Mode-based Reactive Synthesis*

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Abstract. Reactive synthesis aims to automatically generate systems from high-level formal specifications, but its inherent complexity limits its scalability to real-world scenarios. Scalability can be improved by decomposing the specification into independent parts for parallel synthesis, but the dependency between variables limits this approach.
At the same time, specifications used in Requirements Engineering (RE) often include high-level state machine descriptions, known as modes, which structure the specification.
This paper introduces a novel method for the sequential decomposition of reactive synthesis problems based on modes. Our approach automatically uses modes to break down a specification into smaller sub-specifications, synthesizes each independently, and then integrates the solutions into a cohesive global model. We present an algorithm that exploits mode transitions and ensures consistency across synthesized components leveraging off-the-shelf reactive synthesis tools.
We prove the correctness of our approach and show empirically that our method significantly improves scalability when decomposing real-world specifications, outperforming state-of-the-art monolithic tools. As the first sequential decomposition approach, our method offers a promising alternative for scalable reactive synthesis.

1 Introduction

Reactive systems [50], which continuously interact with their environment, are essential in domains like cyber-physical and embedded systems. These systems are crucial for tasks such as model checking [19], property monitoring [8], and model-based testing [31]. Linear-Time Temporal Logic (LTL) [59] is commonly used to specify properties of reactive systems, typically in an *assume-guarantee*

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format ($A \rightarrow G$). Here, Assumptions (A) describe the uncontrollable environment and Guarantees (G) define the desired system behavior. This separation into *environment-controlled* and *system-controlled* variables facilitates effective analysis and synthesis [11].

Reactive synthesis automates the construction of a *controller* from a specification, ensuring that for all valid environment inputs, the controller behaves as required. Despite progress in the field, synthesizing controllers for complex specifications remains computationally challenging. Even with efficient LTL fragments like GR(1) [58,10], deciding realizability and generating a controller can lead to exponential blow-ups [40] in the reduction from the specification to GR(1). Several decomposition techniques have been proposed [28,45,7,20,55] which try to perform synthesis independently for different variables, but these methods face difficulties when sub-specifications share *controllable variables.* Requirements Engineering (RE) methodologies provide useful mechanisms to modularize and specify the system behavior. A typical way to organize system requirements is through the use of high-level state machine descriptions in which the states, referred to as *modes*, encapsulate specific system behavior under particular situations and transitions represent how the system execution evolves and react to environmental events [39,62,70,24].

Mode-based synthesis leverages these modes to synthesize complex systems by decomposing global specifications into sub-specifications for individual modes. However, the only previous approach [13] requires manual intervention, limiting its automation. In this work we present a fully automatic mode-based synthesis process, eliminating the need for manual engineering input. Our approach takes as input the system's global specification, a description of its modes, and optionally, the possible transitions between them. Through a process we call *sequential decomposition*, the synthesis problem is addressed incrementally: each sub-problem is solved independently, focusing on one mode at a time. A key aspect of our method is the treatment of *initial conditions* for each mode. Initial conditions ensure coherence between sub-specifications and the global specification guaranteeing the correct connection among modes. By maintaining alignment with global requirements, our method guarantees consistency across mode transitions, avoiding conflicts and ensuring the system's correctness.

We formalize *mode projection* and *mode-based synthesis*, proving consistency between modes and the global system. Crucially, our approach computes a set of initial conditions that ensure that (1) each sub-specification is realizable, (2) the initial conditions of each mode support the other modes. This allows to conclude that the global specification is also realizable. The resulting structured controllers enhance transparency and interpretability while improving synthesis scalability, as shown empirically against state-of-the-art monolithic tools. The paper is organized as follows: Section 2 covers preliminaries, Section 3 details our approach, Section 4 presents empirical results, and Section 5 concludes.

Related Work. Reactive synthesis [60,10] for LTL is 2EXPTIME-complete [60]. The use of tractable fragments, such as GR(1), reduces synthesis complexity to polynomial time [10]. However, significant challenges remain, particularly in constructing deterministic automata for large Safety-LTL formulas [77]. Compositional approaches improve scalability by decomposing synthesis tasks [66,60]. Dureja and Rozier [26] reduce model-checking tasks via dependency analysis, while Finkbeiner *et al.* [30] extend this idea to synthesis by focusing on *controllable* variables. However, these *simultaneous decomposition* methods, which address the synthesis problem in parallel, struggle when requirements share many controlled variables, limiting their applicability [41,30,54]. In RE, high-level state machines or *modes*, are commonly used to structure system specifications [35,70,71,63,4]. Languages like EARS [53], SCR [38,37], TLSF [42], SPIDER [43], NASA's FRET [33], and Spectra [51] leverage modes for organizing computation and requirements. This aligns with IEEE standard 29148, which notes that *"some systems behave quite differently depending on the mode of operation. For example, a control system may have different features depending on its mode: training, normal, or emergency."* [1]. State-based techniques like Statecharts [36], Broy's hierarchical service modeling [15], and the SCR method further formalize mode-based specifications. Feature modeling [25,67] and safety analysis methods like FTA and FMEA [46,2] also utilize modes. However, translating these mode-based specifications into LTL can increase complexity, hindering *simultaneous methods*. For example, NASA's FRET, which heavily uses state variables (modes), poses challenges for decomposition tools, leading to issues like *false positives* [54] and *goal-conflicts* [21,16,12,22,52,11]. This motivates the need for more efficient synthesis approaches in *real-world* applications. Recent RE research explores using Large Language Models (LLMs) for requirements specification [73,5,57,76,75,72]. LLMs, like GPT series [14,56], LaMDA [68], LLaMa [69], PaLM [18], and BERT [23], can assist in articulating mode-based requirements, potentially simplifying translation to LTL [47,3]. Related to us, Balkan *et al.* [6] use modes in a GR(1) subfragment for control design of *continuous* systems, focusing on quantitative performance. This contrasts with our focus on *discrete* systems and logical correctness in reactive synthesis. More directly, Brizzio *et al.* [13] proposed a mode-based decomposition for a fragment of safety, but require manual specification of initial conditions, which is tedious, error-prone, and deviates from standard RE practices. In contrast, our novel mode-based synthesis *automatically* generates initial conditions, ensuring consistency and eliminating manual intervention. Unlike simultaneous decomposition, we employ a sequential approach. By addressing one sub-problem at a time and leveraging natural mode transitions, our *sequential* method simplifies synthesis, reduces potential conflicts, and inherently ensures consistency. To the best of our knowledge, this is the *first fully automated sequential* mode-based synthesis method.

2 Preliminaries

LTL is a logical formalism widely used to specify reactive systems [59,49]. Given a set of propositional variables AP, LTL formulas are defined using standard logical connectives and the temporal operators $\bigcirc$ (next) and $\mathcal{U}$ (until) as follows:

$$\varphi ::= \texttt{true} \mid a \mid \varphi \vee \varphi \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\varphi$$

where $a \in \mathsf{AP}$. Common operators, such as $\texttt{false}$, $\wedge$ (and), $\square$ (always), and $\rightarrow$ (implies), can be derived: $\phi \wedge \psi \equiv \neg(\neg\phi \vee \neg\psi)$, $\square\phi \equiv \neg(\texttt{true}\,\mathcal{U}\,\neg\phi)$, and $\phi \rightarrow \psi \equiv \neg\phi \vee \psi$. Given an LTL formula φ, $\mathit{Vars}(\varphi) \subseteq \mathsf{AP}$ denotes the set of atomic propositions used in φ. The semantics of LTL associate traces $\sigma \in \Sigma^\omega$ with formulae as follows (we omit the Boolean operators which are standard):

$$\begin{array}{lll} \sigma \vDash a & \text{iff} & a \in \sigma(0) \\ \sigma \vDash \bigcirc\varphi & \text{iff} & \sigma^1 \vDash \varphi \\ \sigma \vDash \varphi_1\,\mathcal{U}\,\varphi_2 & \text{iff} & \text{for some } i \geq 0\ \ \sigma^i \vDash \varphi_2, \text{ and for all } 0 \leq j < i, \sigma^j \vDash \varphi_1 \end{array}$$

where $\sigma(i)$ is the i-th letter of σ, and $\sigma^i \in \Sigma^\omega$ is its suffix starting at the i-th position. Given $L \subseteq \Sigma^\omega$ and a formula φ, we use $L \vDash \varphi$ if for all $\sigma \in L$, $\sigma \vDash \varphi$.

A Syntactic Fragment of Safety-LTL. We focus on a fragment of LTL (Safety-LTL) [77] commonly used in requirement engineering. LTL$_G$ consists of formulas of the form $\psi \wedge \square\varphi$, where ψ is propositional and $\varphi \in \text{LTL}_X$ (which uses only the $\bigcirc$ operator). LTL$_G$ is widely used in industrial safety specifications [17,29,34]. Specifically, we work with GX_0 [13], a sub-fragment of Safety-LTL defined as $\alpha \rightarrow (\beta \wedge \square\psi)$, where α, β, and ψ are conjunctions in LTL$_X$. This fragment extends LTL$_G$, still expresses safety properties, and is supported by tools like Strix [55]. A *reactive specification* $\varphi = (A, G)$ consists of $A = (\theta_e, \varphi_e)$ and $G = (\theta_s, \varphi_s)$, where $\theta_{\{e,s\}}$ represent initial conditions for the environment and system, respectively, and $\varphi_{\{e,s\}}$ are the assumptions and guarantees. Similar to previous works that restrict either the environment's or the system's specifications to simpler LTL fragments for efficiency [30,54,74] we simplify our approach. We use propositional formulas over $\mathit{Vars}(\varphi)$ for A to ensure a consistent environment during decomposition, avoiding *false-negatives* [54], and employ $G \in \text{LTL}_X$ for guarantees. Thus, the intended meaning of φ is the GX_0 formula: $(\theta_e \rightarrow (\theta_s \wedge \square(\varphi_e \rightarrow \varphi_s)))$.

Reactive Synthesis. Reactive LTL synthesis [60] is the problem of automatically constructing a system that reacts to the environment guaranteeing an LTL specification φ. The propositions $\mathit{Vars}(\varphi)$ are partitioned into $\mathcal{X} \cup \mathcal{Y}$, where $\mathcal{X}$ are *environment-controlled* variables and $\mathcal{Y}$ are *system-controlled* variables. A system strategy for φ is a function $\rho : (2^{\mathcal{X}})^+ \rightarrow 2^{\mathcal{Y}}$ mapping finite sequences of $\mathcal{X}$ valuations to $\mathcal{Y}$ valuations. Given an infinite sequence $X = X_1, X_2, \ldots \in (2^{\mathcal{X}})^\omega$, the play induced by strategy ρ is the infinite sequence $\sigma_{\rho,X} = (X_1 \cup \rho(X_1))(X_2 \cup \rho(X_1, X_2)) \ldots$ We use $\mathcal{L}(\rho) = \{\sigma_{\rho,X} \mid X \in (2^{\mathcal{X}})^\omega\}$ for the set of plays played according to ρ. A play σ is winning if $\sigma \vDash \varphi$. A strategy is winning if $\mathcal{L}(\rho) \vDash \varphi$. Realizability is the problem of deciding if a specification has a winning strategy, and synthesis is the problem of computing one.

3 Mode-Based Synthesis for GX_0

We now describe our mode-based reactive synthesis method for GX_0 specifications, beginning with some preliminary definitions. A mode m for a GX_0 formula φ is a predicate over $\mathit{Vars}(\varphi)$ describing a set of system states. A set of modes $M = \{m_0, \dots, m_k\}$ is *valid* for φ if *(1)* modes are mutually exclusive ($m_i, m_j \in M$ with $i \neq j$, $m_i \wedge m_j$ is unsatisfiable) and *(2)* they cover all possible states ($\bigvee_{i=0}^{k} m_i$ is valid). A *mode-transition* from m to a *different* mode n in a trace σ occurs at index i if $\sigma(i) \vDash m$ and $\sigma(i+1) \vDash n$. A mode transition from m to n occurs under a strategy ρ if it occurs in some trace $\sigma \in \mathcal{L}(\rho)$.

Definition 1 (Mode-Graph). *A* mode-graph *is a directed graph $\mathcal{G} = (M, \prec)$, where $M = \{m_0, m_1, \dots, m_k\}$ is a set of modes, and $\prec \subseteq M \times M$ is irreflexive.*

The intended meaning of $\prec$ is to restrict the search of strategies to those where mode-transitions are included in $\prec$. In a *complete mode graph*, $\prec = \{(m, n) \mid m, n \in M, m \neq n\}$ all possible mode transitions are included. In RE, it is common to specify modes and mode-transitions as part of the requirements, as discussed in Sections 1 and 2.

3.1 Basic Mode-Based Synthesis with Initial Conditions

We first define a function reduce that, given a mode $m \in M$, and a formula ψ, it returns a reduced version of it which is specific to mode m:

$$\mathsf{reduce}(\psi, m) = \begin{cases} \mathtt{true}, & \text{if } (m \wedge \neg\psi) \vDash \mathtt{false} \ (1) \\ \mathtt{false}, & \text{if } (m \wedge \psi) \vDash \mathtt{false} \ (2) \\ \mathsf{Simpl}(\mathsf{reduce}(\psi_1, m) \bullet \mathsf{reduce}(\psi_2, m)), & \text{if neither (1) nor (2),} \\ & \quad \text{and } \psi = \psi_1 \bullet \psi_2 \\ & \quad\quad \text{for } \bullet \in \{\wedge, \vee, \rightarrow\} \\ \mathsf{Simpl}(\neg\mathsf{reduce}(\psi', m)), & \text{if } \psi = \neg\psi' \\ \Box\mathsf{Simpl}(\mathsf{reduce}(\psi', m)), & \text{if } \psi = \Box\psi' \\ \bigcirc\mathsf{Simpl}(\mathsf{reduce}(\psi', m)), & \text{if } \psi = \bigcirc\psi' \\ \psi, & \text{otherwise} \end{cases}$$

where $\mathsf{Simpl}(\psi)$ is a function that applies standard Boolean simplifications, like $(x \wedge \mathtt{true}) \mapsto x$, $(x \wedge \mathtt{false}) \mapsto \mathtt{false}$, etc.

Example 1. Consider two modes m_0 and m_1 and the following GX_0 specification $\varphi = ((\mathtt{true}, \mathtt{true}), (\mathtt{true}, \varphi_s))$ [1],which, in this example, simplifies to $\varphi = \Box\varphi_s$, since all other components are $\mathtt{true}$. In this case, φ_s is given by:

$$\boldsymbol{G_1} : \Box(e_1 \rightarrow (m_0 \rightarrow \bigcirc s_1)), \quad \boldsymbol{G_2} : \Box(m_1 \rightarrow \bigcirc(\neg s_3 \vee m_0)), \quad \boldsymbol{G_3} : \Box(\neg m_1 \rightarrow s_3)$$

[1]For readability, throughout the paper, we assume that any unspecified components of φ are implicitly set to $\mathtt{true}$, unless explicitly stated otherwise.

Since we only have φ_s, applying reduce (φ, m_0) projects[2] φ on mode m_0, resulting in a new set of guarantees $\boldsymbol{G_1} : \Box(e_1 \rightarrow \bigcirc s_1)$, $\boldsymbol{G_2} : \Box\mathtt{true}$ and $\boldsymbol{G_3} : \Box s_3$. Particularly, in this example, the result of this projection represents the guarantees that remain relevant in mode m_0, forming a mode-specific specification. We denote this reduced version as $\varphi{\downarrow_{m_0}}$.

The key-stone of mode-based synthesis is that one can focus on simplified specifications for each of the given modes independently—which improves the scalability of off-the-shelf reactive synthesis tools. However, during an execution, a system can transition between different modes. In these transitions, the system may leave the satisfaction of sub-formulas pending for the arriving modes. We call these sub-formulas *pending obligations*, as they represent temporal constraints that must be ensured in future modes. The following definition captures those obligations that a mode must ensure are carried forward during transitions.

Definition 2 (Pending Obligations). *Given a* GX_0 *specification* φ *and a mode* m*, the set of (potential) pending obligations for* m *in* φ *is* $\mathcal{O}^p_\varphi(m) = \{\psi \mid \bigcirc\psi \in \textsf{subformulas}(\varphi_{\downarrow_m})\}$.

Cumulative Obligations. Cumulative obligations, denoted $\mathcal{O}^c_\varphi(m)$, represent the obligations a mode m in a specification φ inherits from its predecessors during transitions. They ensure that all pending requirements are satisfied throughout the system. A straightforward, though imprecise, method to compute them is by aggregating obligations from all immediate predecessors. Particularly, for a given specification φ and a mode m_j: $\mathcal{O}^c_\varphi(m_j) = \bigcup_{m_i \prec m_j} \mathcal{O}^p_\varphi(m_i)$. The concept of cumulative obligations is illustrated in the following example.

Example 2. Consider a mode-graph with $M = \{m_0, m_1, m_2\}$ and $m_0 \prec m_1$, $m_1 \prec m_2$ and $m_2 \prec m_0$. Let e_1 and e_2 be environment-controlled variables, with the system controlling $\{s_1, s_2, s_3, s_4, m_0, m_1, m_2\}$. The specification φ has the following guarantees φ_s:

$$\boldsymbol{G_1} : \Box(e_1 \rightarrow (m_0 \rightarrow (\bigcirc m_1 \wedge \bigcirc\bigcirc s_2))) \qquad \boldsymbol{G_2} : \Box(e_2 \rightarrow (\neg m_2 \rightarrow \bigcirc\bigcirc(\neg s_3 \vee s_1)))$$
$$\boldsymbol{G_3} : \Box(\neg e_1 \rightarrow (m_1 \rightarrow \bigcirc(m_2 \wedge s_4))) \qquad \boldsymbol{G_4} : \Box(m_2 \rightarrow \bigcirc(m_0 \wedge s_1))$$

The reduced specifications for modes m_0, m_1, and m_2 are $\varphi_{\downarrow_{m_0}} = \boldsymbol{G_1} \wedge \boldsymbol{G_2}$, $\varphi_{\downarrow_{m_1}} = \boldsymbol{G_2} \wedge \boldsymbol{G_3}$, and $\varphi_{\downarrow_{m_2}} = \boldsymbol{G_4}$. When the system transitions from m_0 to m_1 while e_1 holds, mode m_1 must fulfill the pending obligation $\bigcirc s_2$. The exact obligations, however, depend on the order of events. If e_2 occurs before e_1, the system must satisfy both $\neg s_3 \vee s_1$ and $\bigcirc s_2$. On the other hand, if e_1 and e_2 occur simultaneously, the system must ensure $\bigcirc(\neg s_3 \vee s_1)$ along with $\bigcirc s_2$. If the system is in mode m_1 and e_1 does not hold, the system transitions to m_2 leaving the pending obligation $m_2 \wedge s_4$. The pending obligations for m_0, m_1 and m_2 are:

$$\mathcal{O}^p_\varphi(m_0) = \{m_1, \bigcirc s_2, s_2, \bigcirc(\neg s_3 \vee s_1), \neg s_3 \vee s_1\}$$
$$\mathcal{O}^p_\varphi(m_1) = \{\bigcirc(\neg s_3 \vee s_1), \neg s_3 \vee s_1, m_2 \wedge s_4\} \qquad \mathcal{O}^p_\varphi(m_2) = \{m_0 \wedge s_1\}$$

[2]Throughout the paper, we use *"reduced spec."* and *"projection"* interchangeably.

Algorithm 1: *Fixpoint Algorithm for Cumulative Obligations.*

```
1  Input: $\varphi = (A, G)$, $M = \{m_0, \ldots, m_k\}$ $\mathcal{G} = (M, \prec)$
2  for $m_j \in M$ do
3      $\mathcal{O}^c_\varphi[m_j] \leftarrow \bigcup_{m_i \prec m_j} \mathcal{O}^p_\varphi(m_i)$
4  changed $\leftarrow$ true
5  while changed do
6      changed $\leftarrow$ false
7      for $(m_i, m_j) \in \prec$ do
8          for $u \in \bigcup_{\bigcirc^k p \in \mathcal{O}^c_\varphi[m_i]} \{\bigcirc^{k-1} p, \bigcirc^{k-2} p, \ldots, p\}$ do
9              if $u \notin \mathcal{O}^c_\varphi[m_j]$ then
10                 $\mathcal{O}^c_\varphi[m_j] \leftarrow \mathcal{O}^c_\varphi[m_j] \cup \{u\}$ ; changed $\leftarrow$ true
11 return $\mathcal{O}^c_\varphi$
```

From this, the *cumulative obligations* would be: $\mathcal{O}^c_\varphi(m_0) = \mathcal{O}^p_\varphi(m_2)$; $\mathcal{O}^c_\varphi(m_1) = \mathcal{O}^p_\varphi(m_0)$; $\mathcal{O}^c_\varphi(m_2) = \mathcal{O}^p_\varphi(m_1)$.

However, as mentioned before, this simple aggregation is *imprecise* and can lead to incorrect results. It fails to capture interactions between obligations arising from mode transitions in execution paths as demonstrated in the following example.

Example 3. Consider a specification with the following guarantees:

$$\boldsymbol{G_1} : \square(m_0 \to \bigcirc m_1) \qquad \boldsymbol{G_2} : \square(m_1 \to \bigcirc m_0)$$
$$\boldsymbol{G_3} : \square(m_0 \to \bigcirc\bigcirc p) \qquad \boldsymbol{G_4} : \square(m_0 \to p)$$

Here, $\mathcal{O}^p_\varphi(m_0)$ includes $\{m_1, \bigcirc p, p\}$, while $\mathcal{O}^p_\varphi(m_1)$ only contains $\{m_0\}$ because $\varphi_{\downarrow m_1}$ retains only $\boldsymbol{G_2}$. If we propagate obligations naively without accounting for mode interactions, $\mathcal{O}^c_\varphi(m_0)$ might incorrectly focus on satisfying only m_0, neglecting the obligation to satisfy p. This error occurs because $\boldsymbol{G_2}$ forces m_1 to transition back to m_0 after just one step, leaving p—which originates from the obligation $\bigcirc p$ that m_0 transferred to m_1—unsatisfied. Such propagation would compromise the correctness of the specification leading to false negatives.

Particularly, cumulative obligations encode the temporal dependencies between modes, ensuring that obligations inherited from predecessors are correctly maintained throughout transitions. Unlike pending obligations, which define future requirements for a specific mode, cumulative obligations ensure that a mode is prepared to handle any inherited constraints. Alg. 1 systematically computes all *cumulative obligations* that a mode may need to satisfy, ensuring the global specification is correctly enforced across transitions. This algorithm iterates until a stable set of obligations is computed. Given a valid set of modes $M = \{m_0, m_1, \ldots, m_k\}$, the *cumulative obligations* $\mathcal{O}^c_\varphi(m)$ for each $m \in M$ are the conditions that m may be forced to satisfy based on the pending obligations inherited from all its predecessors. Each $\mathcal{O}^c_\varphi(m)$ is computed using Alg. 1. Revisiting Ex. 3, Alg.1 iteratively determines that when transitioning from m_1 to m_0, m_0 must satisfy both p and itself, resulting in $\mathcal{O}^c_\varphi(m_0) = \{p, m_0\}$.

We refer to the set $\mathcal{O} = \bigcup_{m \in M} \mathcal{O}^c_\varphi(m)$ as the *universe of obligations* across different modes. To systematically explore the universe of obligations, we introduce *obligation variables* that encode each element within this universe, encoding whether the corresponding obligation is considered. We introduce a fresh variable $v(\varphi)$ for each formula $\varphi \in \mathcal{O}$. We use $v(\varphi)$ for the variable corresponding to φ. While Alg. 1 performs the correct propagation of obligations, the set of obligations computed is a superset of the obligations that a mode may be requested to fulfill. Asking an instance of a mode to satisfy a larger subset of the cumulative obligations makes the instance more difficult to be realizable, while it helps predecessor instances to be realizable. The concept of *initial condition* captures this notion of combination of obligations.

Definition 3 (Initial Conditions). *Given a mode $m \in M$ and a specification $\varphi = ((\theta_e, \varphi_e), (\theta_s, \varphi_s))$, the set of* initial conditions $\mathcal{I}_\varphi(m)$ *is the set of all possible conjunctions of subsets of cumulative obligations:*

$$\mathcal{I}_\varphi(m) = \left\{ \bigwedge_{\phi \in \mathcal{S}} \phi \wedge m \;\middle|\; \mathcal{S} \subseteq \mathcal{O}^c_\varphi(m) \right\}.$$

Additionally, for the ***initial mode*** m_0*, the original system constraint θ_s from φ is always included in its initial conditions*

$$\mathcal{I}_\varphi(m_0) = \left\{ \bigwedge_{\phi \in \mathcal{S}} \phi \wedge (\theta_s \wedge m_0) \;\middle|\; \mathcal{S} \subseteq \mathcal{O}^c_\varphi(m_0) \right\}.$$

Example 4. Consider $\mathcal{O}^c_\varphi(m_0) = \{\bigcirc q, \bigcirc r\}$ and $\mathcal{O}^c_\varphi(m_1) = \{s\}$. The initial conditions are: $\mathcal{I}_\varphi(m_0) = \{\bigcirc q \wedge \bigcirc r, \bigcirc q, \bigcirc r, m_0\}$ and $\mathcal{I}_\varphi(m_1) = \{s, m_1\}$. [3]

Mode-Based Synthesis For clarity, in the following definitions we abuse notation by modifying the order of elements in the specification without altering its intrinsic meaning (see Sec. 2). We denote a specification as $(\theta_e, \theta_s, \varphi^R_m)$ where φ^R_m represents the reduction of both *assumptions* and *guarantees* onto mode m. This reduced specification introduces fresh variables to ensure proper transitions between modes, a process handled by Alg. 2. Given a set of valid modes $M = \{m_0, \dots, m_k\}$, a mode-graph $\mathcal{G} = (M, \prec)$, and a GX_0 specification $\varphi = (A, G)$, the *mode-based synthesis* method constructs a set of mode-specific projections, where each projection for a mode m_i is defined as: $\Pi_{m_i} = (\theta_{e_i}, \theta_{s_i}, \varphi^R_{m_i})$. We denote the full set of projections as: $\Pi = \{\Pi_{m_0}, \Pi_{m_1}, \dots, \Pi_{m_k}\}$. If each projection is realizable the resulting strategies can be composed into a strategy satisfying

[3]Notice that, each $x \in \mathcal{I}_\varphi(m)$ is conjoined with m. For readability, it is omitted from the text.

```
env boolean reset, start;
sys Int(1..20) counter; sys boolean trigger;
// Start and reset are not initially pressed -- theta_e
asm (!reset and !start);
// Only reset or start can be active at a time -- varphi_e
asm G !(reset and start);
// Counter is initially at the lowest value -- theta_s
gar counter=1;
// Everything beyond this point corresponds to varphi_s
// Restart signal always reset the count
gar G (reset -> next(counter=1) );
// Always stay at the same number or increase it
gar G ((counter=1 and start)  -> next(counter=2 or reset));
gar G ((counter=2 and !reset) -> next(counter=3 or reset));
...
gar G ((counter=19 and !reset) -> next(counter=20 or reset));
// Only trigger a signal if the bound is reached.
gar G (counter=20 <-> trigger);
// Reach the limit and start again
gar G (counter=20 -> next(counter=1));
```

Fig. 1: Counter machine example written in `Spectra`.

the original specification. When referring to a specific component of a projection, we use $\Pi_{m_i}.\theta_{e_i}$, $\Pi_{m_i}.\theta_{s_i}$, and $\Pi_{m_i}.\varphi^R_{m_i}$ to extract the corresponding initial conditions or the reduced specification, respectively. Importantly, only the projection for the initial mode m_0 includes θ_e, while all other projections replace it with `true`. This follows from the definition of a GX_0 specification (see Sec. 2), where environment assumptions are expressed as propositional formulas. Since the environment does not maintain state across modes, there is no requirement for θ_e to persist beyond the initial mode. Instead, θ_e in m_0 encodes the initial values of the environment, ensuring consistency with the original specification φ. After the initial mode, θ_e is universally quantified, effectively becoming `true`. It is worth bearing in mind that if the mode-transition relation $\prec$ is not explicitly provided by the engineer, the process assumes a complete graph over M, meaning all modes are initially considered reachable from one another. The synthesis procedure will then refine the mode transitions based on realizability constraints. This formulation ensures that the synthesized controller correctly handles mode transitions while maintaining consistency with the original specification. In particular, this process must satisfy the following objectives:

(1) Ensure that each projection $(\theta_{e_i}, \theta_{s_i}, \varphi^R_{m_i}) \in \Pi$ is realizable. Even though the local strategy generated is infinite, it could decide to *jump* into a successor node and move into a winning sink state.
(2) For each $m_i \prec m_j$ the successor mode m_j must satisfy the obligations inherited from its predecessor m_i, denoted as $\mathcal{O}^p_\varphi(m_i) \cap \mathcal{O}^c_\varphi(m_j)$. The connection between projections is managed through the initial condition $\theta_{s_j} \in \mathcal{I}_\varphi(m_j)$ of the successor m_j. Formally, for all $m_i \prec m_j$, $\theta_{s_j} \implies \bigwedge(\mathcal{O}^p_\varphi(m_i) \cap \mathcal{O}^c_\varphi(m_j))$. This ensures that a transition from m_i to m_j is valid only if m_j satisfies the pending obligations of mode m_i, ensuring the correct connection and composition of projections for sequential decomposition.

Alg. 2 adapted from [13], outlines the mode-based synthesis approach. The algorithm generates a projection Π_{m_i} for each mode m_i, ensuring consistency

Algorithm 2: *Sequential Decomposition.*

```
1  Input: $\varphi = ((\theta_e, \varphi_e), (\theta_s, \varphi_s))$, $\mathcal{I}$, $M' \subseteq M$, $\mathcal{G} = (M, \prec)$, $\mathcal{O}$
2  $\mathcal{O}_{\text{sorted}} = \mathsf{sortBySize}(\mathcal{O}, \downarrow)$ ; $\theta'_s \leftarrow \mathtt{true}$
3  for $\psi \in \theta_s$ do
4      $\psi' \leftarrow \psi[f \backslash v(f)]$ for all $f \in \mathcal{O}_{\text{sorted}}$
5      $\theta'_s \leftarrow \theta'_s \wedge \psi'$
6  for $m_i \in M'$ do
7      $\varphi'_s \leftarrow \varphi_s{\downarrow}_{m_i}$ ; $\varphi^R_{m_i} \leftarrow \mathtt{true}$
8      for $\psi \in \varphi'_s$ do
9          $\psi' \leftarrow \psi[f \backslash v(f)]$ for all $f \in \mathcal{O}_{\text{sorted}}$
10         $\varphi^R_{m_i} \leftarrow \varphi^R_{m_i} \wedge (\neg done \rightarrow \psi')$
11     for $\bigcirc^k p \in \mathcal{O}_{sorted}$ do
12         if $k = 1$ then $\varphi^R_{m_i} \leftarrow \varphi^R_{m_i} \wedge ((\neg done \wedge v(\bigcirc p)) \rightarrow \bigcirc(\neg done \rightarrow p))$ ;
13         else $\varphi^R_{m_i} \leftarrow \varphi^R_{m_i} \wedge ((\neg done \wedge v(\bigcirc^k p)) \rightarrow \bigcirc(\neg done \rightarrow v(\bigcirc^{k-1} p)))$ ;
14     for $(m_i, m_j) \in \prec$, and $\bigcirc^k p \in \mathcal{O}_{sorted}$ and $c_j \in \mathcal{I}$ do
15         if $(c_j \wedge \neg v(\bigcirc^{k-1} p))$ is sat then $\varphi^R_{m_i} \leftarrow \varphi^R_{m_i} \wedge (jump_j \rightarrow \neg v(\bigcirc^{k-1} p))$ ;
16     $\varphi^R_{m_i} \leftarrow \varphi^R_{m_i} \wedge ((\bigvee_{(m_i, m_j) \in \prec} jump_j) \rightarrow \bigcirc done)$
17     $\varphi^R_{m_i} \leftarrow \varphi^R_{m_i} \wedge ((\neg \bigvee_{(m_i, m_j) \in \prec} jump_j) \rightarrow (\neg done \rightarrow \bigcirc \neg done))$
18     $\varphi^R_{m_i} \leftarrow (\varphi_e{\downarrow}_{m_i} \rightarrow \varphi^R_{m_i})$
19     $\Pi[i] \leftarrow (\theta_e, \theta'_s, \varphi^R_{m_i})$
20 return $\Pi$
```

Here, $\varphi[\phi \backslash \psi]$ is the formula obtained by replacing occurrences of ϕ in φ, by ψ.
If M' is a singleton set, Alg. 2 returns a projection specialized for that mode only.

with **manually provided** initial conditions (as required in point (2)). To demonstrate the method, consider the specification φ in Fig. 1. Due to its complexity and the number of variables, traditional synthesis tools often struggle with this kind of specifications. Assume that each mode corresponds to a *unique* counter value. For instance, in mode m_{20} (i.e., *counter=20*), the approach projects the global specification onto m_{20} relying on manually specified initial conditions.

Alg. 2 ensures that the projection $\Pi_{m_{20}}$, along with others, is consistent with the global specification φ. A *reduced* version of this projection is shown in Fig. 2. Transitions between modes are modeled using fresh variables *jump* (indicating a transition to a successor mode) and *done* (indicating that the game for the current mode is completed because a jump transition was taken). A transition is allowed only if the initial condition of the successor mode satisfies the obligations at the time of the jump. This guarantees that if the set of reduced specifications is realizable then the strategies for the different modes can be properly connected. Although Alg. 2 significantly improves synthesis speed, it introduces two limitations: (1) Each mode is associated with a single initial condition, and (2) these initial conditions must be manually specified. To address (1), Alg. 2 can be modified to iterate over a sequence of sets of possible initial conditions candidates (*initial-universe*) $\mathcal{I}^u_\varphi = [\{\theta^1_{s_i}, \ldots, \theta^k_{s_i}\}, \{\theta^1_{s_j}, \ldots, \theta^k_{s_j}\}, \ldots]$, updating the jump variable to $jump^i_j$ for each different initial condition required.

```
env boolean reset, start;
sys Int(20) counter;
sys Int(1) jump;
sys boolean trigger, done, o_1;
//varphi_e and theta_s
asm G !(reset and start);
gar (counter=20 and ! done);
// varphi_s
gar G (!done -> trigger);
gar G (!done -> counter=20->o_1);
gar G (!done -> reset -> o_1);
gar G (!done and o_1->next(done));
gar G (jump=1 -> next(done));
gar G (done -> next(done));
```

Fig. 2: Reduced specification $\mathit{\Pi}_{m_{20}}$.

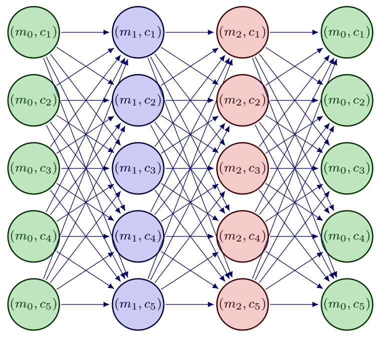
However, the second problem remains a significant challenge due to the considerable manual effort required to provide the initial conditions. To address this, we propose here a method for *automatic mode-based synthesis* that computes these initial conditions automatically. Alg 2 then generates projections using these automatically computed initial conditions $\mathcal{I}_\varphi$, ensuring realizability and consistency.

3.2 A Fixpoint Search Method for Initial Conditions

We now propose an automatic mode-based synthesis method that eliminates the need for manually specifying initial conditions. The key challenge is to discover a set of feasible initial conditions for each mode, ensuring that all projections remain realizable. For this, we consider multiple instances of a mode m_j, each corresponding to an initial condition c_k. An instance of a mode with an initial condition supports its predecessors allowing them to jump, and at the same time it uses its successors to attempt its own realizability. The core task is to verify the realizability of each projection $\mathit{\Pi}_{m_i} = (\theta_{e_i}, \theta_{s_i}, \varphi^R_{m_i})$. Since we have already established that θ_e appears only in the initial mode and is implicitly `true` in all others, this holds throughout the following definitions.

If an instance is found to be unrealizable with a given collection of successor instances it must be removed and its predecessors realizability need to be reestablished. We will provide a fixpoint search technique that iteratively refines the space of possible initial conditions until a stable solution is found. The following definition captures the set of possible instantiations.

Definition 4 (Mode Instance Graph). *Given a set of modes M, a specification φ, a set of initial conditions $\mathcal{I}_\varphi(m)$ for each mode $m \in M$, and a mode graph $\mathcal{G} = (M, \prec)$, a Mode Instance Graph (MIG) is a graph $(V, \mapsto)$, where:*
- $V \subseteq \{(m, c) \mid m \in M$ *and* $c \in \mathcal{I}_\varphi(m)\}$
- $\mapsto = \{((m_i, c_j), (m_k, c_l)) \mid (m_i, c_j) \in V, (m_k, c_l) \in V$ *and* $m_i \prec m_k)\}$



In a mode instance graph, each mode is instantiated with possibly many initial conditions, which are connected to every instance of its successor nodes. For example, consider a mode-graph with three modes m_0, m_1, and m_2 and $m_0 \prec m_1 \prec m_2 \prec m_0$. Assume each mode has five possible initial conditions, denoted as (m_i, c_k). *The universe of exploration in the diagram on the left illustrates the MIG for this simplified example.* Our approach systematically evaluates each configuration to ensure that all projections meet the realizability requirements. Informally, our method iterates through the MIG to find a *proof of realizability* for the specification φ, pruning the search space as unfeasible nodes are found. Formally:

Definition 5 (Realizability Proof). *Given a GX_0 specification defined as follows: $\varphi = ((\theta_e, \varphi_e), (\theta_s, \varphi_s))$, a Mode Instance Graph $MIG = (V, \mapsto)$. A proof of realizability is a subgraph $\mathcal{R} = (V', \mapsto')$, where $V' \subseteq V$ and $\mapsto' \subseteq \mapsto$ such that for every projection $\Pi_{m_i} = (\theta_{e_i}, c_k, \varphi^R_{m_i})$ generated from $(m_i, c_k) \in V'$, the following holds:*

$$(\Pi_{m_i}.\theta_{e_i} \rightarrow (\Pi_{m_i}.c_k \wedge \Box(\Pi_{m_i}.\varphi^R_{m_i})))$$

where

$$\Pi_{m_i}.\theta_{e_i} = \begin{cases} \theta_e & \text{if } m_i = m_0, \\ \mathtt{true} & \text{otherwise.} \end{cases}$$

A realizabilty proof guarantees the realizability of φ (Thm. 2). Alg. 3 systematically searches for subsets of the initial conditions, one for each node, aiming to find a realizability proof. The search begins with a candidate proof by creating the MIG that contains all modes and their initial conditions. The algorithm proceeds in rounds, where at each round all remaining instance modes are checked for realizability. At each round unrealizable instances are removed. The process iterates until a fixpoint is reached, that is when a full round is passed with all remaining instances are realizable. The resulting graph is a realizability proof $\mathcal{R}$ (or a graph that does not contain any instance of the initial mode). A modification of Alg. 3, allows us to bypass many realizability checks by inferring results

Algorithm 3: *Search for Init. Cond.*

```
1  Input: $\varphi = ((\theta_e, \varphi_e), (\theta_s, \varphi_s))$, $M = \{m_0, \ldots, m_n\}$, $\mathcal{G} = (M, \prec)$,
2  $\mathcal{O}^c_\varphi \leftarrow$ Alg 1$(\varphi, M, \mathcal{G})$ ; $\mathcal{O} \leftarrow \bigcup_{m \in M} \mathcal{O}^c_\varphi(m)$
3  for $m_i \in M$ do $\mathcal{I}^u_\varphi[i] \leftarrow \mathcal{I}_\varphi(m_i)$ ;
4  MIG $\leftarrow$ createMIG$(\mathcal{G}, \mathcal{I}^u_\varphi)$
5  do
6      initial $\leftarrow$ false ; finished $\leftarrow$ true
7      for $m_i \in M$ do
8          foreach candidate $c \in \mathcal{I}^u_\varphi[i]$ do
9              if $m_i = m_0$ then
                   $(\theta_{e_i}, \theta_{s_i}, \varphi^R_{m_i}) \leftarrow$ Alg 2$(((\theta_e, \varphi_e), (c, \varphi_s)), \mathcal{I}^u_\varphi, \{m_i\}, \mathcal{G}, \mathcal{O})$ ;
10             else $(\theta_{e_i}, \theta_{s_i}, \varphi^R_{m_i}) \leftarrow$ Alg 2$(((\mathtt{true}, \varphi_e), (c, \varphi_s)), \mathcal{I}^u_\varphi, \{m_i\}, \mathcal{G}, \mathcal{O})$ ;
11             if realizable$(\theta_{e_i}, \theta_{s_i}, \varphi^R_{m_i})$ then
12                 if $m_i = m_0$ then initial $\leftarrow$ true;
13             else
14                 finished $\leftarrow$ false
15                 MIG $\leftarrow$ MIG $\setminus (m_i, c)$
16                 $\mathcal{I}^u_\varphi[i] \leftarrow \mathcal{I}^u_\varphi[i] \setminus c$
17 while $\neg$finished $\wedge$ $\neg$initial;
18 return MIG
```

based on previous checks in the same iteration, using *memoization* [9]. For instance, if a mode with candidate initial condition $p \wedge q$ is realizable, the same mode will also be realizable for initial conditions p, q and `true`. Conversely, if p is unrealizable, all initial conditions that imply p are also unrealizable. For example, in our case study (Fig. 1), we defined two modes, m_0 that spans from 1 to 10, while m_1 spans from 11 to 20. Our algorithm efficiently computes the initial conditions as follows: for m_0, it deduces the initial condition $\theta_0 =$ `counter=1`, and for m_1, it derives the initial condition $\theta_1 =$ `counter=11`. These results align with those manually derived in [13].

3.3 Correctness

This section establishes the soundness of the mode-based synthesis algorithm. We begin by stating a previously established result.

Theorem 1 (Soundness of Alg. 2 [13]). *Let φ be a GX_0 specification, $\mathcal{G} = (M, \prec)$ a mode-graph, and $\mathcal{I}_\varphi(m)$ the set of initial conditions for each mode $m \in M$. If every mode projection Π_m is realizable, then the original specification φ is realizable.*

The following theorem formalizes the soundness of Alg. 3.

Theorem 2 (Soundness of Alg. 3). *Let $\varphi = ((\theta_e, \varphi_e), (\theta_s, \varphi_s))$ be a GX_0 specification. If Alg. 3 returns a realizability proof $\mathcal{R}$, then φ is realizable.*

Proof (sketch). We proceed by induction on the length of the environment sequence $X_1, \ldots, X_T$ in a trace σ. We define a function $\gamma(\sigma, t)$ that returns the mode active at time t in the trace σ. That is, at every step t, the system is in mode $m_t = \gamma(\sigma, t)$. The algorithm constructs a realizability proof $\mathcal{R}$, where each mode m_j is associated with a projection $\Pi_{m_j} = (\theta_{e_j}, \theta_{s_j}, \varphi^R_{m_j})$ that is realizable. The set $\mathcal{I}_\varphi(m_j)$ ensures that θ_{s_j} is a valid initial condition, satisfying all required obligations. The proof proceeds as follows:

- **Base case:** The system starts in mode $m_0 = \gamma(\sigma, 0)$ with the initial environment valuation X_1. The algorithm constructs the realizability proof $\mathcal{R}$ by iterating over all candidate initial conditions $\theta_{s_0} \in \mathcal{I}_\varphi(m_0)$, checking the realizability of $\Pi_{m_0} = (\theta_e, \theta_{s_0}, \varphi^R_{m_0})$. If θ_{s_0} is realizable, the algorithm keeps (m_0, θ_{s_0}) in $\mathcal{R}$. Since the algorithm removes all unrealizable candidates, the existence of $(m_0, \theta_{s_0}) \in \mathcal{R}$ guarantees that the system has a valid strategy satisfying Π_{m_0} at $t = 0$. Therefore, the base case holds.
- **Inductive step:** Assume that for all previous steps $t < T$, the system was realizable under its respective projections, i.e., for all $X_1, \ldots, X_t$, the corresponding mode projections $\Pi_{\gamma(\sigma,t)}$ were realizable. Now consider the next time step T, and let $m_j = \gamma(\sigma, T)$ be the mode active at T. The algorithm ensures that the realizability of Π_{m_j} holds for some initial condition candidate $\theta_{s_j} \in \mathcal{I}_\varphi(m_j)$. There are two possible cases:

1. *The system remains in mode m_j.* Since the system does not transition, the active mode remains $m_j = \gamma(\sigma, T)$. By the inductive hypothesis, the game has been realizable up to step $T-1$. By the algorithm's construction, $(m_j, c_k) \in \mathcal{R}$ only if Π_{m_j} was realizable for some $c_k \in \mathcal{I}_\varphi(m_j)$. This means that there exists a valid strategy for m_j that satisfies Π_{m_j} at time T. If the system were not realizable at T, then the environment would have a winning strategy leading to an unrealizable state. However, such a strategy would necessarily include the initial state, contradicting our hypothesis that the game was realizable up to $T-1$. Therefore, staying in the same mode ensures that the system remains realizable, ensuring that the specification is not violated at step T.
2. *The system transitions to a new mode m_k.* The algorithm only allows a transition from m_j to m_k, i.e., $\gamma(\sigma, T-1) \neq \gamma(\sigma, T)$, if there exists an initial condition $\theta_{s_k} \in \mathcal{I}_\varphi(m_k)$ that satisfies the pending obligations from m_j, i.e., $\mathcal{O}^p_\varphi(m_j) \cap \mathcal{O}^c_\varphi(m_k)$. If no such initial condition existed, the system would remain in m_j, contradicting the assumption that a transition to m_k occurred. As stated in Def. 3, $\mathcal{I}_\varphi(m_k)$ includes all possible obligations that m_k may inherit and since the algorithm removes all unrealizable candidates from $\mathcal{I}_\varphi(m_k)$, any transition to m_k necessarily maintains realizability.

At each step, the system either (1) remains in the same mode with a winning strategy or (2) transitions to a realizable mode, ensuring correctness throughout the execution. Hence, φ is realizable. $\square$

Thm. 2 establishes soundness but not completeness. Our approach forces the system to choose the next mode based solely on the current history, disregarding the next environment input. This can lead to false negatives. Consider the specification $\square(m_0 \to \bigcirc(e \vee m_1))$, where m_0, m_1 are modes and e is an environment input. This requires that if the system is in m_0, then in the next step, either e is `true` or the system transitions to m_1. Our approach must decide whether to transition to m_1 *before* observing e. However, a winning strategy might depend on e: stay in m_0 if e is `true`, and transition to m_1 otherwise. For instance, if e alternates, a winning strategy is to stay in m_0 while e is `true` and move to m_1 when e is `false`. Our method, however, must choose between always staying in m_0 (violating the spec when e is `false`) or always transitioning to m_1 (unnecessarily when e is `true`), incorrectly reporting unrealizability. This illustrates that premature decision-making can lead to false negatives. To mitigate incompleteness, we define a sufficient condition: *mode-determinism*, ensuring that the current variable valuation uniquely determines the next mode without violating the specification. For a GX_0 formula $\square\psi$ [4] with variables $\overline{z} = \mathit{Vars}(\psi) = \mathcal{X} \cup \mathcal{Y}$, let $\varphi(\overline{z}, \overline{z}')$ be the relation between pre- and post-state variables satisfying φ.

Definition 6 (Mode-deterministic). *A specification $\square\psi$ with $\overline{z} = \mathit{Vars}(\psi)$ is* mode-deterministic *if there are no modes m_1, m_2 and m_3 (with $m_2 \neq m_3$)*

[4] Nested $\bigcirc$ operators in ψ are handled by introducing fresh variables $v_{\bigcirc\alpha} \iff \bigcirc\alpha$.

s.t.:

$$\exists \overline{z}, \overline{z}_2, \overline{z}_3.\big(m_1(\overline{z}) \wedge (\psi(\overline{z}, \overline{z}_2) \wedge m_2(\overline{z}_2)) \wedge (\psi(\overline{z}, \overline{z}_3) \wedge m_3(\overline{z}_3))\big).$$

This condition ensures that no state in m_1 can transition simultaneously to m_2 and m_3. Mode-determinism can be verified using k^3 SAT queries, where k is the number of modes. For a mode-deterministic specification φ the *feasible mode jumps* $J(\varphi) \subseteq M \times M$ are defined as: $(m_i, m_j) \in J$ if: $\exists \overline{z}, \overline{z}'.\big(m_i(\overline{z}) \wedge \psi(\overline{z}, \overline{z}') \wedge m_j(\overline{z}')\big)$. This captures possible mode transitions within a φ trace.

Theorem 3. *Let φ be a mode-deterministic specification and let $(M, \prec)$ be a mode-graph with $J(\varphi) \subseteq \prec$. If the mode-based synthesis algorithm returns unrealizable, then φ is unrealizable.*

It is always possible to choose a proper $\prec$ either by computing $J(\varphi)$ or by using the complete mode-graph.

4 Empirical Evaluation

We evaluate our approach against the following research questions:

- **RQ1:** How effectively does our method compute initial conditions?
- **RQ2:** Does our mode-based technique improve controller synthesis time compared to traditional methods?
- **RQ3:** How well does our heuristic prune the search space?

Specifications. For our evaluation, we use benchmarks from [13] alongside new specifications from recent work [61,52], written in languages like `Spectra` and `FRET`. Additionally, we adapt each specification to *mode-based determinism* and we introduce *goal-conflicts* to make them unrealizable to test whether our method correctly identifies those scenarios. Table 1 summarizes the specifications, including assumptions (#A), guarantees (#G), and modes (#M).

Spec.	$\#A - \#G$	$\#M$
counter(n)	2-(n+5)	2,4,7
sis-1500	2-7	3
thermostat(n)	3-4	3
cruise-fse	3-15	4
altlayer(n)	1-9	3
lift(n)	1-187	3
fret-lift	2-14	4
double-counter(n)	2-(2n + 5)	2,4,7

Table 1: Specs and Modes.

Setting and Evaluation. We implemented our approach in Java, leveraging the widely-used `OwL` library [44] for parsing and manipulating LTL formulas. For verifying realizability, we used `Strix` [55]. The experiments were conducted on a cluster featuring Intel Xeon processors clocked at 2.6GHz and equipped with 16GB of RAM, running GNU/Linux. Our tool, case studies, and instructions for replicating the experiments are in our replication package [5].

[5]https://sites.google.com/view/mode-decomposition

Specifications		**Time (s)**			**#Strix**		**Detailed time (fp)**	
Name.	$\#M$	Strix	fp	fph	fp	fph	Proj.	Real.
counter-10	2	8	0.56	0.49	2	2	0.41	0.15
counter-14	7	`timeout`	1.23	1.31	7	7	0.82	0.41
counter-20	4	`timeout`	1.01	1.06	4	3	0.80	0.27
lift-15	3	`timeout`	4.03	4.00	5	4	3.44	0.59
sis-1500	3	`timeout`	46.40	43.00	3	2	42.84	3.92
double-counter-10	2	6	0.80	0.73	3	3	0.60	0.20
double-counter-14	7	94	3.23	2.96	13	10	2.23	1.00
double-counter-20	4	`timeout`	2.62	2.56	7	5	2.14	0.48
cruise-fse	4	`timeout`	65.37	63.89	11	9	45.86	19.51
altlayer	3	`timeout`	18.13	18.52	4	4	15.83	2.30
fret-lift	4	`timeout`	27.48	27.12	4	4	21.80	5.68
thermostat-80	3	60	29.64	29.66	3	2	27.40	2.24
thermostat-150	3	ERROR	60.53	55.41	3	2	52.68	7.85

Table 2: All experimental results (Realizable cases).

Effectiveness of Initial Condition Computation. We evaluated three approaches: monolithic synthesis (*Strix*), our fixpoint method (*fp*), and our memoization fixpoint method (*fph*). Addressing **RQ1** (effectiveness of initial condition computation), both *fp* and *fph* successfully generated initial conditions for all tested specifications, resulting in successful synthesis in all realizable cases and consistently outperforming *Strix* (see Table 2). Moreover, in mode-deterministic unrealizable cases, our method correctly concluded unrealizability (by generating empty solutions), whereas *Strix* timed out in most of the cases (see Table 3).

Effectiveness of Search Space Pruning. Addressing **RQ3**, memoization's impact varied by specification structure. It modestly reduced realizability checks for specifications with mutually exclusive candidates (e.g., SCR). However, for larger search spaces with *well-defined lattice structures*, it significantly pruned the search space and improved execution time. Further evaluation using randomly generated conjunctive formulas (`Spot` [27]) as initial candidates confirmed that solver calls could be reduced by up to 50%. These results highlight memoization's potential, particularly in non-mutually exclusive specifications, with similar trends in unrealizable cases (Table 3), reinforcing approach's robustness.

Impact on Synthesis Time. Addressing **RQ2** (synthesis time improvement), our comparative analysis revealed substantial differences. The monolithic method (*Strix*) consistently timed out (10 minutes) on larger instances, often failing due to formula size. In contrast, our methods, *fp* and *fph*, completed synthesis well within the time limit, reducing synthesis time by over 90%. Critically, as noted

Specifications		**Time (s)**			**#Strix**		**Detailed time (fph)**	
Name	$\#M$	Strix	fp	fph	fp	fph	Proj.	Real.
counter-10	2	8	0.61	0.59	3	2	0.39	0.20
counter-14	7	`timeout`	4.33	2.66	21	16	1.69	0.98
counter-20	4	`timeout`	2.24	4.60	11	10	3.28	1.33
lift-15	3	`timeout`	6.92	10.93	9	7	9.50	1.44
sis-1500	3	`timeout`	44.72	58.19	3	3	53.58	4.62
double-counter-10	2	13	1.28	1.83	7	7	1.14	0.61
double-counter-14	7	114	2.52	3.18	13	10	2.32	0.84
double-counter-20	4	`timeout`	6.83	9.60	23	19	7.75	1.84
cruise-fse	4	`timeout`	99.83	134.72	23	18	101.42	33.30
altlayer	3	`timeout`	65.40	65.45	11	10	57.77	7.68
fret-lift	4	`timeout`	79.80	81.80	10	10	64.18	17.61
thermostat-80	3	74	110.67	100.84	9	7	94.20	6.64
thermostat-150	3	ERROR	193.17	185.69	9	6	173.87	11.83

Table 3: All experimental results (Unrealizable cases). All cases executed using mode-determinism.

by Mavridou *et al.* [54], state-of-the-art *simultaneous* decomposition tools [41,30] failed on all our specifications due to the frequent use of state variables/modes and many shared controllable variables. Our *sequential* mode-based approaches, however, excelled in these environments, demonstrating their adaptability where simultaneous methods fall short. This dramatic reduction in synthesis time shows that our method complements existing approaches by enabling synthesis for a broader class of complex *real-world* specifications.

5 Conclusion and Future Work

This paper presents a novel, fully automated mode-based reactive synthesis method. Taking an LTL (GX_0) specification and a set of modes (with optional transitions) as input, our iterative algorithm efficiently searches for initial condition combinations that realize the overall specification or concludes unrealizability. This automatic search for suitable initial conditions is a key feature of our approach, simplifying the synthesis process for engineers.

A current limitation is that completeness for unrealizability requires mode-based determinism and a subsuming mode-graph, which we plan to address in future work. However, our method achieves significantly faster synthesis compared to monolithic methods, enabling more effective derivation of controllers for complex real-world specifications, as supported by a thorough empirical evaluation. Future work will also explore controller explainability, extend to richer LTL fragments such as Safety-LTL, and investigate mode-synthesis with theories [64,32,65,48].

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