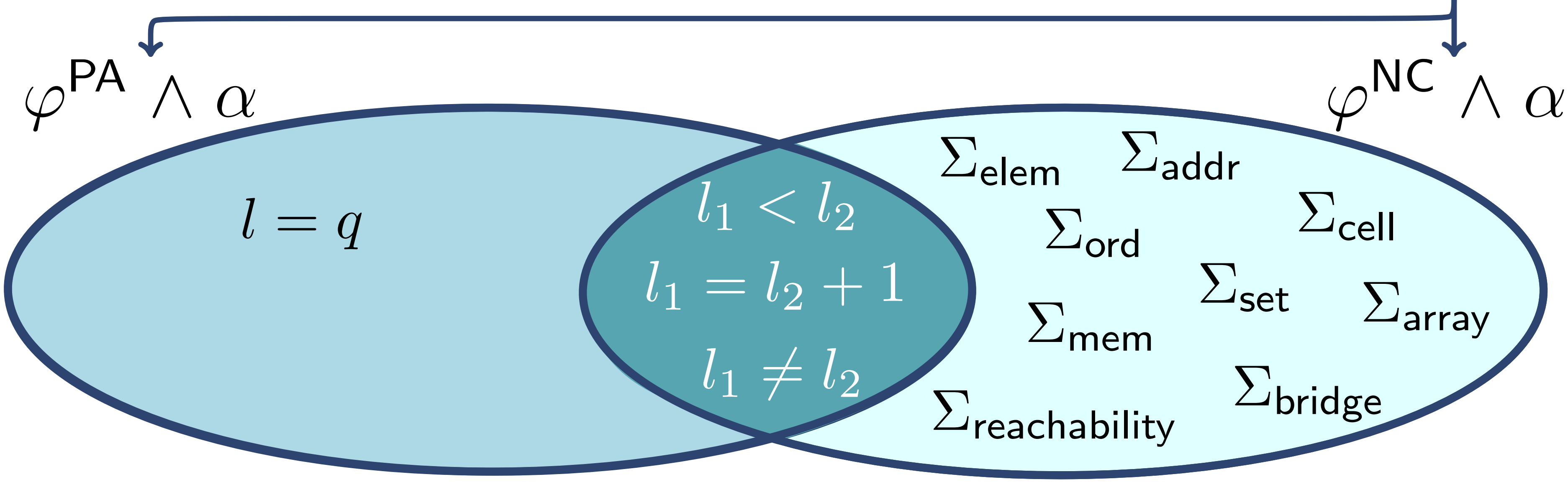

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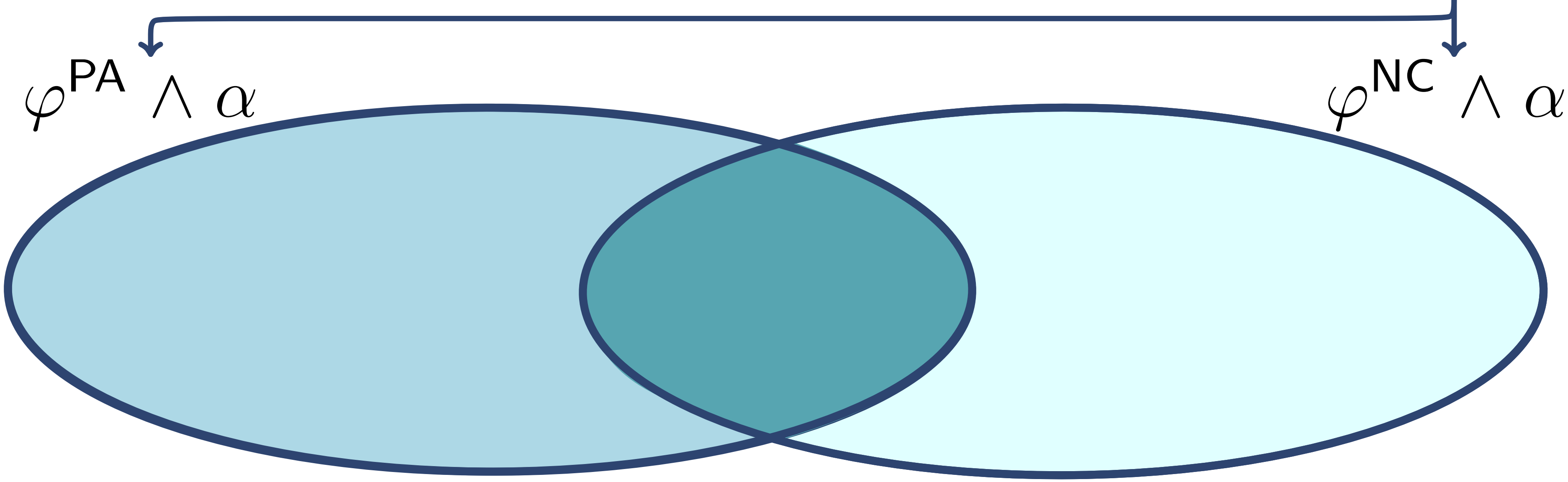

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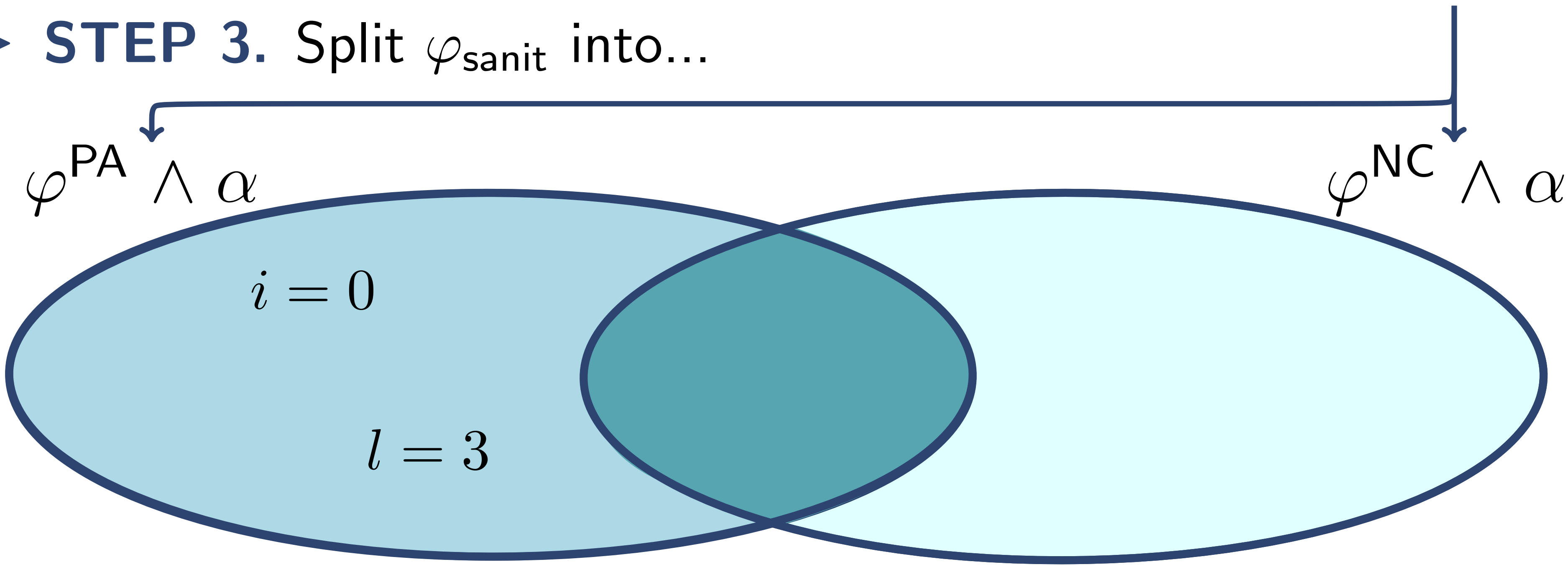

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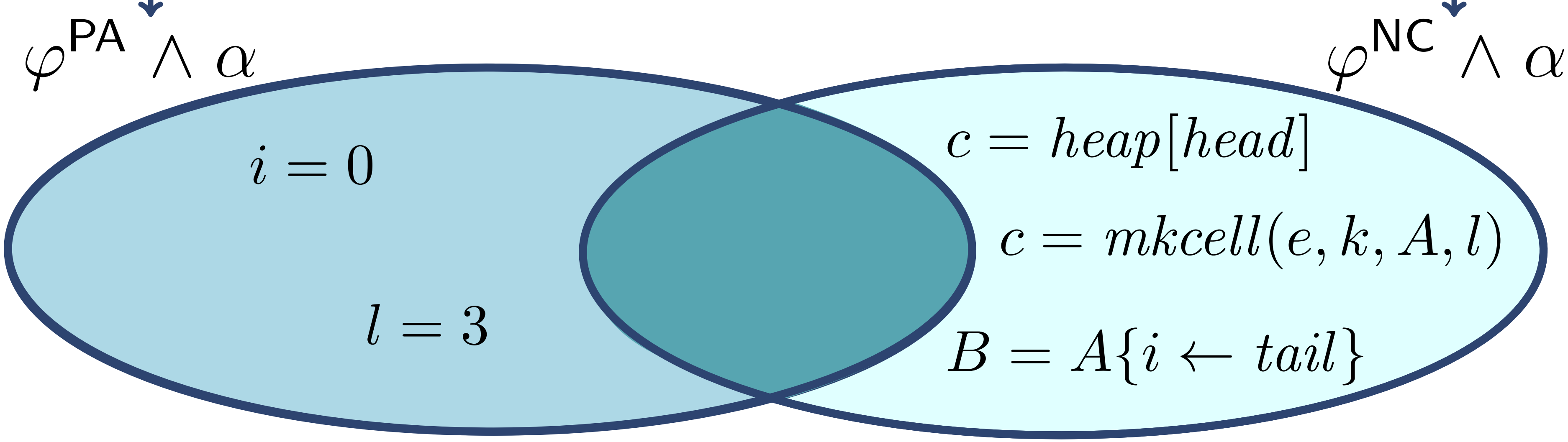

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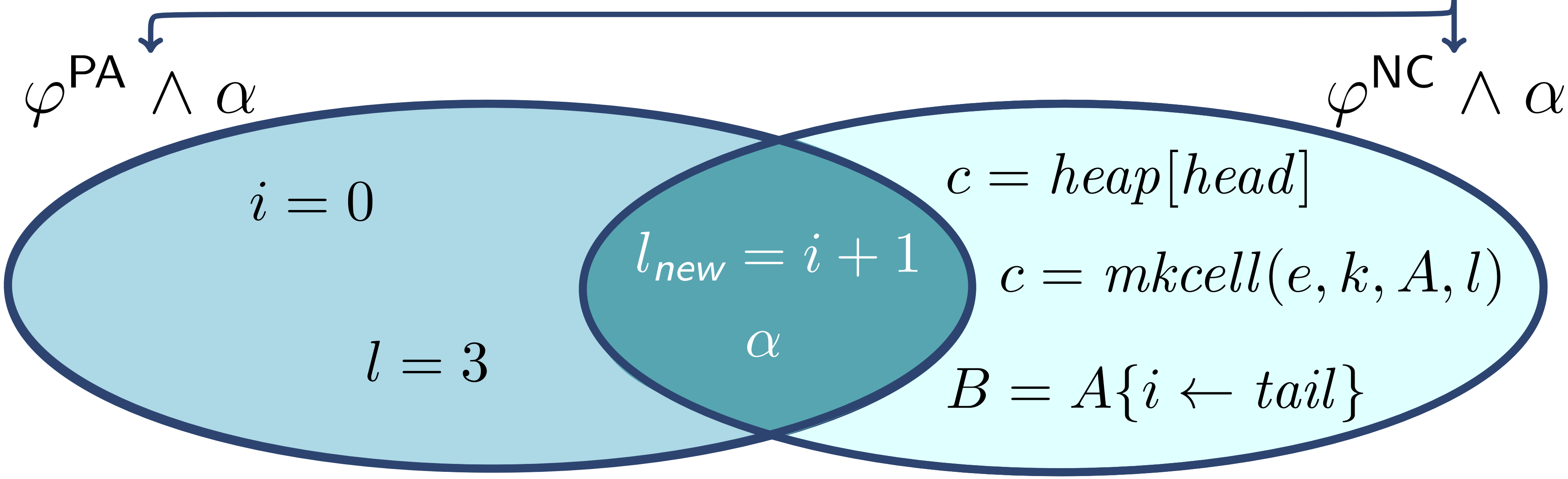

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$$\varphi^{\text{PA}} \wedge \alpha \qquad\qquad \varphi^{\text{NC}} \wedge \alpha$$

- **STEP 4.** Check satisfiability of $(\varphi^{\text{PA}} \wedge \alpha)$ and $(\varphi^{\text{NC}} \wedge \alpha)$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

φ_{norm}

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

$$\varphi \to \varphi_{\text{norm}} \longleftrightarrow \varphi_{\text{sanit}}$$

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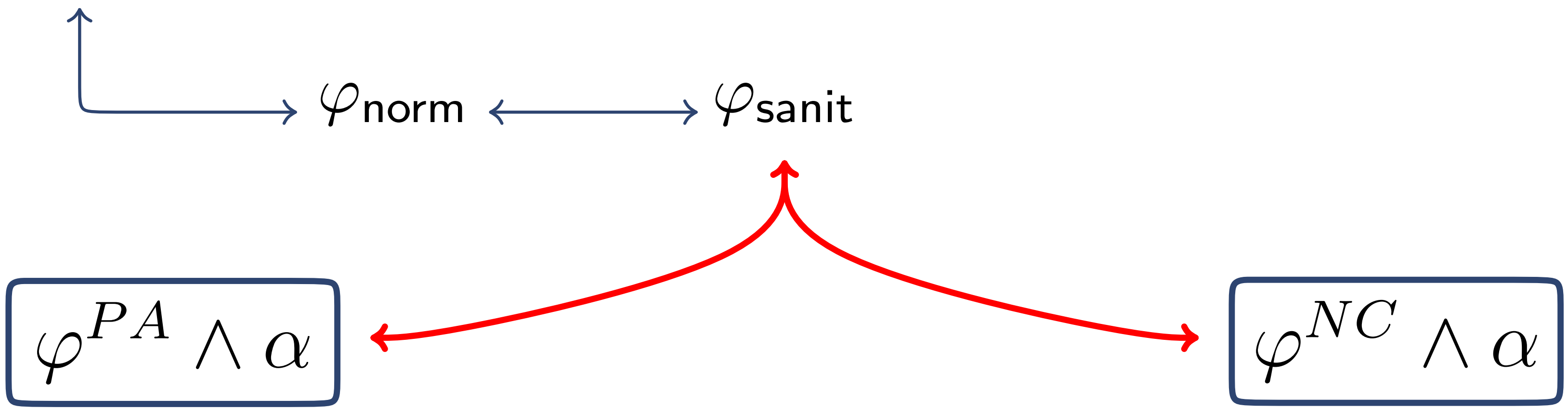

Theorem:

φ : TSL formula is satisfiable

iff

for some arrangement α, both
$(\varphi^{PA} \wedge \alpha)$ and $(\varphi^{NC} \wedge \alpha)$ are satisfiable

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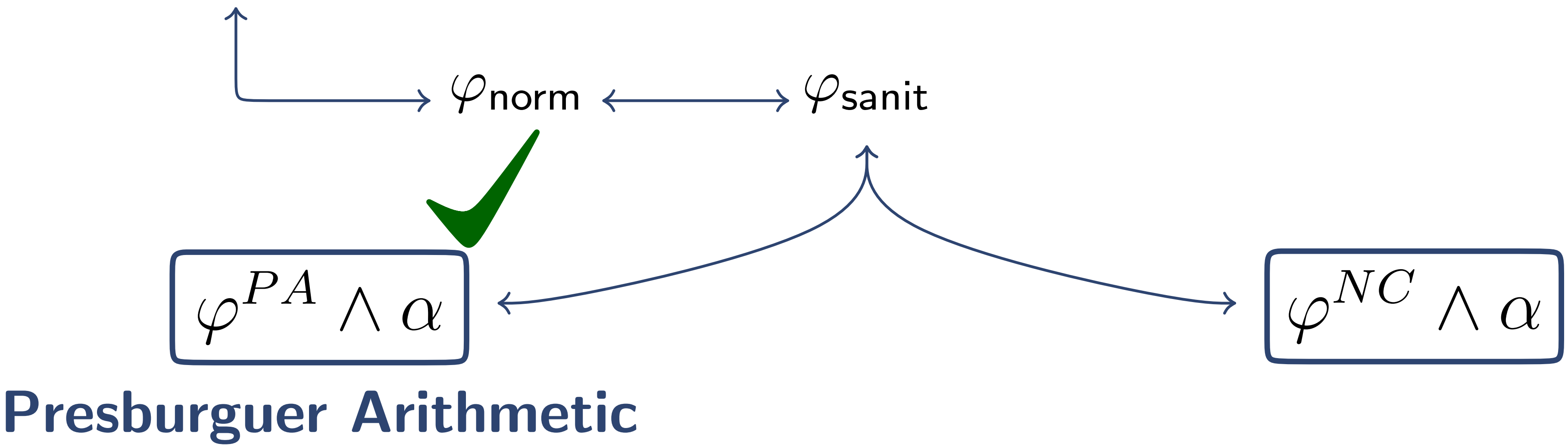

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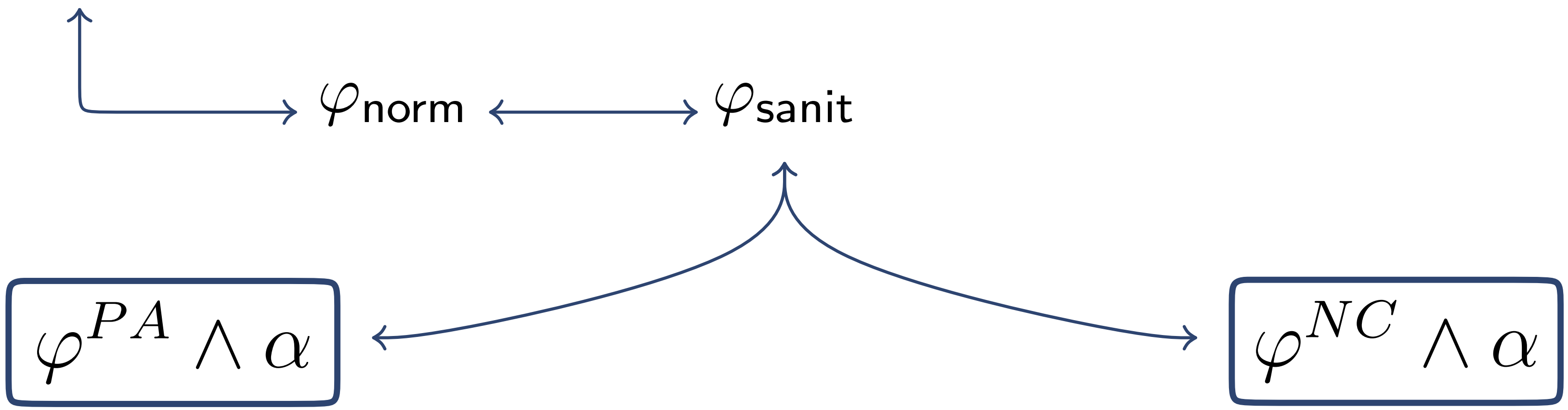

- **Gapless model**: we stay only with **interesting levels**

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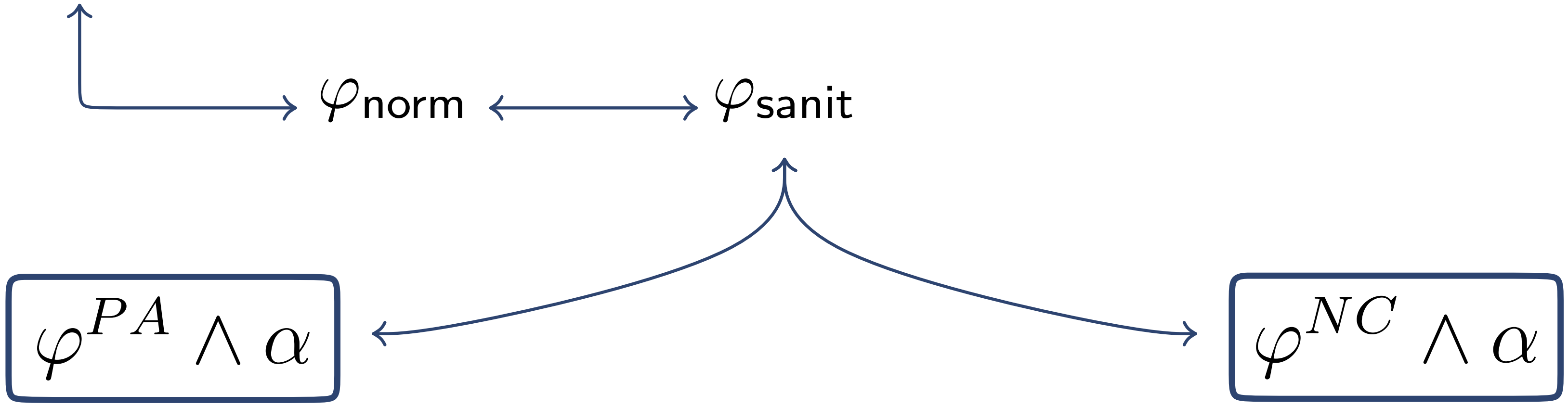

$$V_{\mathsf{level}}(\varphi^{NC} \wedge \alpha) = \{l_1, l_3\}$$

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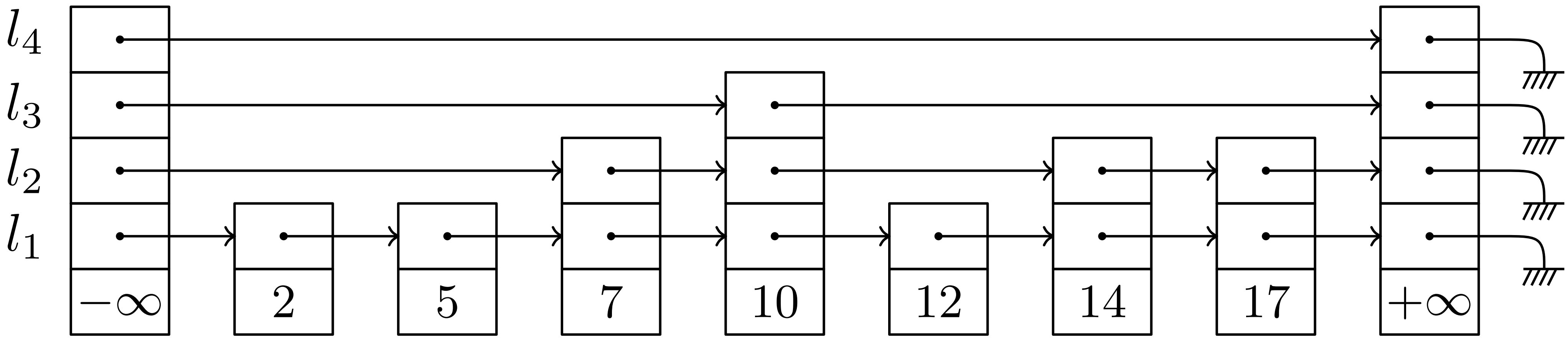

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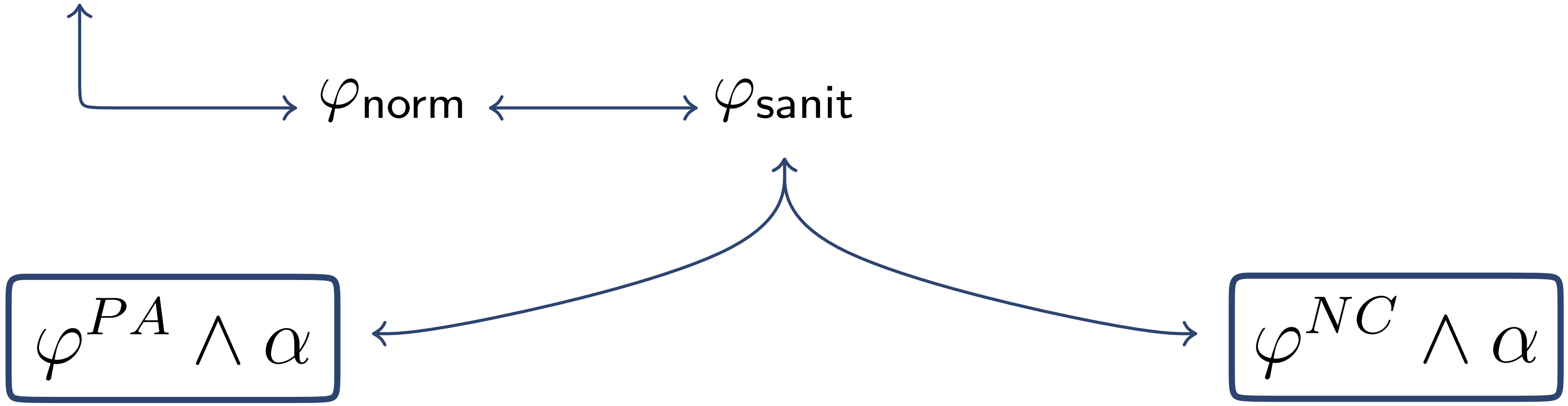

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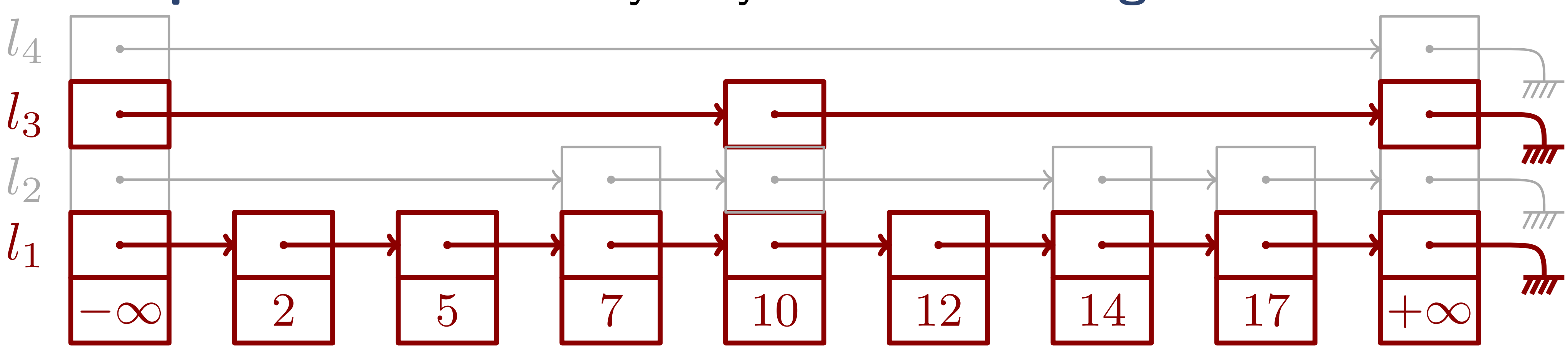

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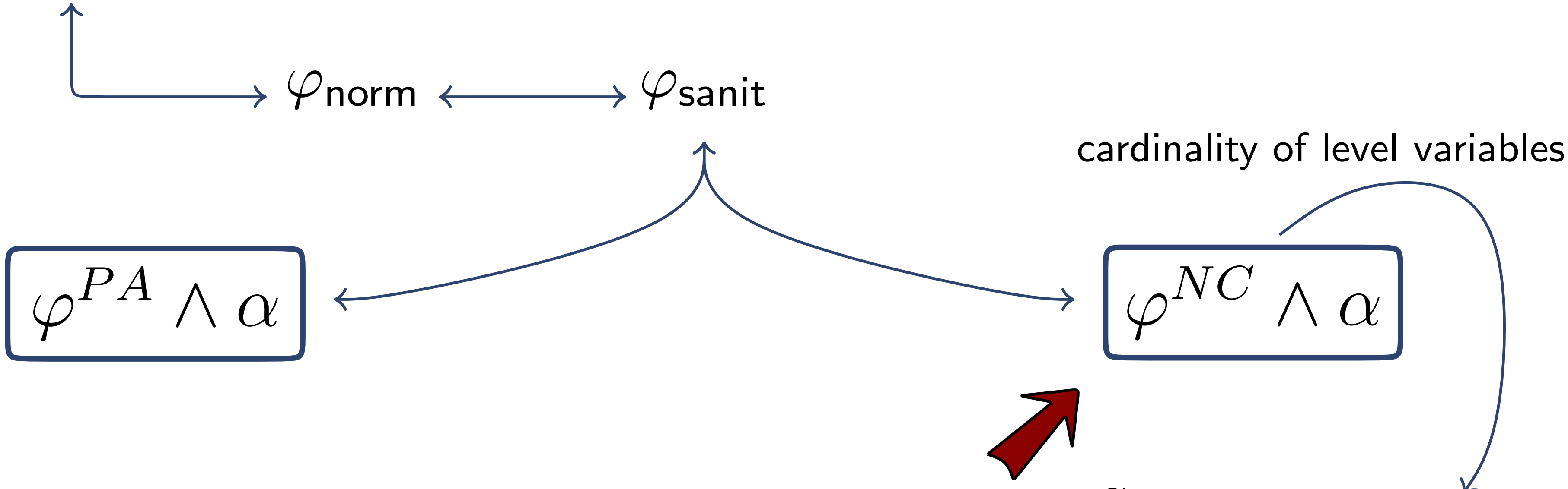
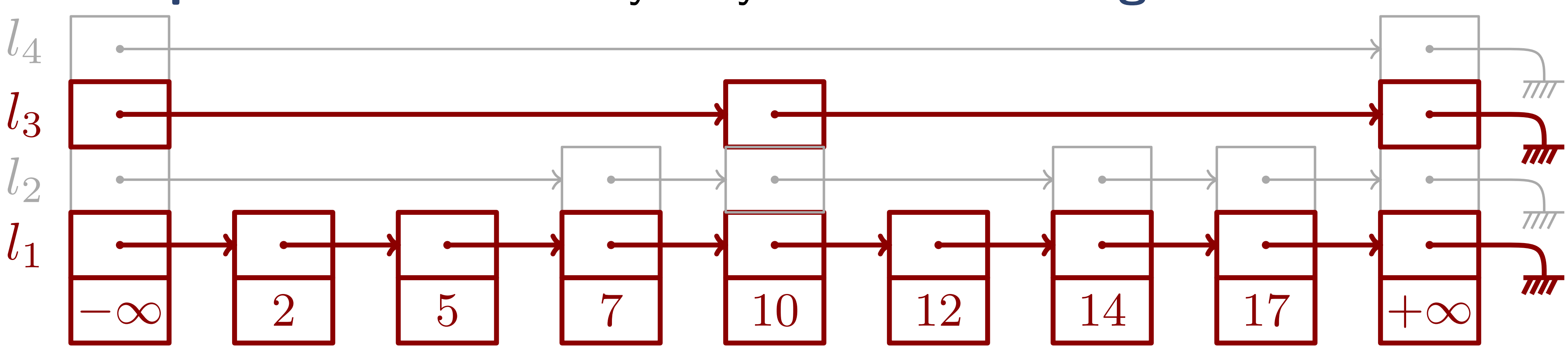


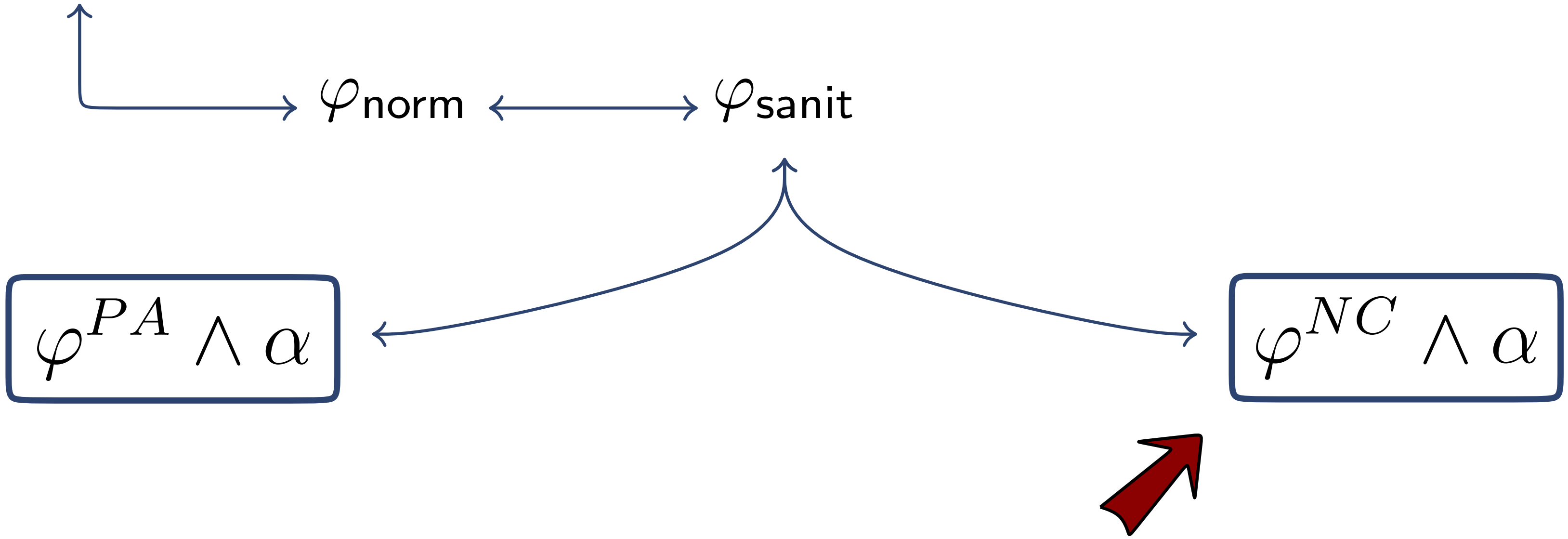
We reduce the formula to $\ulcorner \varphi^{NC} \wedge \alpha \urcorner : \text{TSL}_K$

- **Gapless model**: we stay only with **interesting levels**



Decision Procedure for TSL: Correctness

- Let φ be a TSL formula



We reduce the formula to $\ulcorner \varphi^{NC} \land \alpha \urcorner : \mathsf{TSL}_{\mathsf{K}}$

Theorem:

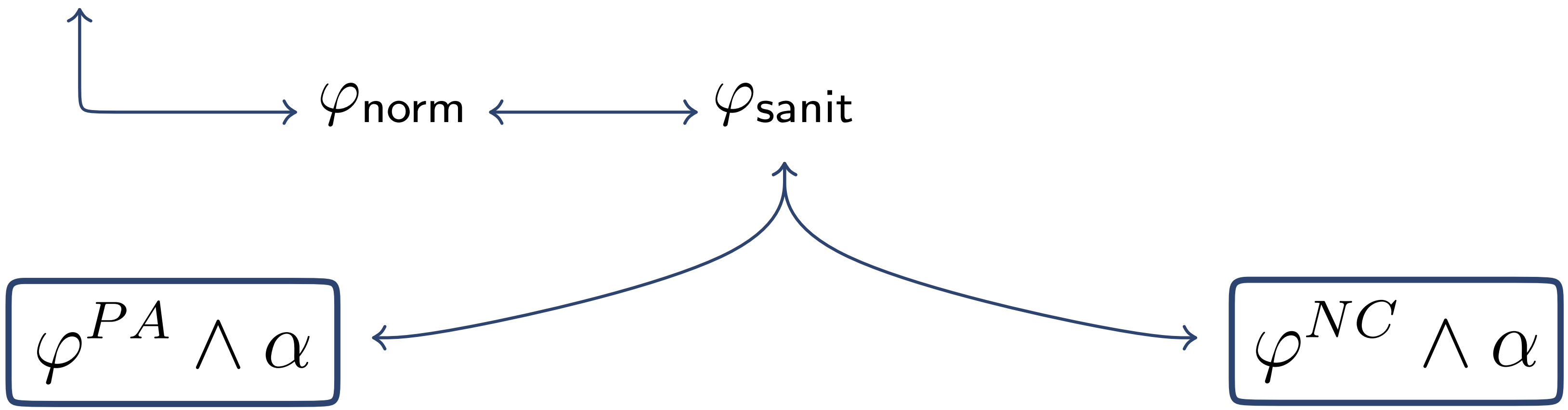
ψ a sanitized TSL formula without constant levels is satisfiable

iff

$\ulcorner \psi \urcorner : \mathsf{TSL}_{\mathsf{K}}$ is satisfiable

Decision Procedure for TSL: Correctness

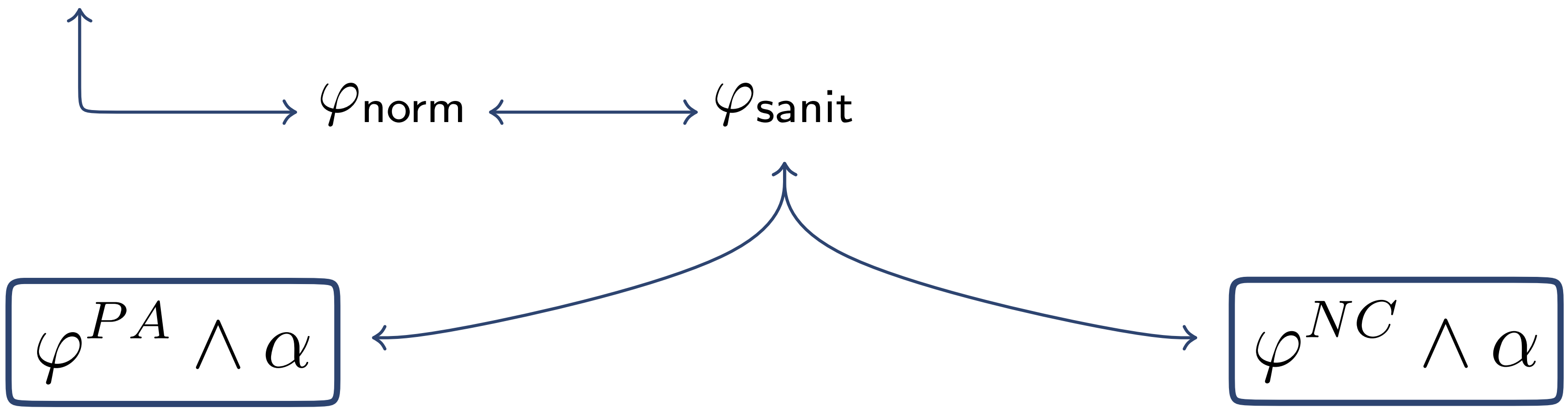
- Let φ be a TSL formula



- Reduction TSL $\longrightarrow$ $\mathsf{TSL}_{\mathsf{K}}$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula



- Reduction TSL $\longrightarrow$ TSL$_K$

$$\ulcorner c = mkcell(e, k, A, l) \urcorner \qquad c = (e, k, v_{A[0]}, \ldots, v_{A[K-1]})$$

$$v_{A[l]} = A(l)$$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

$\varphi_{\text{norm}} \longleftrightarrow \varphi_{\text{sanit}}$

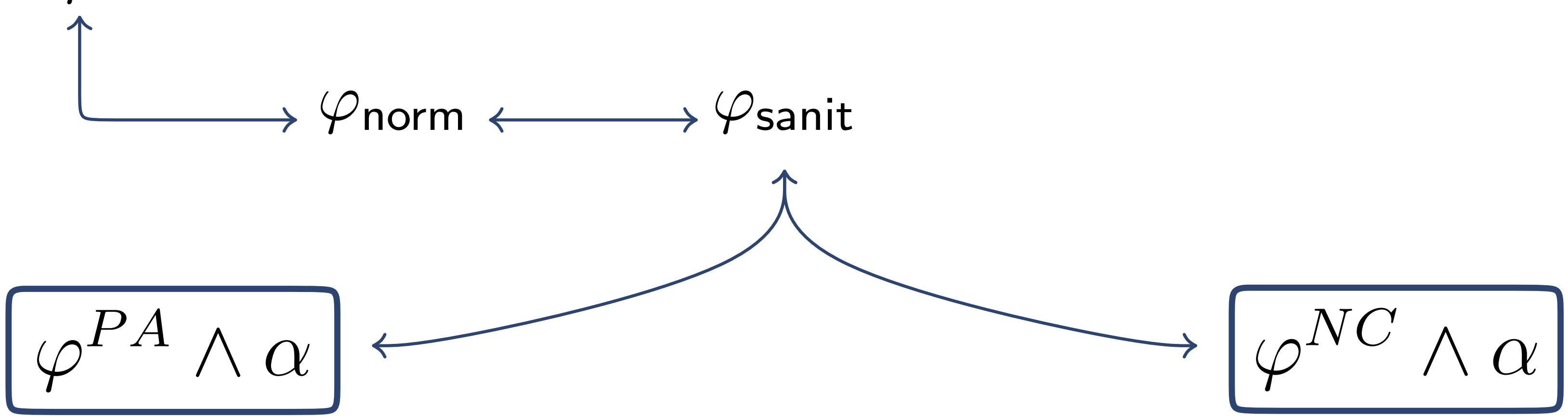
$\boxed{\varphi^{PA} \wedge \alpha}$ $\qquad$ $\boxed{\varphi^{NC} \wedge \alpha}$

- Reduction $\quad \mathsf{TSL} \longrightarrow \mathsf{TSL}_{\mathsf{K}}$

$\ulcorner c = mkcell(e, k, A, l) \urcorner$	$c = (e, k, v_{A[0]}, \ldots, v_{A[K-1]})$
$\ulcorner a = A[l] \urcorner$	$\bigwedge_{i=0 \ldots \mathsf{K}-1} l = i \rightarrow a = v_{A[i]}$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

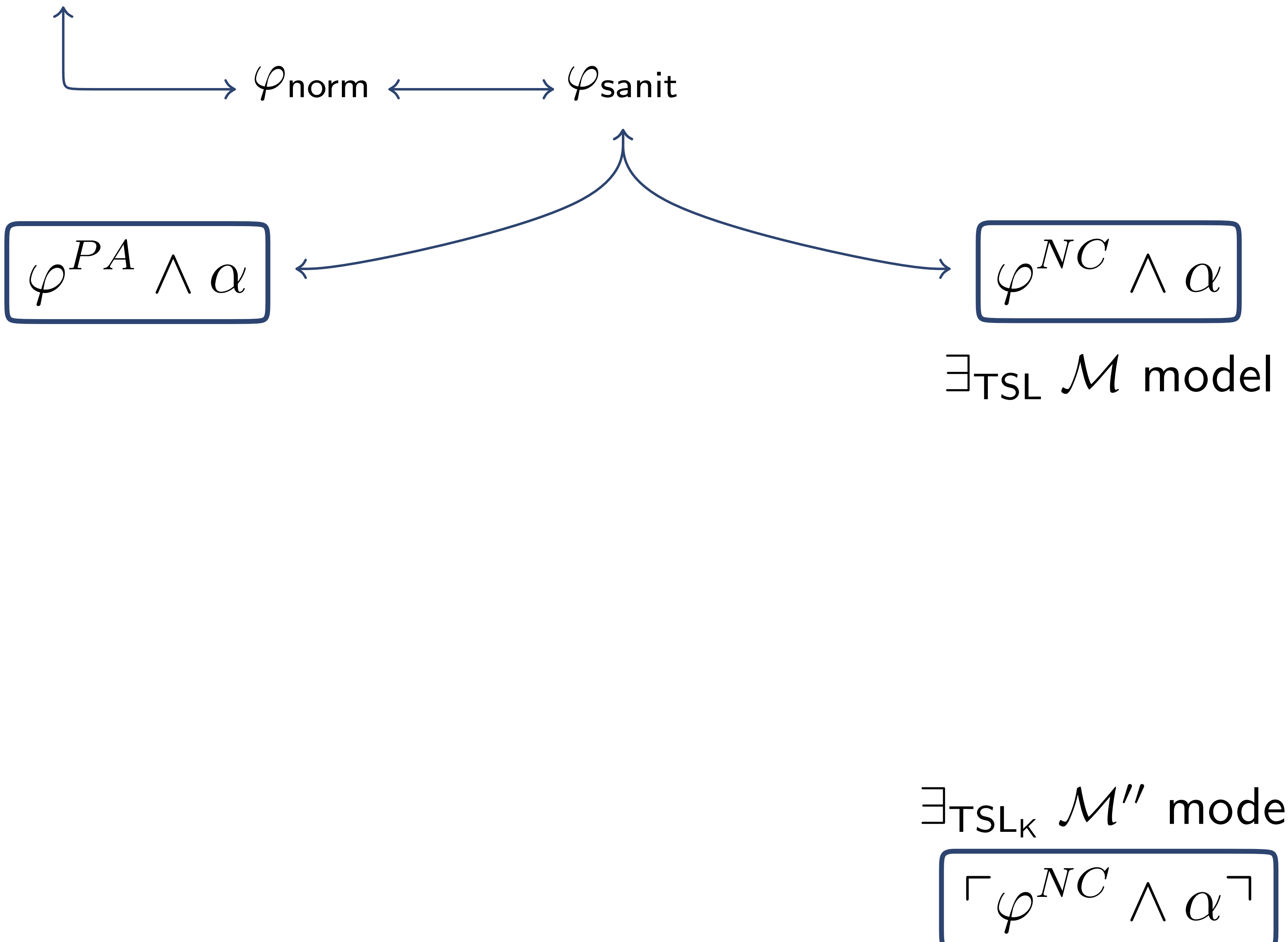


- Reduction

TSL	$\longrightarrow$ TSL$_K$
$\ulcorner c = mkcell(e, k, A, l) \urcorner$	$c = (e, k, v_{A[0]}, \ldots, v_{A[K-1]})$
$\ulcorner a = A[l] \urcorner$	$\bigwedge_{i=0 \ldots K-1} l = i \rightarrow a = v_{A[i]}$
$\ulcorner B = A\{l \leftarrow a\} \urcorner$	$\left(\bigwedge_{i=0 \ldots K-1} l = i \rightarrow a = v_{B[i]} \right) \wedge \left(\bigwedge_{j=0 \ldots K-1} l \neq j \rightarrow v_{B[j]} = v_{A[j]} \right)$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula



Decision Procedure for TSL: Correctness

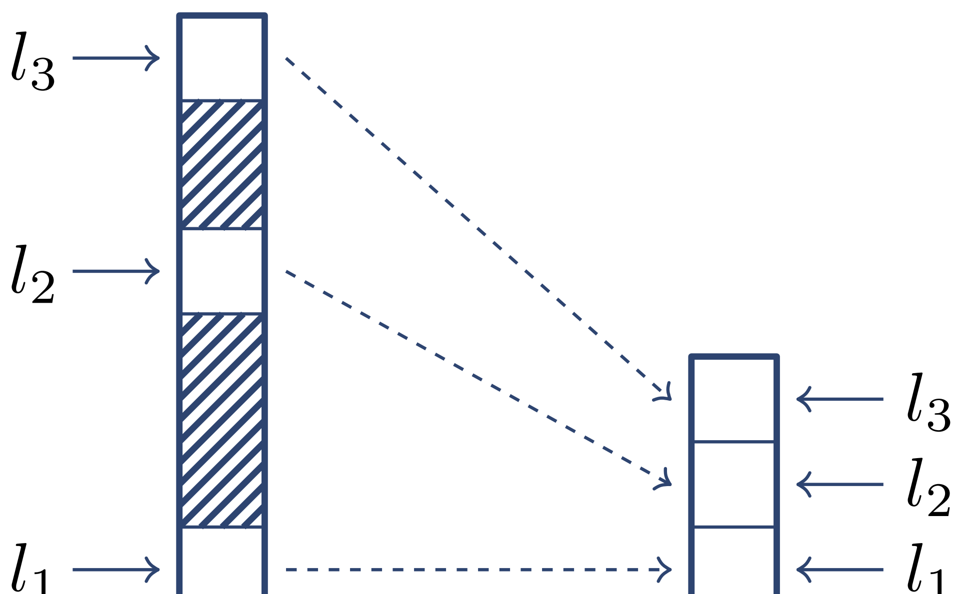
- Let φ be a TSL formula

φ_{norm} $\quad$ φ_{sanit}

$\varphi^{PA} \wedge \alpha$ $\quad$ $\varphi^{NC} \wedge \alpha$

$\exists_{\text{TSL}}\ \mathcal{M}$ model

$\exists_{\text{TSL}(gapless)}\ \mathcal{M}'$ model

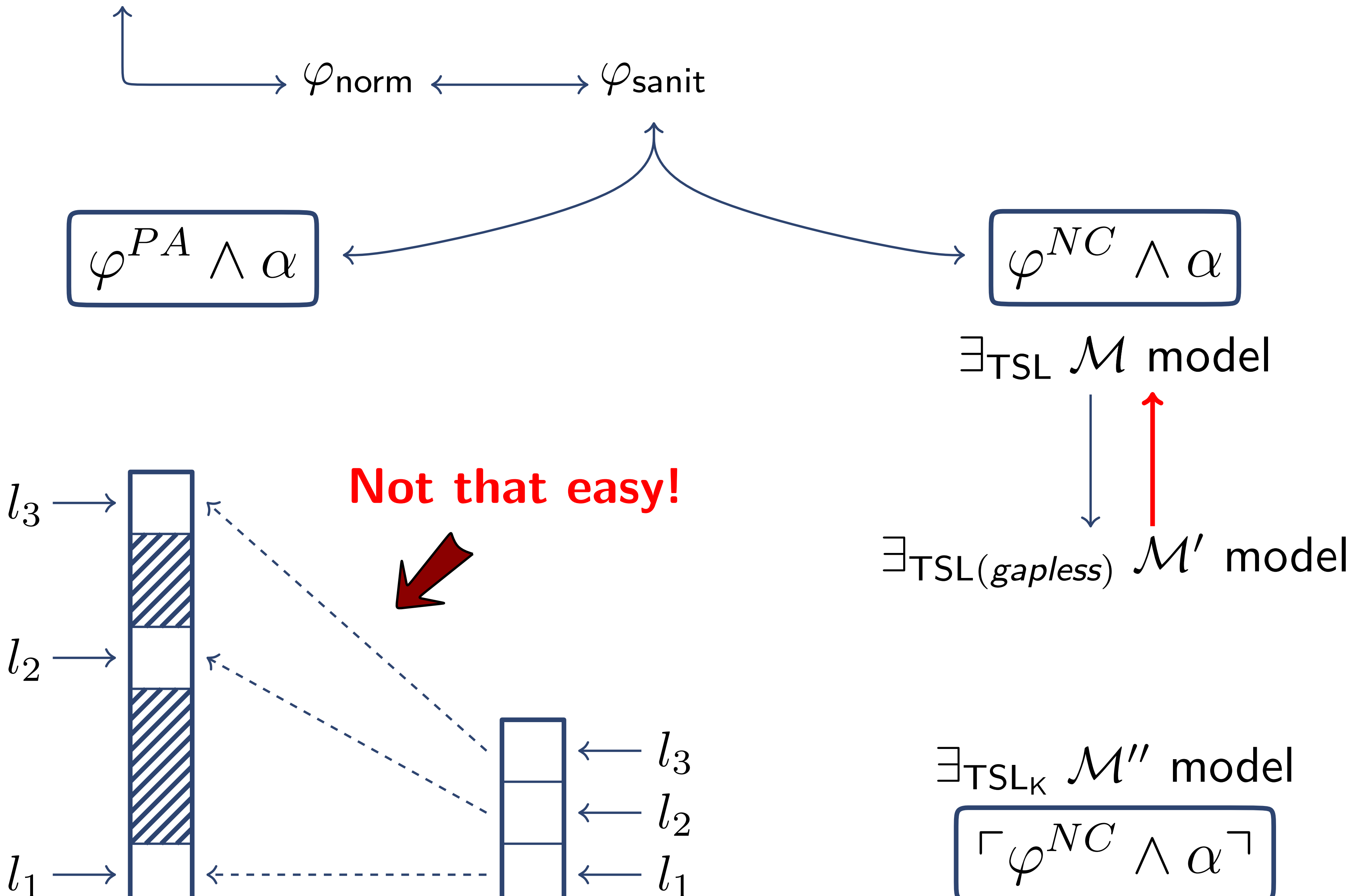


$\exists_{\text{TSL}_{\text{K}}}\ \mathcal{M}''$ model

$\ulcorner \varphi^{NC} \wedge \alpha \urcorner$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula



Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

$\varphi_{\text{norm}} \longleftrightarrow \varphi_{\text{sanit}}$

$\varphi^{PA} \wedge \alpha$

$\varphi^{NC} \wedge \alpha$

$\exists_{\mathsf{TSL}}\ \mathcal{M}$ model

$B = A\{l_1 \leftarrow a\}$

$\exists_{\mathsf{TSL}(gapless)}\ \mathcal{M}'$ model

$\exists_{\mathsf{TSL_K}}\ \mathcal{M}''$ model

$\ulcorner \varphi^{NC} \wedge \alpha \urcorner$

- Let φ be a TSL formula

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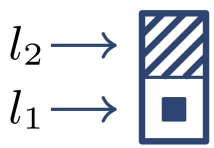
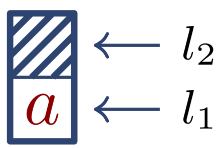


A



B

$\exists_{\mathsf{TSL}_{\mathsf{K}}}\ \mathcal{M}''$ model

$\boxed{\ulcorner \varphi^{NC} \wedge \alpha \urcorner}$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

$\varphi_{\text{norm}} \longleftrightarrow \varphi_{\text{sanit}}$

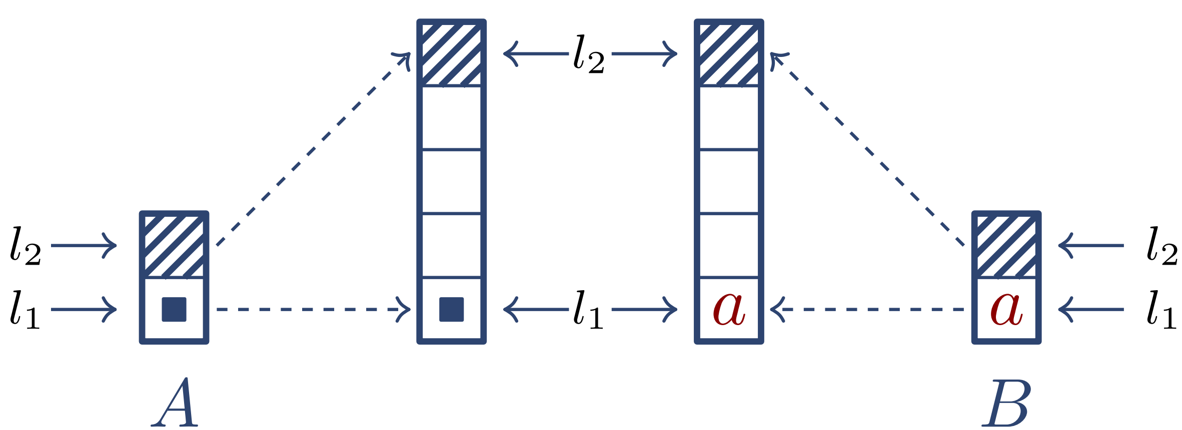
$\varphi^{PA} \wedge \alpha$

$\varphi^{NC} \wedge \alpha$

$\exists_{\text{TSL}}\ \mathcal{M}$ model

$B = A\{l_1 \leftarrow a\}$

$\exists_{\text{TSL}(gapless)}\ \mathcal{M}'$ model

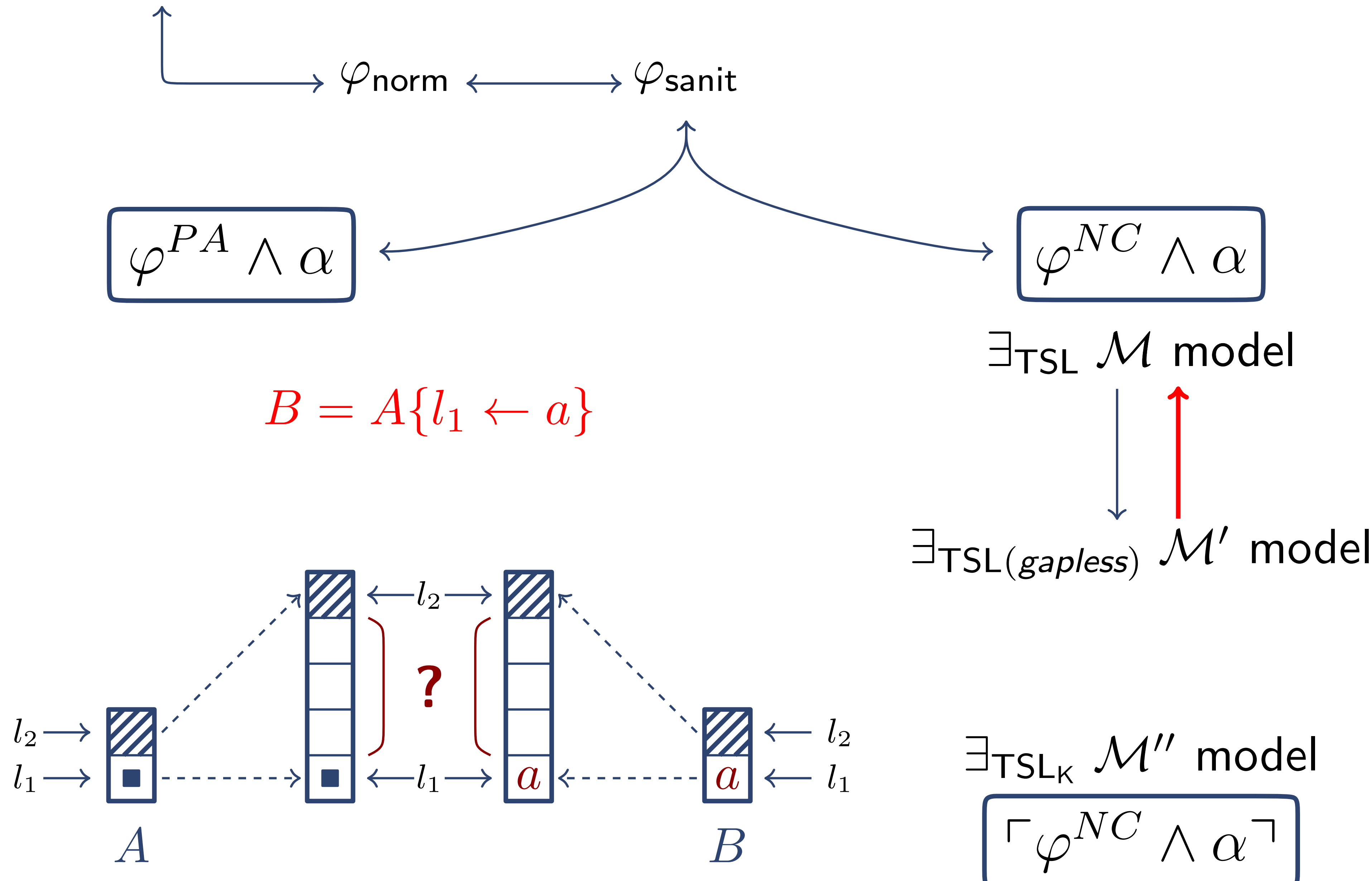


$\exists_{\text{TSL}_K}\ \mathcal{M}''$ model

$\ulcorner \varphi^{NC} \wedge \alpha \urcorner$

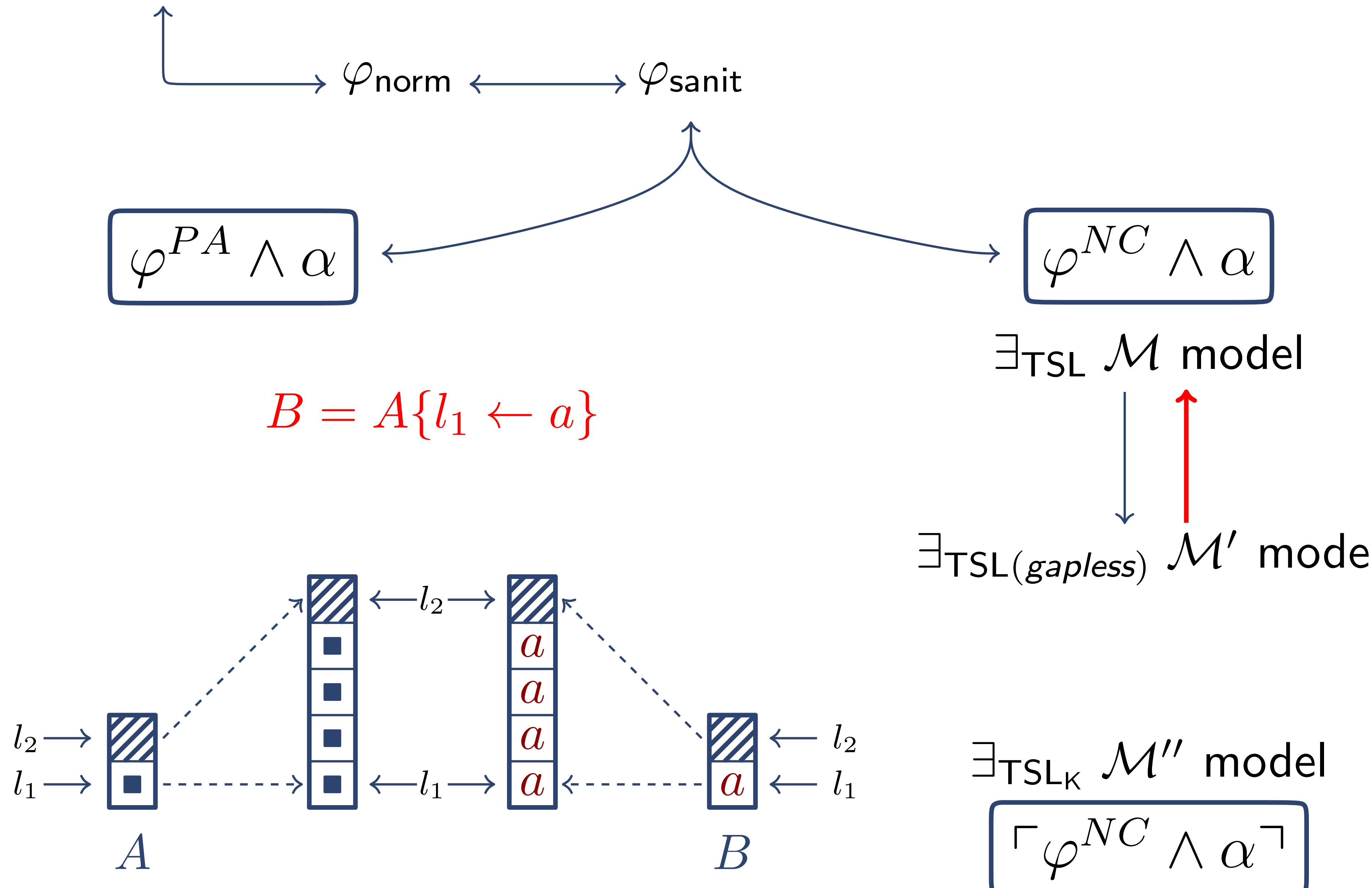
Decision Procedure for TSL: Correctness

- Let φ be a TSL formula



Decision Procedure for TSL: Correctness

- Let φ be a TSL formula



Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

$\varphi_{\text{norm}} \longleftrightarrow \varphi_{\text{sanit}}$

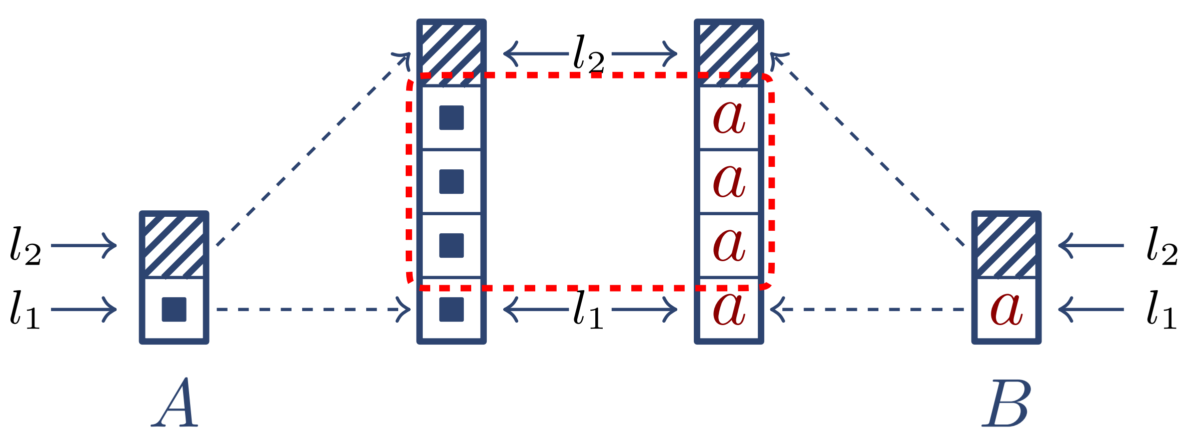
$\boxed{\varphi^{PA} \wedge \alpha}$

$\boxed{\varphi^{NC} \wedge \alpha}$

$\exists_{\text{TSL}}\ \mathcal{M}$ model

~~$B = A\{l_1 \leftarrow a\}$~~

$\exists_{\text{TSL}(\textit{gapless})}\ \mathcal{M}'$ model



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Decision Procedure for TSL: Correctness

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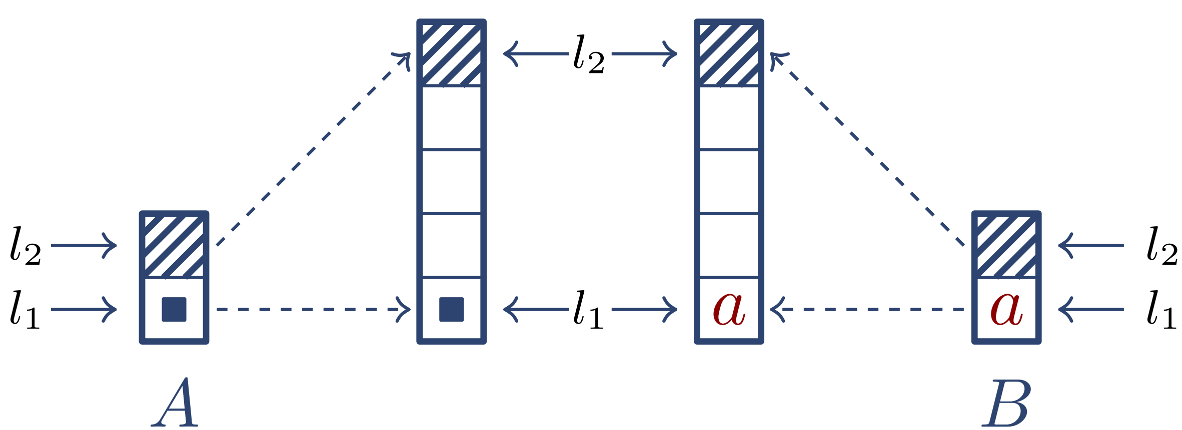

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$\varphi^{PA} \wedge \alpha$

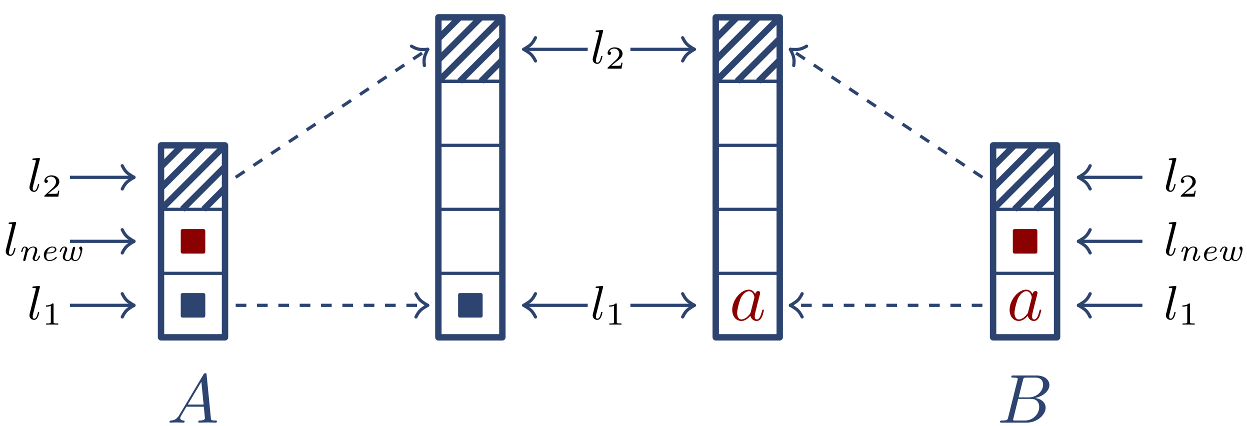
$\varphi^{NC} \wedge \alpha$

$\exists_{\mathsf{TSL}}\ \mathcal{M}$ model

$\exists_{\mathsf{TSL}(\mathit{gapless})}\ \mathcal{M}'$ model

$B = A\{l_1 \leftarrow a\}$

Add $l_{new} = l_1 + 1$



$\exists_{\mathsf{TSL_K}}\ \mathcal{M}''$ model

$\ulcorner \varphi^{NC} \wedge \alpha \urcorner$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

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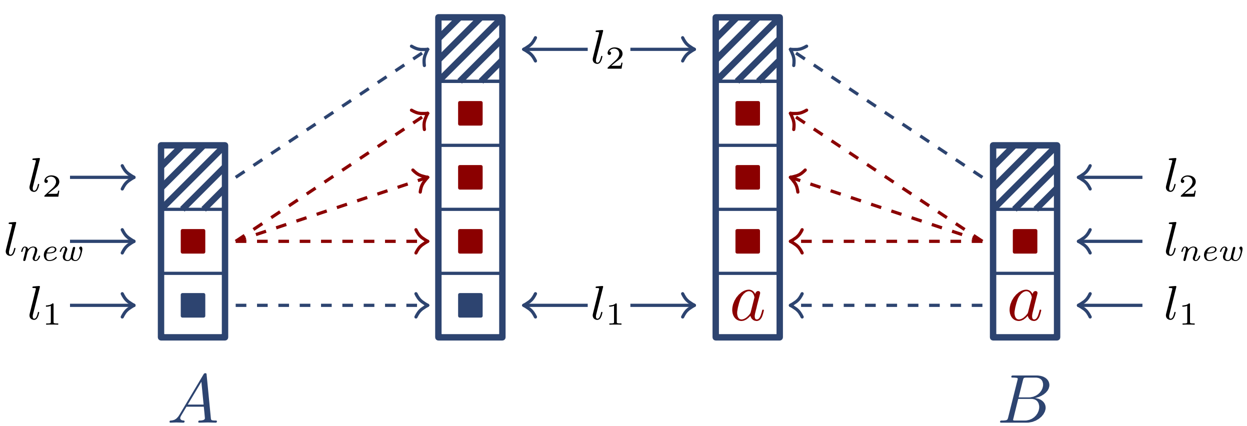

$\exists_{\textsf{TSL}_\textsf{K}}\ \mathcal{M}''$ model

$\boxed{\ulcorner \varphi^{NC} \wedge \alpha \urcorner}$

Decision Procedure for TSL: Correctness

- Let φ be a TSL formula

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$\exists_{\mathsf{TSL_K}}\ \mathcal{M}''$ model

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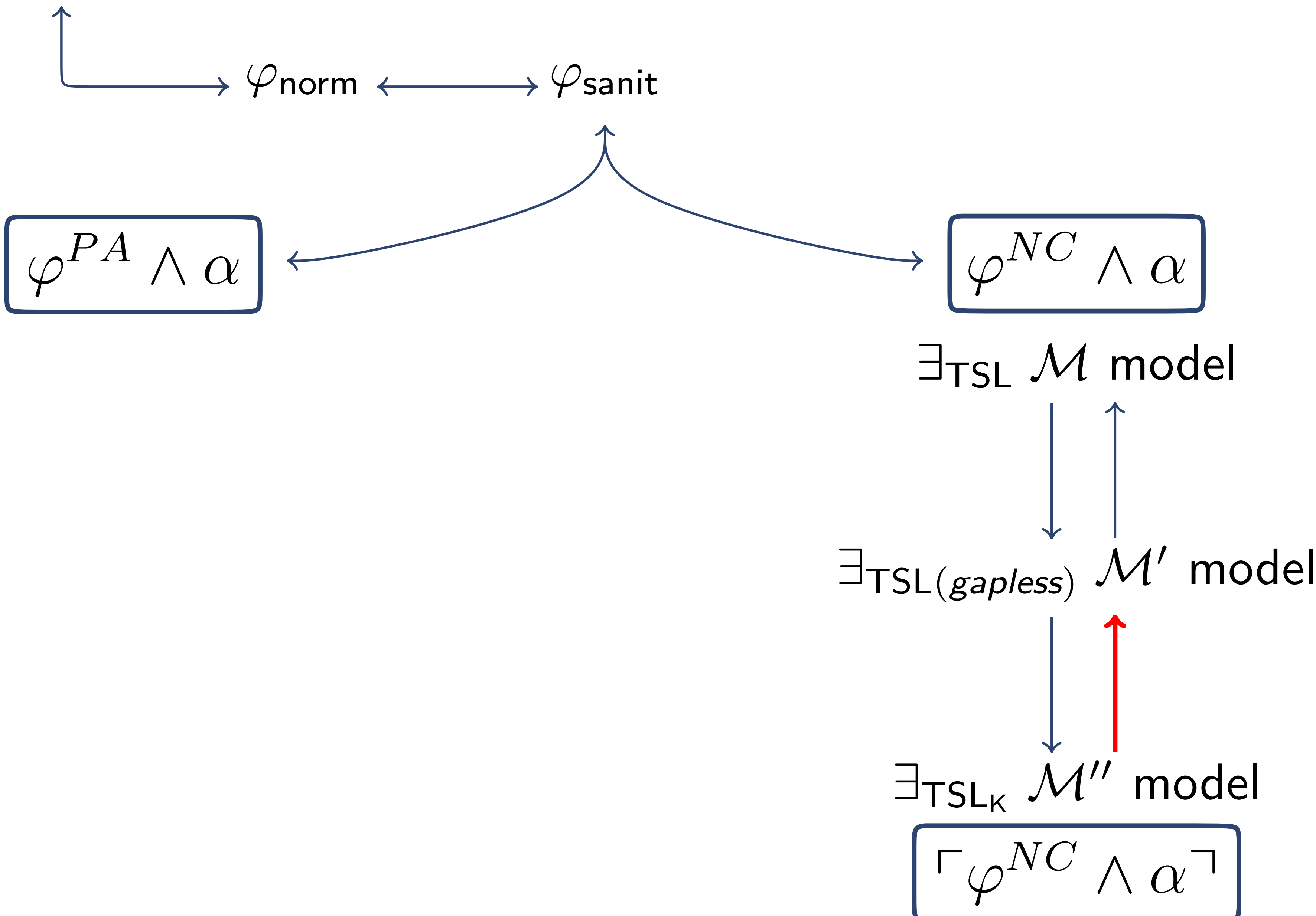

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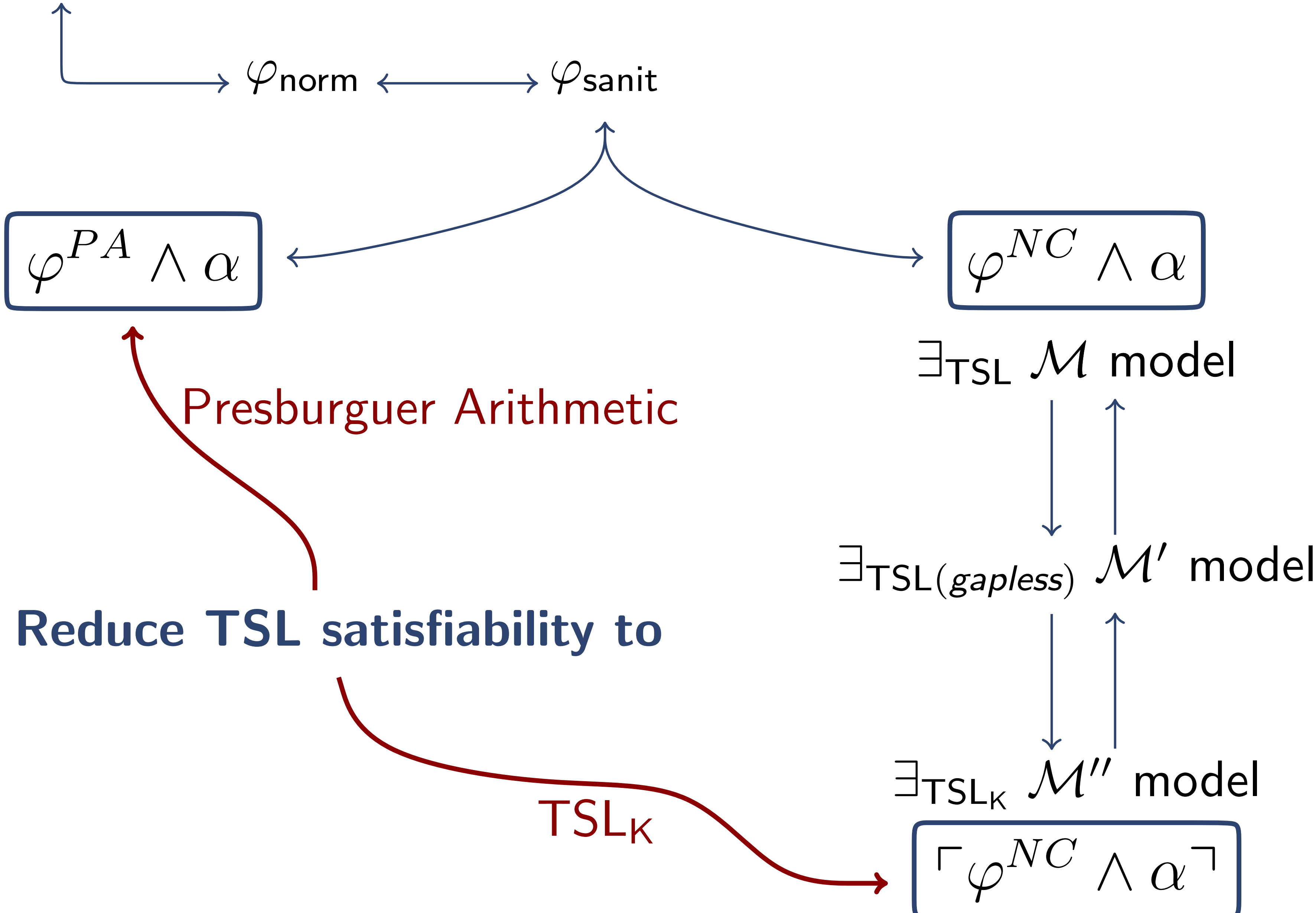

Empirical Evaluation

- We have **implemented** TSL decision procedure in **LEAP**
- We verify **shape preservation** and **functional properties**
- We **compare** TSL with previous TSL_K **performance**

Empirical Evaluation

	Form.	#Calls to DPs					VC time (s.)		Time (s.)
	$\#\varphi$	TSL	TSL_1	TSL_2	TSL_3	TSL_4	slowest	avg	DP
skiplist	560	28	45	92	38	14	5.40	0.24	19.64
region	1583	56	111	185	76	–	22.66	0.54	42.93
next	1899	30	39	55	22	–	0.32	0.02	1.60
order	2531	57	167	286	116	4	2.35	0.84	6.75
$skiplist_{KDE}$	214	14	37	61	32	12	5.93	0.24	13.14
$nodes_{KDE}$	585	32	99	174	76	–	3.10	0.17	9.36
$pointers_{KDE}$	1115	27	38	42	16	–	0.22	0.01	0.76
$values_{KDE}$	797	34	120	194	76	–	0.64	0.06	3.06
funcInsert	75	7	9	2	–	–	0.02	0.01	0.04
funcRemove	75	8	9	15	2	–	0.04	0.01	0.10
$skiplist_1$	119	–	32	–	–	–	0.10	0.01	0.32
$region_1$	119	–	27	–	–	–	0.14	0.01	0.28
$skiplist_2$	137	–	–	47	–	–	2.15	0.05	4.13
$region_2$	122	–	–	27	–	–	1.08	0.03	2.44
$skiplist_3$	154	–	–	–	62	–	776.45	15.27	1221.52
$region_3$	124	–	–	–	27	–	17.36	0.34	26.92
$skiplist_4$	171	–	–	–	–	77	**T.O.**	**T.O.**	**T.O.**
$region_4$	126	–	–	–	–	27	226.08	4.30	348.44

Empirical Evaluation

	Form.	#Calls to DPs					VC time (s.)		Time (s.)
	$\#\varphi$	TSL	TSL_1	TSL_2	TSL_3	TSL_4	slowest	avg	DP
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$skiplist_4$	171	–	–	–	–	77	**T.O.**	**T.O.**	**T.O.**
$region_4$	126	–	–	–	–	27	226.08	4.30	348.44

TSL decision procedure vs TSL_K decision procedure

Empirical Evaluation

	Form.	#Calls to DPs					VC time (s.)		Time (s.)
	#φ	TSL	TSL_1	TSL_2	TSL_3	TSL_4	slowest	avg	DP
skiplist	560	28	45	92	38	14	5.40	0.24	19.64
region	1583	56	111	185	76	–	22.66	0.54	42.93
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$region_3$	124	–	–	–	27	–	17.36	0.34	26.92
$skiplist_4$	171	–	–	–	–	77	**T.O.**	**T.O.**	**T.O.**
$region_4$	126	–	–	–	–	27	226.08	4.30	348.44

A TSL queries is **decomposed into multiple calls** to TSL_K

Empirical Evaluation

	Form. $\#\varphi$	#Calls to DPs TSL	TSL_1	TSL_2	TSL_3	TSL_4	VC time (s.) slowest	avg	Time (s.) DP
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$region_3$	124	–	–	–	27	–	17.36	0.34	26.92
$skiplist_4$	171	–	–	–	–	77	**T.O.**	**T.O.**	**T.O.**
$region_4$	126	–	–	–	–	27	226.08	4.30	348.44

While **TSL_K did not scale beyond a skiplist with 4 levels**...

Empirical Evaluation

	Form.	#Calls to DPs					VC time (s.)		Time (s.)
	$\#\varphi$	TSL	TSL_1	TSL_2	TSL_3	TSL_4	slowest	avg	DP
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$region_3$	124	–	–	–	27	–	17.36	0.34	26.92
$skiplist_4$	171	–	–	–	–	77	**T.O.**	**T.O.**	**T.O.**
$region_4$	126	–	–	–	–	27	226.08	4.30	348.44

...**TSL** verified the examples in **less than a minute for every height**

Conclusions

- We presented **TSL**, a theory for skiplists of arbitrary height
- **TSL** can reason about memory, cells, pointers, regions, reachability, ordered lists and sublists
- We proved **TSL decidable** and presented a **decision procedure**
- Decision procedure has been **implemented** as part of **LEAP**
- We used TSL to **verify real world implementations**
- **Future work**
  - Extend TSL for concurrent skiplists
  - Verify further implementations
  - Improve automation by generating and propagating invariants

LEAP and examples availblable at

`software.imdea.org/leap`